Motional Influences on Magnetic Reproduction: An Analysis Based on the Reciprocity Theorem

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Abstract—**Motional influences in magnetic reproduction are analyzed with the aid of the reciprocity theorem of magnetic recording theory. Alternative expressions for the induced READ voltage are derived for the case of a carrier (disk or tape) of spatially recorded magnetization in rigid (translatory and/or rotary) motion. The expressions exhibit, through different physical interpretations, the influence of the motion of a magnetic tape or disk on the induced READ voltage.**

I. INTRODUCTION

IN this paper we investigate the influence of the motion of a magnetic tape or disk on the induced READ voltage in magnetic reproduction. The analysis is carried out with the aid of the reciprocity theorem of magnetic recording theory. In its basic form, as we use it, this theorem yields an expression for the electromotive force that is induced in the READ coil. It expresses this quantity as the time derivative of a volume integral representing the interaction between recorded magnetization and a certain auxiliary field that is characteristic for the reproduce head (with possible shields). The auxiliary field is the magnetic field that a time-independent current in the READ coil would generate in space, the recorded magnetization being absent. For the case of a rigid motion (translatory and/or rotary) of the tape or disk, alternative expressions for the READ voltage are derived, which seem to be new. They contain either the magnetization charge (of both volume and surface types) or the magnetization currents (of both volume and surface types) that are associated with the recorded pattern of magnetization, as well as the instantaneous value of the velocity of transportation. Also, they contain a term associated with the instantaneous value of the angular velocity of the rotational part of the rigid motion. This term, too, seems to be new, and its contribution to the head's response needs further investigation.

Throughout the analysis only the (vectorial) magnetic field quantities occur. In this respect, it was thought appropriate to include a brief, but complete, derivation of the reciprocity theorem of magnetic recording theory entirely based on the full vectorial form of the magnetic field equations (rather than on the use of the magnetic scalar potential as is usually done [1]-[6]). Among others, our method has the advantage that the presence of arbitrary current distributions and (perfectly) conducting shields can be accounted for. Reference [6] elucidates how to interpret the "auxiliary field" and the "impressed magnetization" when the partly reversible action of the recording media is taken into account. It is noted that if *finitely conducting* media are present in the configuration, the more general reciprocity theorem, applying to the complete electromagnetic field, has to be employed [7], [8]. The reason for this is the occurrence of an additional time dependence due to the diffusion character of eddy currents. Our alternative forms can be especially useful in considerations of the design of reproduce heads and for weighing the pros and cons of "longitudinal" versus "transverse" (or "perpendicular") magnetization to be employed in the recording process.

II. DESCRIPTION OF THE CONFIGURATION

The configuration consists of a magnetic reproduce head with reading coil, in the neighborhood of which electric and/or magnetic shields may be present. These parts of the configuration occupy a bounded domain in space. As to the different subdomains, a nomenclature is adopted that reflects their physical properties. This nomenclature is shown in Table I. It is assumed that the domains V_μ, V_1, V_2 are nonoverlapping (Fig. 1).

To locate a point in the configuration, we employ orthogonal Cartesian coordinates x, y, z with respect to a given orthogonal

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TABLE I
NOMENCLATURE ADOPTED FOR THE DIFFERENT SUBDOMAINS OF THE CONFIGURATION

Domain	Boundary Surface	Physical Property
V_μ	S_μ	Nonconducting material of finite permeability.
V_1		Nonconducting material of infinite permeability.
V_2	S_2	Perfectly conducting material.
V_0^*		Vacuum domain.
V_J		Nonmagnetic material capable of carrying external currents.
V_M	S_M	Nonconducting material capable of carrying recorded magnetization.

*V_0 is the domain outside S_μ, S_1, and S_2.

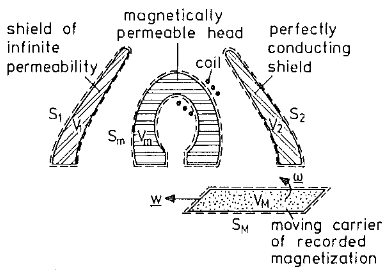

Fig. 1. Magnetic reproduce head with coil and shields and moving carrier of recorded magnetization.

Cartesian reference frame that is at rest with respect to the reproduce head and the shields. The reference frame is specified by its origin O and the three mutually perpendicular base vectors of unit length $\boldsymbol{i}_x$, $\boldsymbol{i}_y$, $\boldsymbol{i}_z$. In the given order, the base vectors form a right-handed system. The position vector $\boldsymbol{r}$ is then given by

$$\boldsymbol{r} = x\boldsymbol{i}_x + y\boldsymbol{i}_y + z\boldsymbol{i}_z. \tag{1}$$

The time coordinate is denoted by t.

III. THE MAGNETIC FIELD IN THE CONFIGURATION

We employ the quasi-static approximation of the field equations. Then the magnetic state in the configuration involves the following quantities:

$\boldsymbol{H}$ magnetic field strength (A/m),
$\boldsymbol{B}$ magnetic flux density (T),
$\boldsymbol{M}$ magnetization (A/m),
$\boldsymbol{J}$ volume density of electric current (A/m^2),
while the induced electric field is
$\boldsymbol{E}$ electric field strength (V/m).

The pertaining SI units have been indicated in parentheses.

A. Field Equations

At any interior point of a domain where the magnetic properties vary continuously with position, the field quantities satisfy the field equations

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}, \tag{2}$$

$$\nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B}, \tag{3}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{4}$$

$$\nabla \cdot \boldsymbol{J} = 0, \tag{5}$$

while

$$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M}), \tag{6}$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m is the permeability *in vacuo*. The operator ∇ is given by

$$\nabla = \boldsymbol{i}_x\partial_x + \boldsymbol{i}_y\partial_y + \boldsymbol{i}_z\partial_z \tag{7}$$

and partial differentiation is denoted by ∂.

We now assume that the magnetic state present in the configuration has been reached by starting from a situation where no field is present, and switching on the sources at an instant in the finite past. For those states, we introduce the vector potential $\boldsymbol{A}$ as (taking $t = 0$ as instant of switching)

$$\boldsymbol{A} = -\int_0^t \boldsymbol{E}\, d\tau. \tag{8}$$

Substitution of (8) in (3) leads to

$$\nabla \times \boldsymbol{A} = \boldsymbol{B}. \tag{9}$$

Another quantity of interest is the electromotive force e induced along an oriented closed contour C. This is given by

$$e = \oint_C \boldsymbol{\tau} \cdot \boldsymbol{E}\, ds, \tag{10}$$

where $\boldsymbol{\tau}$ is the unit vector along the tangent to C, drawn in the chosen direction of circulation.

B. The Magnetic Constitutive Relations

To specify the magnetic constitutive relations, we separate the magnetization into a field-dependent, induced part $\boldsymbol{M}_i$ and a field-independent, permanent part $\boldsymbol{M}_p$:

$$\boldsymbol{M} = \boldsymbol{M}_i + \boldsymbol{M}_p. \tag{11}$$

Now, for the reciprocity theorem to hold, the media in the configuration have to be linear, time-invariant, and instantaneously and locally reacting in their magnetic behavior. For media of this kind, we have

$$\boldsymbol{M}_i(\boldsymbol{r}, t) = \kappa(\boldsymbol{r}) \cdot \boldsymbol{H}(\boldsymbol{r}, t), \tag{12}$$

where κ denotes the magnetic susceptibility, a tensor of rank two. Equation (12) applies to anisotropic media of the class under consideration. For isotropic media of the same kind, (12) reduces to

$$\boldsymbol{M}_i(\boldsymbol{r}, t) = \kappa(\boldsymbol{r})\, \boldsymbol{H}(\boldsymbol{r}, t), \tag{13}$$

where κ is the scalar magnetic susceptibility.

C. Boundary Conditions

At surfaces across which the properties of the media show abrupt changes, the field equations (2)-(5) have to be supplemented by boundary conditions. These are

$$\{\boldsymbol{n} \times \boldsymbol{H}, \boldsymbol{n} \times \boldsymbol{E} \text{ (and hence } \boldsymbol{n} \times \boldsymbol{A}), \boldsymbol{n} \cdot \boldsymbol{B}\}$$

$$\text{continuous across } S_\mu, \tag{14}$$

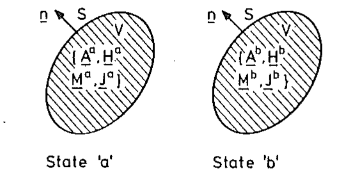

Fig. 2. Illustration of the reciprocity theorem for quasi-static magnetic fields.

$$\boldsymbol{n} \times \boldsymbol{H} \to \boldsymbol{0} \text{ upon approaching } S_1, \tag{15}$$

$$\boldsymbol{n} \times \boldsymbol{E} \to \boldsymbol{0} \text{ (and hence } \boldsymbol{n} \times \boldsymbol{A} \to \boldsymbol{0}) \text{ upon approaching } S_2, \tag{16}$$

where $\boldsymbol{n}$ is the unit vector along the normal to the relevant surface.

D. Conditions at Infinity

For any configuration of the type under consideration, that occupies a bounded domain in space, we have

$$\boldsymbol{E} \text{ (and hence } \boldsymbol{A}) = \text{order } (|\boldsymbol{r}|^{-2}) \text{ as } |\boldsymbol{r}| \to \infty, \tag{17}$$

$$\{\boldsymbol{H}, \boldsymbol{B}\} = \text{order } (|\boldsymbol{r}|^{-3}) \text{ as } |\boldsymbol{r}| \to \infty. \tag{18}$$

IV. The Magnetic Field Reciprocity Theorem

A reciprocity theorem interrelates two admissible "states" that could be present in one and the same domain V in space (Fig. 2). Let V be the bounded domain interior to the bounded closed surface S and let $\boldsymbol{n}$ be the unit vector along the normal to S, pointing away from V. The field quantities in the two states "a" and "b" are denoted by the superscripts a and b, respectively. We now consider the expressions for

$$\nabla \cdot (\boldsymbol{A}^a \times \boldsymbol{H}^b) \quad \text{and} \quad \nabla \cdot (\boldsymbol{A}^b \times \boldsymbol{H}^a).$$

Using (2), (9), (6), and (11), we obtain

$$\nabla \cdot (\boldsymbol{A}^a \times \boldsymbol{H}^b) = \mu_0 \boldsymbol{H}^b \cdot \boldsymbol{H}^a + \mu_0 \boldsymbol{H}^b \cdot (\boldsymbol{M}_i^a + \boldsymbol{M}_p^a) - \boldsymbol{A}^a \cdot \boldsymbol{J}^b \tag{19}$$

and

$$\nabla \cdot (\boldsymbol{A}^b \times \boldsymbol{H}^a) = \mu_0 \boldsymbol{H}^a \cdot \boldsymbol{H}^b + \mu_0 \boldsymbol{H}^a \cdot (\boldsymbol{M}_i^b + \boldsymbol{M}_p^b) - \boldsymbol{A}^b \cdot \boldsymbol{J}^a. \tag{20}$$

If now we impose on κ the condition that it be a symmetrical tensor, we have

$$\boldsymbol{H}^b \cdot \boldsymbol{M}_i^a = \boldsymbol{H}^a \cdot \boldsymbol{M}_i^b \tag{21}$$

and hence

$$\nabla \cdot (\boldsymbol{A}^a \times \boldsymbol{H}^b - \boldsymbol{A}^b \times \boldsymbol{H}^a) = \mu_0 \boldsymbol{H}^b \cdot \boldsymbol{M}_p^a - \boldsymbol{A}^a \cdot \boldsymbol{J}^b - \mu_0 \boldsymbol{H}^a \cdot \boldsymbol{M}_p^b + \boldsymbol{A}^b \cdot \boldsymbol{J}^a. \tag{22}$$

Integration of (22) over the domain V and application of Gauss' divergence theorem to the resulting left side lead to

$$\iint_S \boldsymbol{n} \cdot (\boldsymbol{A}^a \times \boldsymbol{H}^b - \boldsymbol{A}^b \times \boldsymbol{H}^a) \, dA = \iiint_V (\mu_0 \boldsymbol{H}^b \cdot \boldsymbol{M}_p^a - \boldsymbol{A}^a \cdot \boldsymbol{J}^b - \mu_0 \boldsymbol{H}^a \cdot \boldsymbol{M}_p^b + \boldsymbol{A}^b \cdot \boldsymbol{J}^a) \, dV. \tag{23}$$

Equation (23) is the desired reciprocity theorem. Note that due to the boundary conditions (14)-(16), none of the surfaces S_μ, S_1, or S_2 contributes to the left side of (23). As to the contribution from a sufficiently large sphere S_Δ of radius Δ and center at the origin, we observe that (17) and (18) lead to

$$\iint_{S_\Delta} \boldsymbol{n} \cdot (\boldsymbol{A}^a \times \boldsymbol{H}^b - \boldsymbol{A}^b \times \boldsymbol{H}^a) \, dA = \text{order } (\Delta^{-3}) \text{ as } \Delta \to \infty \tag{24}$$

and hence this contribution vanishes in the limit $\Delta \to \infty$.

The volume integral containing the current densities can be further reduced in case the currents flow in a thin-wire loop C by using the relation

$$\boldsymbol{J} \, dV = I \boldsymbol{\tau} \, ds, \tag{25}$$

where I denotes the electric current in the loop, $\boldsymbol{\tau}$ is the unit vector along the tangent to C, and s is the arc length along C. Since I does not vary along the wire, we have

$$\iiint_V (-\boldsymbol{A}^a \cdot \boldsymbol{J}^b + \boldsymbol{A}^b \cdot \boldsymbol{J}^a) \, dV = -\Phi^a I^b + \Phi^b I^a, \tag{26}$$

where we have used the relation (cf. (8) and (10))

$$\oint_C \boldsymbol{\tau} \cdot \boldsymbol{A} \, ds = -\int_0^t e \, d\tau = \Phi, \tag{27}$$

in which Φ denotes the magnetic flux linked to the loop.

V. Application of the Reciprocity Theorem to Magnetic Recording Theory

In this section we investigate the implications of the reciprocity theorem derived in Section IV for the magnetic recording situation described in Section III. To this end, we apply (23) to the domain V_0, taking the reading coil to be a filamentary one. Taking into account (24) and (26), (23) leads to

$$\Phi^a I^b - \Phi^b I^a = \iiint_{V_M} \mu_0 (\boldsymbol{H}^b \cdot \boldsymbol{M}_p^a - \boldsymbol{H}^a \cdot \boldsymbol{M}_p^b) \, dV, \tag{28}$$

where the right side contains the "permanent" or "source" part of the magnetization only.

In (28), state "a" is identified with an auxiliary state that is characterized by $I^a \neq 0$, $\boldsymbol{M}_p^a = \boldsymbol{0}$. Since nowhere in the configuration is finitely conducting material present, $\boldsymbol{H}^a$ follows I^a instantaneously in its time dependence, and $\boldsymbol{H}^a/I^a$ is a time-independent quantity. Further, state "b" is identified with the reading state that is characterized by $I^b = 0$, $\boldsymbol{M}_p^b = \boldsymbol{M}_p^R$, where $\boldsymbol{M}_p^R$ is the recorded magnetization pattern. The reading state will be denoted by the superscript R. Substitution of these data in (28) yields

$$\Phi^R = \iiint_{V_M} \mu_0 \boldsymbol{h}^a \cdot \boldsymbol{M}_p^R \, dV, \tag{29}$$

in which

$$\boldsymbol{h}^a = \boldsymbol{H}^a / I^a. \tag{30}$$

The reading electromotive force e^R is then obtained as

$$e^R = -\partial_t \iiint_{V_M} \mu_0 \boldsymbol{h}^a \cdot \boldsymbol{M}_p^R \, dV. \tag{31}$$

Equation (31) is the fundamental expression upon which our further investigations are based. Taking into account that through the motion of the recorded magnetization, V_M and $\boldsymbol{M}_p^R$ depend on t, we arrive at

$$e^R = -\iiint_{V_M} \mu_0 \boldsymbol{h}^a \cdot \partial_t \boldsymbol{M}_p^R \, dV - \iint_{S_M} (\boldsymbol{n} \cdot \boldsymbol{v}) \mu_0 \boldsymbol{h}^a \cdot \boldsymbol{M}_p^R \, dA, \tag{32}$$

where $\boldsymbol{v}$ denotes the local velocity of the carrier of magnetization at a point of its boundary surface S_M, and $\boldsymbol{n}$ is the unit vector along the normal to S_M, pointing away from V_M. Alternative forms of (32) that express in more detail and through different physical interpretations the influence that the motion of the tape or disk has on the induced READ voltage will be derived in Section VI. We consider the case of rigid motion (translatory and/or rotary) and assume that the time dependence of $\boldsymbol{M}_p^R$ is only due to this motion.

VI. Expressions for the Read EMF from a Rigidly Moving Carrier of Spatially Recorded Magnetization

For a rigidly moving carrier of spatially recorded magnetization, the space-time dependence of $\boldsymbol{M}_p^R$ is expressed as

$$\boldsymbol{M}_p^R = \boldsymbol{M}_0 \left[\boldsymbol{r} - \int_0^t \boldsymbol{v}(\boldsymbol{r}, \tau) \, d\tau \right] \text{ when } \boldsymbol{r} \in V_M, \tag{33}$$

where $\boldsymbol{M}_0(\boldsymbol{r})$ represents the pattern of recorded magnetization at the instant $t = 0$, and $\boldsymbol{v}$ is the instantaneous velocity of the pattern. Now, the local velocity of a *rigidly* moving object can be written as

$$\boldsymbol{v}(\boldsymbol{r}, t) = \boldsymbol{w}(t) + \boldsymbol{\omega}(t) \times [\boldsymbol{r} - \boldsymbol{r}_\omega(t)], \tag{34}$$

where $\boldsymbol{w}$ denotes the translational part of $\boldsymbol{v}$, $\boldsymbol{\omega}$ denotes the instantaneous angular velocity, and $\boldsymbol{r}_\omega$, the position vector of the momentary axis of rotation. The vectors $\boldsymbol{w}$, $\boldsymbol{\omega}$, and $\boldsymbol{r}_\omega$ are constant throughout V_M. From (34) we obtain the following results, which will be needed in our further discussion:

$$\nabla \cdot \boldsymbol{v} = 0, \tag{35}$$

$$\nabla \times \boldsymbol{v} = 2\boldsymbol{\omega}, \tag{36}$$

$$\nabla(\boldsymbol{v} \cdot \boldsymbol{P}) = -\boldsymbol{\omega} \times \boldsymbol{P} \text{ for any constant vector } \boldsymbol{P}, \tag{37}$$

$$\nabla \times (\boldsymbol{v} \times \boldsymbol{P}) = \boldsymbol{\omega} \times \boldsymbol{P} \text{ for any constant vector } \boldsymbol{P}. \tag{38}$$

We return to (33) and obtain

$$\partial_t \boldsymbol{M}_p^R = -(\boldsymbol{v} \cdot \nabla) \boldsymbol{M}_0.^{1} \tag{39}$$

Substitution of (39) in the first term on the right side of (32) yields

$$e^R = \iiint_{V_M} \mu_0 \boldsymbol{h}^a \cdot [(\boldsymbol{v} \cdot \nabla) \boldsymbol{M}_0] \, dV - \iiint_{V_M} \nabla \cdot [\boldsymbol{v}(\mu_0 \boldsymbol{h}^a \cdot \boldsymbol{M}_0)] \, dV, \tag{40}$$

where, in the second term on the right side, we have applied Gauss' divergence theorem. However, on account of (35), we have

$$\begin{aligned} \nabla \cdot [\boldsymbol{v}(\mu_0 \boldsymbol{h}^a \cdot \boldsymbol{M}_0)] &= (\boldsymbol{v} \cdot \nabla)(\mu_0 \boldsymbol{h}^a \cdot \boldsymbol{M}_0) \\ &= \boldsymbol{M}_0 \cdot [(\boldsymbol{v} \cdot \nabla) \mu_0 \boldsymbol{h}^a] + \mu_0 \boldsymbol{h}^a \cdot [(\boldsymbol{v} \cdot \nabla) \boldsymbol{M}_0]. \end{aligned} \tag{41}$$

Substitution of (41) in (40) leads to

$$e^R = -\iiint_{V_M} \boldsymbol{M}_0 \cdot [(\boldsymbol{v} \cdot \nabla) \mu_0 \boldsymbol{h}^a] \, dV. \tag{42}$$

Equation (42) is an alternative form of the reciprocity theorem for motions of the type (34).

Next, we restart from (32) and (39) and use in the volume integral the following identity

$$\begin{aligned} &\mu_0 \boldsymbol{h}^a \cdot [(\boldsymbol{v} \cdot \nabla) \boldsymbol{M}_0] \\ &\quad = \mu_0 \boldsymbol{h}^a \cdot [-\nabla \times (\boldsymbol{v} \times \boldsymbol{M}_0) + \boldsymbol{v}(\nabla \cdot \boldsymbol{M}_0) + \boldsymbol{\omega} \times \boldsymbol{M}_0], \end{aligned} \tag{43}$$

in which we have used (35). Further, since $\nabla \times \boldsymbol{h}^a = \boldsymbol{0}$ when $\boldsymbol{r} \in V_M$, we can rewrite (43) as

$$\begin{aligned} \mu_0 \boldsymbol{h}^a \cdot [(\boldsymbol{v} \cdot \nabla) \boldsymbol{M}_0] &= \nabla \cdot [\mu_0 \boldsymbol{h}^a \times (\boldsymbol{v} \times \boldsymbol{M}_0)] \\ &\quad + (\mu_0 \boldsymbol{h}^a \cdot \boldsymbol{v})(\nabla \cdot \boldsymbol{M}_0) + \mu_0 \boldsymbol{h}^a \cdot (\boldsymbol{\omega} \times \boldsymbol{M}_0). \end{aligned} \tag{44}$$

Substitution of (44) in the resulting right side of (32) and application of Gauss' divergence theorem to the term originating from the first term on the right side of (44), lead, after some vector manipulation in the resulting surface integral, to

$$e^R = -\iint_{S_M} (\mu_0 \boldsymbol{h}^a \cdot \boldsymbol{v})(\boldsymbol{n} \cdot \boldsymbol{M}_0) \, dA + \iiint_{V_M} (\mu_0 \boldsymbol{h}^a \cdot \boldsymbol{v})(\nabla \cdot \boldsymbol{M}_0) \, dV + \iiint_{V_M} \mu_0 \boldsymbol{h}^a \cdot (\boldsymbol{\omega} \times \boldsymbol{M}_0) \, dV. \tag{45}$$

[1] In all spatial differentiations operating on $\boldsymbol{M}_0$, it is understood that they act on $\boldsymbol{M}_0(\boldsymbol{r})$; in the result, $\boldsymbol{r}$ is then replaced by $\boldsymbol{r} - \int_0^t \boldsymbol{v}(\boldsymbol{r}, \tau) \, d\tau$.

Equation (45) is a second alternative form of the reciprocity theorem for motions of the type (34). The first and the second term on the right side indicate that the magnetization charges (with surface density $\boldsymbol{n} \cdot \boldsymbol{M}_0$ on S_M and volume density $-\nabla \cdot \boldsymbol{M}_0$ in V_M) are "sensed" by $-\mu_0 \boldsymbol{h}^a \cdot \boldsymbol{v}$ to yield their contribution to the output electromotive force. The third term shows the influence of a rigid rotation of the carrier of recorded magnetization.

Finally, we again restart from (32) and (39) and use in the volume integral the following identity

$$\mu_0 \boldsymbol{h}^a \cdot [(\boldsymbol{v} \cdot \nabla)\boldsymbol{M}_0] = \mu_0 \boldsymbol{h}^a \cdot [-\boldsymbol{v} \times (\nabla \times \boldsymbol{M}_0) + \nabla(\boldsymbol{v} \cdot \boldsymbol{M}_0) + \boldsymbol{\omega} \times \boldsymbol{M}_0], \tag{46}$$

in which we have used (37). Further, since $\nabla \cdot \boldsymbol{h}^a = 0$ when $\boldsymbol{r} \in V_M$, we can rewrite (46) as

$$\mu_0 \boldsymbol{h}^a \cdot [(\boldsymbol{v} \cdot \nabla)\boldsymbol{M}_0] = (\boldsymbol{v} \times \mu_0 \boldsymbol{h}^a) \cdot (\nabla \times \boldsymbol{M}_0) + \nabla \cdot [\mu_0 \boldsymbol{h}^a(\boldsymbol{v} \cdot \boldsymbol{M}_0)] + \mu_0 \boldsymbol{h}^a \cdot (\boldsymbol{\omega} \times \boldsymbol{M}_0). \tag{47}$$

Substitution of (47) in the right side of (32) and application of Gauss' divergence theorem to the term originating from the second term on the right side of (47), lead, after some vector manipulation in the resulting surface integral, to

$$e^R = -\iint_{S_M} (\boldsymbol{v} \times \mu_0 \boldsymbol{h}^a) \cdot (\boldsymbol{n} \times \boldsymbol{M}_0)\, dA + \iiint_{V_M} (\boldsymbol{v} \times \mu_0 \boldsymbol{h}^a) \cdot (\nabla \times \boldsymbol{M}_0)\, dV + \iiint_{V_M} \mu_0 \boldsymbol{h}^a \cdot (\boldsymbol{\omega} \times \boldsymbol{M}_0)\, dV. \tag{48}$$

Equation (48) is a third alternative form of the reciprocity theorem for motions of the type (34). The first and the second term on the right side indicate that the magnetization currents (with surface density $-\boldsymbol{n} \times \boldsymbol{M}_0$ on S_M and volume density $\nabla \times \boldsymbol{M}_0$ in V_M) are sensed by $\boldsymbol{v} \times \mu_0 \boldsymbol{h}^a$ to yield their contribution to the output electromotive force. The third term, as in (45), shows the extra influence of a rigid rotation of the carrier of recorded magnetization. The rotational term yields, for example, a contribution if a magnetized tape is moved along a gapped head whose front surface is curved. The magnitude of this contribution is difficult to estimate, but it cannot be neglected beforehand.

Each of the alternative expressions (42), (45), and (48) for the induced READ voltage can have its own specific advantages in analyzing the output electromotive force due to a rigidly moving pattern of recorded magnetization. In this respect, we mention the discussion on advantages and disadvantages of longitudinal versus transverse recording.

VII. Conclusion

Several equivalent forms of the reciprocity theorem of magnetic recording theory were presented. The fundamental form of the theorem holds for any distribution of magnetization in time and space present in a time-dependent bounded domain. The latter domain is exterior to the configuration of reproduce head (with reading coil) and, if present, electric and/or magnetic shields. The magnetically permeable media are assumed to be linear, nonconducting, time-invariant, and locally and instantaneously reacting; they may be anisotropic and inhomogeneous. In addition, media of infinite permeability may be present. The conducting media are assumed to be perfectly conducting. The alternative forms of the reciprocity theorem have been derived for the case where the change of magnetization in time is due to the rigid motion of a carrier of spatially recorded magnetization. They express, through different physical interpretations, the influence that the motion of the tape or the disk has on the induced READ voltage. Translational as well as rotational motion are considered in combination. The results are believed to be of importance to the design of the geometry of a reproduce head, with or without electric and/or magnetic shields, such that its combination with a particular pattern of recorded magnetization yields an output electromotive force of the reading coil of a desired shape. Further, the results can be used in the study of the influence of unwanted motions of the magnetic tape or disk on the output of the reading coil, e.g., if the transport velocity shows random deviations from a desired value.

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