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Time-domain far-field scattering of plane scalar waves in the Born approximation

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The low-contrast or (first-order) Born approximation is applied to the time-domain scattering of a plane scalar wave by an object of bounded extent present in a homogeneous embedding. Closed-form analytic expressions are obtained for the spherical-wave far-field scattering amplitude related to homogeneous objects of the following shapes: an ellipsoid, an elliptic cylinder of finite height, and a tetrahedron. Dispersion is included. Apart from their intrinsic interest, the results may be useful as test cases for time-domain inverse-scattering algorithms.

1. INTRODUCTION

In the field of inverse scattering, much research is focused on the development of computer algorithms that serve to reconstruct geometrical and/or physical parameters of certain configurations from measured data related to the scattered wave arising from a known incident wave that probes the configuration.[1–10] Since any wave phenomenon is a phenomenon in space–time, it seems more or less natural to perform the inversion in space–time too. Many inversion algorithms are, furthermore, at least in their initial phases, tested for the linearized inversion problem that results on applying the low-contrast or (first-order) Born approximation. These considerations motivate the search for time-domain scattering problems that can be solved in closed form, provided that the objects show enough features to make them interesting as test cases for time-domain inversion algorithms.

In the present paper, closed-form analytic expressions are obtained for the spherical-wave, far-field scattering amplitude in the case in which a plane wave is incident upon an object of bounded extent present in a homogeneous embedding. Homogeneous objects of the following shapes are considered: an ellipsoid, an elliptic cylinder of finite height, and a tetrahedron. The analysis is entirely carried out in the space–time domain. Dispersion is included; the Lorentzian absorption line is discussed as an example. Scattering, in the Born approximation, by a geometry that is the union of the elementary geometries considered here can be dealt with by superposition. Superposition also applies to the presence of several absorption lines.

2. FORMULATION OF THE SCATTERING PROBLEM

In three-dimensional space a scattering object is present that occupies a bounded domain V. The domain exterior to V is denoted by V'. To specify a position in the configuration, we employ the coordinates $\{x, y, z\}$ with respect to an orthogonal Cartesian reference frame with origin O and the three mutually perpendicular base vectors $\{\mathbf{i}_x, \mathbf{i}_y, \mathbf{i}_z\}$ of unit length each. In the indicated order, the base vectors form a right-handed system. When appropriate, the space coordinates are collectively denoted by the position vector $\mathbf{r} = x\mathbf{i}_x + y\mathbf{i}_y + z\mathbf{i}_z$. The time coordinate is denoted by t. In the configuration, scattering of waves takes place whose physical effects are accounted for by the scalar wave function $u = u(\mathbf{r}, t)$.

The domain V' is assumed to be homogeneous and dispersion free. In it, the waves propagate with speed c_0. In any source-free subdomain of V', u satisfies the homogeneous, three-dimensional wave equation

$$(\nabla \cdot \nabla)u - c_0^{-2}\partial_t^2 u = 0, \tag{2.1}$$

where $\nabla = \mathbf{i}_x\partial_x + \mathbf{i}_y\partial_y + \mathbf{i}_z\partial_z$ and ∂ denotes partial differentiation. The scattering object shows a contrast in physical properties with respect to the embedding. This contrast is accounted for by a scalar, causal, contrast-susceptibility relaxation function $\chi = \chi(\mathbf{r}, t)$. Then, in V, u satisfies the wave equation

$$(\nabla \cdot \nabla)u - c_0^{-2}\partial_t^2[u(\mathbf{r}, t) + \int_0^\infty \chi(\mathbf{r}, t')u(\mathbf{r}, t - t')\mathrm{d}t'] = 0. \tag{2.2}$$

Here, $\chi(\mathbf{r}, t)$ represents the (causal) time response at position $\mathbf{r}$ in the medium in the case in which the local value of the wave function u would be a unit pulse (delta function) in time. For a homogeneous medium, χ is independent of $\mathbf{r}$. The most common example in optics is the dielectric response of a collection of polarized atoms, according to the classical Lorentz model, and characteristic for a spectral absorption line. In this case we have

$$\chi = \begin{cases} 0 & \text{when } -\infty < t < 0 \\ (\omega_p^2/\Omega)\exp(-\Gamma t)\sin(\Omega t) & \text{when } 0 < t < \infty \end{cases} \tag{2.3}$$

with

$$\omega_p = (Nq/m\epsilon_0)^{1/2} \tag{2.4}$$

and

$$\Omega = (\omega_0^2 + \omega_p^2/3 - \Gamma^2)^{1/2}. \tag{2.5}$$

In Eqs. (2.3)–(2.5), the symbols have the following meaning:

- N is the number density,
- q is the absolute value of electric charge,
- m is the mass,
- ϵ_0 is the permittivity *in vacuo*,
- ω_0 is the angular resonance frequency associated with the restoring force (Coulomb force),
- ω_p is the angular plasma frequency,
- Γ is the phenomenological damping coefficient,
- Ω is the natural angular frequency,

all associated with the atom's moving electric-charge distribution. In the model, the spherical-cavity Lorentz correction has been taken into account. [The corresponding frequency-domain susceptibility $\hat{\chi}$ is given by

$$\hat{\chi} = \omega_p^2(-\omega^2 - 2i\omega\Gamma + \omega_0^2 + \omega_p^2/3)^{-1}, \tag{2.6}$$

with the complex time factor $\exp(-i\omega t)$ understood.] Obviously, $\chi = 0$ when $\mathbf{r} \in V'$.

The object is irradiated by a uniform plane wave with wave shape $f = f(t)$ and propagating in the direction of the unit vector $\hat{\alpha}$. The corresponding wave function u^i is given by

$$u^i = f(t - \hat{\alpha} \cdot \mathbf{r}/c_0). \tag{2.7}$$

Next, the wave function of the scattered wave is introduced as

$$u^s = u - u^i. \tag{2.8}$$

In view of Eqs. (2.1), (2.2), (2.7), and (2.8), the scattered wave function satisfies the equation

$$(\nabla \cdot \nabla)u^s - c_0^{-2}\partial_t^2 u^s = -q^s, \tag{2.9}$$

where

$$q^s = -c_0^{-2}\partial_t^2 \int_0^\infty \chi(\mathbf{r}, t')u(\mathbf{r}, t - t')\mathrm{d}t'. \tag{2.10}$$

From Eq. (2.9) it follows that the scattered wave that is causally related to the incident wave can be expressed as

$$u^s(\mathbf{r}, t) = \int_{\mathbf{r}' \in V} \frac{q^s(\mathbf{r}', t - |\mathbf{r} - \mathbf{r}'|/c_0)}{4\pi|\mathbf{r} - \mathbf{r}'|}\mathrm{d}V. \tag{2.11}$$

In the far-field region, Eq. (2.11) reduces to

$$u^s(\mathbf{r}, t) \sim \frac{A(\hat{\beta}, t - |\mathbf{r}|/c_0)}{4\pi|\mathbf{r}|}, \qquad (|\mathbf{r}| \to \infty), \tag{2.12}$$

with

$$A(\hat{\beta}, t) = \int_{\mathbf{r}' \in V} q^s(\mathbf{r}', t + \hat{\beta} \cdot \mathbf{r}'/c_0)\mathrm{d}V, \tag{2.13}$$

in which $\hat{\beta} = \mathbf{r}/|\mathbf{r}|$ denotes the unit vector in the direction of observation. Note that in expression (2.12) the time argument in A has been delayed by the travel time from the origin (which is located in the neighborhood of the object) to the point of observation. It is clear that if q^s were known, expressions (2.11)–(2.13) would solve the scattering problem. In Section 3, q^s will be obtained by applying the low-contrast or (first-order) Born approximation.

3. THE BORN APPROXIMATION

In the weak-scattering, low-contrast or (first-order) Born approximation, the wave function u in the right-hand side of Eq. (2.10) is replaced by u^i. In our case, this amounts to

$$q^s \overset{B}{=} -c_0^{-2}\partial_t^2 \int_0^\infty \chi(\mathbf{r}, t')f(t - t' - \hat{\alpha} \cdot \mathbf{r}/c_0)\mathrm{d}t'. \tag{3.1}$$

In our further analysis, the expression that results on substituting Eq. (3.1) into Eq. (2.13) will be evaluated for a number of homogeneous objects of different shapes. For such objects we write

$$A(\hat{\beta}, t) = -V \int_0^\infty \chi(t')\Upsilon(\hat{\beta} - \hat{\alpha}, t - t')\mathrm{d}t', \tag{3.2}$$

where V is the volume of the object and Υ is a shape factor that depends on the shape and the dimensions of the object, on the wave shape of the incident wave, and on $\hat{\beta} - \hat{\alpha}$. As far as the dependence on the wave shape of the incident wave is concerned, this wave shape itself occurs as well as its differentiated and integrated forms. For the differentiated wave shapes we employ the notation

$$D^n f = \partial_t^n f \qquad \text{with} \qquad n = 1, 2 \ldots, \tag{3.3}$$

and for the integrated wave shape

$$If = \int_{t_0}^t f(t')\mathrm{d}t', \tag{3.4}$$

where t_0 is the instant at which the incident wave reaches the origin of the chosen coordinate system.

The general expression for Υ follows from Eqs. (3.1) and (3.2) as

$$\Upsilon(\mathbf{s}, t) = c_0^{-2}V^{-1} \int_{\mathbf{r} \in V} \partial_t^2 f(t + \mathbf{s} \cdot \mathbf{r})\mathrm{d}V, \tag{3.5}$$

where we have put $\mathbf{s} = (\hat{\beta} - \hat{\alpha})/c_0$. From Eq. (3.5) it immediately follows that when $\mathbf{s} = \mathbf{0}$ (i.e., for observation behind the object) we have

$$\Upsilon(0, t) = c_0^{-2}D^2 f(t) \tag{3.6}$$

for any object. Another general property of $\Upsilon(\mathbf{s}, t)$ follows by observing that f satisfies the homogeneous wave equation (2.1). Replacing in Eq. (3.5) $c_0^{-2}\partial_t^2 f$ by $(\nabla \cdot \nabla)f$ and applying Gauss's divergence theorem, we arrive at

$$\Upsilon(\mathbf{s}, t) = V^{-1} \int_{\mathbf{r} \in \partial V} (\hat{\nu} \cdot \mathbf{s})Df(t + \mathbf{s} \cdot \mathbf{r})\mathrm{d}A, \tag{3.7}$$

where ∂V denotes the boundary surface of V and $\hat{\nu}$ is the unit vector along the outward normal to ∂V. Equation (3.7) shows that the far-field, plane-wave scattering by a homogeneous object can, in the Born approximation, be regarded as a surface effect.

For an arbitrary incident wave we have

$$A(\hat{\beta}, t) = -V \int_0^\infty \chi(t')\Upsilon(\hat{\beta}, t - t')\mathrm{d}t', \tag{3.8}$$

where now

$$\Upsilon(\hat{\beta}, t) = c_0^{-2}V^{-1} \int_{\mathbf{r} \in V} \partial_t^2 u^i \mathrm{d}V. \tag{3.9}$$

Proceeding as before, we obtain

$$\Upsilon(\hat{\beta}, t) = V^{-1} \int_{\mathbf{r} \in \partial V} (\hat{\upsilon} \cdot \nabla) u^i \mathrm{d}A, \qquad (3.10)$$

with the same conclusion as to the Born scattering being a surface effect.

4. THE SHAPE FACTOR OF AN ELLIPSOID

In this section we calculate the shape factor Υ, defined in Eq. (3.5), for the ellipsoid

$$V = \{\mathbf{r}; 0 \leqslant x^2/a^2 + y^2/b^2 + z^2/c^2 < 1\}. \qquad (4.1)$$

The volume of this object is

$$V = 4\pi abc/3. \qquad (4.2)$$

To carry out the integration at the right-hand side of Eq. (3.5), we first introduce as variables of integration

$$\xi = x/a, \qquad \eta = y/b, \qquad \zeta = z/c. \qquad (4.3)$$

In ξ, η, ζ space the domain of integration is the solid sphere $0 \leqslant \xi^2 + \eta^2 + \zeta^2 < 1$ of unit radius. In this space we introduce as variables of integration the spherical polar coordinates $\{\rho, \theta, \phi\}$ with $0 \leqslant \rho < 1$, $0 \leqslant \theta \leqslant \pi$, $0 \leqslant \phi < 2\pi$, around $s_x a\mathbf{i}_x + s_y b\mathbf{i}_y + s_z c\mathbf{i}_z$ as the axis. Then,

$$\begin{aligned}\mathbf{s} \cdot \mathbf{r} &= (s_x a)\xi + (s_y b)\eta + (s_z c)\zeta \\ &= \gamma\rho \cos(\theta), \end{aligned} \qquad (4.4)$$

where

$$\gamma = (s_x{}^2a^2 + s_y{}^2b^2 + s_z{}^2c^2)^{1/2} > 0, \qquad (4.5)$$

while

$$\mathrm{d}V = abc\rho^2 \sin(\theta)\mathrm{d}\rho\mathrm{d}\theta\mathrm{d}\phi. \qquad (4.6)$$

Replacing differentiations of f with respect to t by differentiations with respect to the appropriate spatial variables of integration, the resulting integral can be evaluated in closed form. We obtain

$$\begin{aligned}\Upsilon = (3c_0{}^{-2}/2\gamma^3)\{\gamma[f(t + \gamma) + f(t - \gamma)] \\ - If(t + \gamma) + If(t - \gamma)\}. \end{aligned} \qquad (4.7)$$

When $\gamma \to 0$, i.e., $\mathbf{s} \to \mathbf{0}$, the right-hand side approaches the value $c_0{}^{-2}D^2f$, which is the correct limit in view of Eq. (3.6).

5. THE SHAPE FACTOR OF AN ELLIPTIC CYLINDER OF FINITE HEIGHT

In this section we calculate the shape factor Υ, defined in Eq. (3.5), for the elliptic cylinder of finite height

$$V = \{\mathbf{r}; 0 \leqslant x^2/a^2 + y^2/b^2 < 1, -h/2 < z < h/2\}. \qquad (5.1)$$

The volume of this object is

$$V = \pi abh. \qquad (5.2)$$

In the right-hand side of Eq. (3.5) we first carry out the integration with respect to z, which is elementary. To carry out the integration with respect to x and y, we first introduce as variables of integration

$$\xi = x/a, \qquad \eta = y/b. \qquad (5.3)$$

In the ξ, η plane the domain of integration is the interior of the circle $0 \leqslant \xi + \eta^2 < 1$ of unit radius. In this plane we introduce as variables of integration the polar coordinates $\{\rho, \theta\}$ with $0 \leqslant \rho < 1$, $0 \leqslant \theta < 2\pi$, around $s_x a\mathbf{i}_x + s_y b\mathbf{i}_y$ as the axis. Then,

$$\begin{aligned}s_x x + s_y y &= (s_x a)\xi + (s_y b)\eta \\ &= \gamma\rho \cos(\theta), \end{aligned} \qquad (5.4)$$

where

$$\gamma = (s_x{}^2a^2 + s_y{}^2b^2)^{1/2} > 0, \qquad (5.5)$$

while

$$\mathrm{d}x\mathrm{d}y = ab\rho\mathrm{d}\rho\mathrm{d}\theta. \qquad (5.6)$$

Replacing differentiations of f with respect to t by differentiations with respect to the appropriate spatial variables of integration, reducing the integral with respect to θ to one over the interval $(0, \pi)$, introducing $\tau = \cos(\theta)$ as the variable of integration, and integrating by parts, we end up with

$$\begin{aligned}\Upsilon = [2/\pi c_0{}^2 s_z h] \int_{-1}^{1} (1 - \tau^2)^{1/2}\{Df[t + \gamma\tau + s_z h/2] \\ - Df[t + \gamma\tau - s_z h/2]\}\mathrm{d}\tau. \end{aligned} \qquad (5.7)$$

Special cases occur when $s_z = 0$ (i.e., $\beta_z = \alpha_z$) and when $\gamma = 0$ (i.e., $\beta_x = \alpha_x$ and $\beta_y = \alpha_y$). These are discussed below.

Special Case $s_z = 0$

By letting $s_z \to 0$ in the right-hand side of Eq. (5.7), we arrive at

$$\Upsilon = (2/\pi c_0{}^2) \int_{-1}^{1} D^2f(t + \gamma\tau)(1 - \tau^2)^{1/2}\mathrm{d}\tau. \qquad (5.8)$$

Special Case $\gamma = 0$

By letting $\gamma \to 0$ in the right-hand side of Eq. (5.7) we arrive at

$$\Upsilon = (c_0{}^{-2}/s_z h)[Df(t + s_z h/2) - Df(t - s_z h/2)]. \qquad (5.9)$$

The special case $\mathbf{s} = \mathbf{0}$ (i.e., $\hat{\beta} = \hat{\alpha}$) follows from either Eq. (5.8) or Eq. (5.9) as $\Upsilon = c_0{}^{-2}D^2f(t)$, which is the correct limit in view of Eq. (3.6).

6. THE SHAPE FACTOR OF A TETRAHEDRON

In this section we calculate the shape factor Υ, defined in Eq. (3.5), for the tetrahedron

$$V = \left\{\mathbf{r}; \mathbf{r} = \sum_{i=1}^{4} \lambda_i \mathbf{r}_i, 0 \leqslant \lambda_i \leqslant 1, \sum_{i=1}^{4} \lambda_i = 1\right\}. \qquad (6.1)$$

In Eq. (6.1), $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$, and $\mathbf{r}_4$ are the position vectors of the vertices, and λ_1, λ_2, λ_3, and λ_4 are the barycentric coordinates of a point in the interior of the tetrahedron or on its boundary. The volume of this object is

$$\begin{aligned}V = |-\mathbf{r}_1 \cdot (\mathbf{r}_2 \times \mathbf{r}_3) + \mathbf{r}_2 \cdot (\mathbf{r}_3 \times \mathbf{r}_4) \\ - \mathbf{r}_3 \cdot (\mathbf{r}_4 \times \mathbf{r}_1) + \mathbf{r}_4 \cdot (\mathbf{r}_1 \times \mathbf{r}_2)|/6. \end{aligned} \qquad (6.2)$$

The integration is carried out by putting $\lambda_1 = 1 - \lambda_2 - \lambda_3 - \lambda_4$ and extending the resulting integration over the range $0 < \lambda_2$

< 1, $0 < \lambda_3 < 1 - \lambda_2$, $0 < \lambda_4 < 1 - \lambda_2 - \lambda_3$. Taking into account that the Jacobian $\partial(x, y, z)/\partial(\lambda_2, \lambda_3, \lambda_4) = 6V$, we end up with

$$\Upsilon = 6c_0^{-2}[(\gamma_{12}\gamma_{13}\gamma_{14})^{-1}If(t + \mathbf{s}\cdot\mathbf{r}_1) + (\gamma_{21}\gamma_{23}\gamma_{24})^{-1}If(t + \mathbf{s}\cdot\mathbf{r}_2) + (\gamma_{31}\gamma_{32}\gamma_{34})^{-1}If(t + \mathbf{s}\cdot\mathbf{r}_3) + (\gamma_{41}\gamma_{42}\gamma_{43})^{-1}If(t + \mathbf{s}\cdot\mathbf{r}_4)], \quad (6.3)$$

in which

$$\gamma_{ij} = \mathbf{s}\cdot(\mathbf{r}_i - \mathbf{r}_j) = -\gamma_{ji}. \quad (6.4)$$

The special cases in which $\mathbf{s}$ is perpendicular to an edge, or to two crossing edges, or to a face of the tetrahedron will be discussed below. These special cases are most easily dealt with by redoing the integrations that need modification.

Special Case $\gamma_{12} = \gamma_{21} = 0$

This covers the case in which $\hat{\beta} - \hat{\alpha}$ is perpendicular to the edge from vertex $\mathbf{r}_1$ to vertex $\mathbf{r}_2$. The result is

$$\Upsilon = 6c_0^{-2}\{(\gamma_{13}\gamma_{14})^{-1}[f(t + \mathbf{s}\cdot\mathbf{r}_1) - (\gamma_{13}^{-1} + \gamma_{14}^{-1}) \times If(t + \mathbf{s}\cdot\mathbf{r}_1)] + (\gamma_{31}^{2}\gamma_{34})^{-1}If(t + \mathbf{s}\cdot\mathbf{r}_3) + (\gamma_{41}^{2}\gamma_{43})^{-1}If(t + \mathbf{s}\cdot\mathbf{r}_4\}. \quad (6.5)$$

Special Case $\gamma_{12} = \gamma_{21} = 0$ and $\gamma_{13} = \gamma_{31} = 0$

This covers the case in which $\hat{\beta} - \hat{\alpha}$ is perpendicular to the face having the vertices $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$. The result is

$$\Upsilon = 6c_0^{-2}\{\gamma_{14}^{-3}[\tfrac{1}{2}\gamma_{14}^{2}Df(t + \mathbf{s}\cdot\mathbf{r}_1) - \gamma_{14}f(t + \mathbf{s}\cdot\mathbf{r}_1) + If(t + \mathbf{s}\cdot\mathbf{r}_1)] + \gamma_{41}^{-3}If(t + \mathbf{s}\cdot\mathbf{r}_4)\}. \quad (6.6)$$

Special Case $\gamma_{12} = \gamma_{21} = 0$ and $\gamma_{34} = \gamma_{43} = 0$

This covers the case in which $\hat{\beta} - \hat{\alpha}$ is perpendicular to two nonintersecting edges bounded by the vertices $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$, $\mathbf{r}_4$, respectively. The result is

$$\Upsilon = 6c_0^{-2}\{\gamma_{31}^{-2}[f(t + \mathbf{s}\cdot\mathbf{r}_3) + f(t + \mathbf{s}\cdot\mathbf{r}_1)] - 2\gamma_{31}^{-3}[If(t + \mathbf{s}\cdot\mathbf{r}_3) - If(t + \mathbf{s}\cdot\mathbf{r}_1)]\}. \quad (6.7)$$

Similar results hold for other edges and faces.

7. CONCLUSION

The explicit expressions derived in the foregoing sections show that the time-domain spherical-wave far-field scattering amplitude for plane-wave scattering in the Born approximation contains, in general, the wave shape of the incident wave, its (once or twice) differentiated forms, and its integrated form. In special directions the overall wave shape differs from the one that applies to the general case.

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