Computational Acoustics – Volume 1
R.L. Lau, D. Lee and A.R. Robinson (Editors)
Elsevier Science Publishers B.V. (North-Holland)

Analysis of Time-Domain Acoustic Wave Fields in Arbitrarily Continuously Layered Configurations Based on the Modified Cagniard Method

Martin D. Verweij and Adrianus T. de Hoop
Laboratory of Electromagnetic Research
Department of Electrical Engineering
Delft University of Technology
P.O. Box 5031, 2600 GA Delft, The Netherlands

Abstract: A combination of the Neumann series solution and the modified Cagniard method is used to derive a theoretically exact space-time domain solution for the 3-D acoustic wave propagation problem in continuously layered media. First, transformations are applied to the basic acoustic differential equations. Most importantly, this includes a one-sided temporal Laplace transformation with real positive transformation parameter. Secondly, the transform domain differential equations are converted into a system of integral equations, which in turn leads to a convergent Neumann series solution. Finally, the individual terms are transformed back to the space-time domain using the modified Cagniard method. In contrast to the standard angular wave number/frequency domain analysis, difficulties due to "turning rays" are avoided.

1. INTRODUCTION

In this paper we investigate the influence of vertical inhomogeneity of a fluid on the propagation of transient acoustic waves. The applied method is an integral transform method. It consists of three basic ingredients: transformations with respect to the time coordinate as well as the horizontal spatial coordinates, the solution of the resulting transform domain differential equations, and the transformation back to the space-time domain. It is now standard practice to apply integral transform methods to configurations consisting of piecewise homogeneous layers [e.g., Helmberger (1968), Wiggins & Helmberger (1974)]. It is less common practice to use integral transform methods in the investigation of wave propagation in purely continuously layered configurations; this approach was originated by Chapman (1974, 1976). We, too, shall follow the approach in which the continuous behaviour of the medium parameters is understood right from the start.

As far as the time coordinate is concerned, the Laplace transformation with real and positive transformation parameter will be employed, which takes causality explicitly into account. The Fourier transformation will be used for the spatial transformation with respect to the horizontal coordinates. In the transform domain then a wave propagation problem in the vertical spatial coordinate remains. To solve the relevant one-dimensional differential equations we will apply the WKBJ iterative solution [Chapman (1981)]. In the frequency domain approach the WKBJ iterative solution can break down due to a zero vertical propagation coefficient (which occurs at a turning point). Owing to the use of the time Laplace transformation this difficulty is avoided. The final operation is the transformation back to the space-time domain, which we will perform using the modified Cagniard method. This method is also known as the Cagniard-De Hoop method or the generalized ray method [Cagniard (1939, 1962), De Hoop (1960, 1988)].

The resulting overall method turns out to be applicable to all horizontally layered media with parameter profiles that are everywhere continuous and piecewise continuously differentaible with respect to the vertical coordinate.

2. CONFIGURATION AND BASIC EQUATIONS

The configuration to be investigated consists of a point source and a point receiver of acoustic waves. Both are situated in an isotropic fluid that is invariant in the horizontal Carthesian directions x_1 and x_2, while its properties are continuous and piecewise continuously differentiable in the vertical direction x_3. Using the subscript notation and the summation convention, where lower-case Latin subscripts range from 1 to 3, the space-time domain linearized acoustic equations can be written as

$$\partial_k v_k(x_i,t) + \kappa(x_3)\,\partial_t p(x_i,t) = q(x_i,t). \tag{2.1}$$

$$\partial_k p(x_i,t) + \rho(x_3)\,\partial_t v_k(x_i,t) = f_k(x_i,t), \tag{2.2}$$

Here, p is the acoustic pressure, v_k is the particle velocity, $\rho(x_3)$ is the volume density of mass of the fluid and $\kappa(x_3)$ is its compressibility. Further, ∂_t and ∂_k denote differentiation with respect to time and the coordinate x_k, respectively. The action of the source is accounted for by f_k, its volume density of volume force, and q, its volume density of volume injection rate. Throughout our investigation the source is taken to be a point source, and without loss of generality we may assume that

$$\{q(x_i,t), f_k(x_i,t)\} = \delta(x_1,x_2,x_3-x_3^S)\,\{Q^S(t), F_k^S(t)\}. \tag{2.3}$$

3. TRANSFORMATION OF THE BASIC EQUATIONS

First, the space-time domain quantities are transformed to the space/temporal Laplace domain according to

$$\hat{p}(x_i, s) = \int_{0-}^{\infty} p(x_i, t)\, \exp(-st)\, dt. \tag{3.1}$$

Following Cagniard (1939, 1962), the transformation parameter s is taken to be real and positive; this choice is characteristic for the modified Cagniard method. Secondly, a two-dimensional spatial Fourier transformation with respect to x_1 and x_2, defined by

$$\hat{\tilde{p}}(\alpha_1, \alpha_2, x_3, s) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{p}(x_i, s)\, \exp[is(\alpha_1 x_1 + \alpha_2 x_2)]\, dx_1\, dx_2, \tag{3.2}$$

brings us at the final transform domain. The corresponding inverse Fourier transformation is

$$\hat{p}(x_i, s) = \left(\frac{s}{2\pi}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{\tilde{p}}(\alpha_1, \alpha_2, x_3, s)\, \exp[-is(\alpha_1 x_1 + \alpha_2 x_2)]\, d\alpha_1\, d\alpha_2. \tag{3.3}$$

Note that the actual Fourier transformation parameters are $s\alpha_1$ and $s\alpha_2$. Applying the respective transformations to the basic acoustic equations (2.1) and (2.2), and eliminating $\hat{\tilde{v}}_1$ and $\hat{\tilde{v}}_2$, we arrive at the transform domain differential equations; in matrix notation these are

$$\partial_3 \begin{bmatrix} \hat{\tilde{v}}_3 \\ \hat{\tilde{p}} \end{bmatrix} + s \begin{bmatrix} 0 & \gamma(x_3)Y(x_3) \\ \gamma(x_3)Y^{-1}(x_3) & 0 \end{bmatrix} \begin{bmatrix} \hat{\tilde{v}}_3 \\ \hat{\tilde{p}} \end{bmatrix} = \delta(x_3 - x_3^S) \begin{bmatrix} \hat{\tilde{Q}}^S \\ \hat{\tilde{F}}^S \end{bmatrix}, \tag{3.4}$$

in which

$$\gamma(x_3) = [c^{-2}(x_3) + \alpha_1^2 + \alpha_2^2]^{1/2} \tag{3.5}$$

is the local vertical propagation coefficient or local vertical slowness,

$$Y(x_3) = \frac{\gamma(x_3)}{\rho(x_3)} \tag{3.6}$$

is the local vertical acoustic wave admittance, and

$$c(x_3) = [\rho(x_3)\, \kappa(x_3)]^{-1/2} \tag{3.7}$$

is the local acoustic wave speed. The quantities $\hat{\tilde{Q}}^S$ and $\hat{\tilde{F}}^S$ are the transform domain notional source strengths. Equation (3.4) can be written as

$$\partial_3 \hat{\tilde{\mathbf{b}}}_I + s\mathbf{A}_{IJ}(x_3)\, \hat{\tilde{\mathbf{b}}}_J = \hat{\tilde{\mathbf{u}}}_I, \tag{3.8}$$

where the upper case Latin subsripts take on the values 1 and 2. In this equation $\hat{\hat{\mathbf{b}}}_I$ is denoted as the acoustic state vector, $\mathbf{A}_{IJ}(x_3)$ is the system matrix and $\hat{\hat{\mathbf{u}}}_I$ is the source strength vector.

4. SOLUTION OF THE SYSTEM OF TRANSFORM DOMAIN DIFFERENTIAL EQUATIONS

As a first step in solving the transform domain equation (3.8), we decompose the system matrix $\mathbf{A}_{IJ}(x_3)$ according to $\mathbf{A}_{IJ}(x_3) = \mathbf{N}_{IK}(x_3)\,\mathbf{\Lambda}_{KL}(x_3)\,\mathbf{N}^{-1}_{LJ}(x_3)$. In this equation

$$\mathbf{\Lambda}_{KL}(x_3) = \begin{bmatrix} \gamma(x_3) & 0 \\ 0 & -\gamma(x_3) \end{bmatrix} \tag{4.1}$$

is the eigenvalue matrix of $\mathbf{A}_{IJ}(x_3)$, whereas

$$\mathbf{N}_{IK}(x_3) = \tfrac{1}{2}\sqrt{2}\begin{bmatrix} Y^{1/2}(x_3) & -Y^{1/2}(x_3) \\ Y^{-1/2}(x_3) & Y^{-1/2}(x_3) \end{bmatrix} \tag{4.2}$$

is the matrix of normalized eigencolumns of $\mathbf{A}_{IJ}(x_3)$, also denoted as the composition matrix. We define the wave vector $\hat{\hat{\mathbf{w}}}_J$ through the composition relation

$$\hat{\hat{\mathbf{b}}}_I = \mathbf{N}_{IJ}(x_3)\,\hat{\hat{\mathbf{w}}}_J. \tag{4.3}$$

Using eqs. (3.8) and (4.3), the wave vector differential equation is found as

$$\partial_3\hat{\hat{\mathbf{w}}}_I + s\mathbf{\Lambda}_{IJ}(x_3)\,\hat{\hat{\mathbf{w}}}_J = \mathbf{\Delta}_{IK}(x_3)\hat{\hat{\mathbf{w}}}_K + \mathbf{N}^{-1}_{IJ}(x_3)\,\hat{\hat{\mathbf{u}}}_J, \tag{4.4}$$

where the so-called coupling matrix $\mathbf{\Delta}_{IK}(x_3)$ is given by

$$\mathbf{\Delta}_{IK}(x_3) \;=\; -\mathbf{N}^{-1}_{IJ}(x_3)\,[\partial_3\mathbf{N}(x_3)]_{JK} = \begin{bmatrix} 0 & \chi(x_3) \\ \chi(x_3) & 0 \end{bmatrix}, \tag{4.5}$$

in which

$$\chi(x_3) = \partial_3 Y(x_3)/2Y(x_3) = \tfrac{1}{2}\partial_3[\ln Y(x_3)] = -\frac{\partial_3 c(x_3)}{2c^3(x_3)\gamma^2(x_3)} - \frac{\partial_3\rho(x_3)}{2\rho(x_3)} \tag{4.6}$$

is the local reflection coefficient or inhomogeneity function. Using standard Green's function methods, eq. (4.4) is recast into the integral operator equation

$$\hat{\hat{w}}_I = \mathsf{L}_{IJ}\,\hat{\hat{w}}_J + \hat{\hat{h}}_I, \tag{4.7}$$

in which the integral operator L_{IJ} is defined by

$$\mathsf{L}_{IJ}\hat{\tilde{w}}_J = \begin{bmatrix} \int_{-\infty}^{x_3} \chi(x_3') \exp(-s\int_{x_3'}^{x_3} \gamma\, d\zeta)\, \hat{\tilde{w}}_2\, dx_3' \\ -\int_{x_3}^{\infty} \chi(x_3') \exp(-s\int_{x_3}^{x_3'} \gamma\, d\zeta)\, \hat{\tilde{w}}_1\, dx_3' \end{bmatrix}. \tag{4.8}$$

The source vector $\hat{\tilde{h}}_I$ at the observation level x_3 is given by

$$\hat{\tilde{h}}_I = \begin{bmatrix} \frac{1}{2}\sqrt{2}\, A\, H(x_3 - x_3^S) \exp(-s\int_{x_3^S}^{x_3} \gamma\, d\zeta) \\ -\frac{1}{2}\sqrt{2}\, B\, H(x_3^S - x_3) \exp(-s\int_{x_3}^{x_3^S} \gamma\, d\zeta) \end{bmatrix}, \tag{4.9}$$

with

$$A = \hat{\tilde{F}}^S\, Y^{1/2}(x_3^S) + \hat{\tilde{Q}}^S\, Y^{-1/2}(x_3^S), \tag{4.10}$$

$$B = \hat{\tilde{F}}^{S'}\, Y^{1/2}(x_3^S) - \hat{\tilde{Q}}^S\, Y^{-1/2}(x_3^S), \tag{4.11}$$

where $H(x_3) = \{1, \frac{1}{2}, 0\}$ for $\{x_3 > 0, x_3 = 0, x_3 < 0\}$ is the Heaviside unit step function, which occurs in view of causality. The solution of eq. (4.7) can be found by using the Neumann series solution, also known as the WKBJ iterative solution, given by

$$\hat{\tilde{w}}_I = \hat{\tilde{w}}_I^{(0)} + \hat{\tilde{w}}_I^{(1)} + \hat{\tilde{w}}_I^{(2)} + \cdots, \tag{4.12}$$

in which

$$\hat{\tilde{w}}_I^{(i)} = \begin{cases} \hat{\tilde{h}}_I, & (i = 0), \\ \mathsf{L}_{IJ}\hat{\tilde{w}}_J^{(i-1)}, & (i = 1, 2, \cdots). \end{cases} \tag{4.13}$$

Using eq. (4.8), the second line of eq.(4.13)can be written as

$$\hat{\tilde{w}}_1^{(i)} = \int_{-\infty}^{x_3} \chi(x_3') \exp(-s\int_{x_3'}^{x_3} \gamma\, d\zeta)\, \hat{\tilde{w}}_2^{(i-1)}\, dx_3', \qquad (i = 1, 2, \ldots), \tag{4.14}$$

$$\hat{\tilde{w}}_2^{(i)} = -\int_{x_3}^{\infty} \chi(x_3') \exp(-s\int_{x_3}^{x_3'} \gamma\, d\zeta)\, \hat{\tilde{w}}_1^{(i-1)}\, dx_3', \qquad (i = 1, 2, \ldots). \tag{4.15}$$

In view of the fact that the Laplace transformation paramater s is real and can be chosen arbitrarily large, the norm of the operator L_{IJ} in eq. (4.8) can be made less than unity provided that $\gamma(x_3)$ remains bounded away from zero and that $\chi(x_3)$ remains bounded [De Hoop (1990)]. For all real values of α_1 and α_2, i.e., those values used in connection with the inverse spatial Fourier transformation, these requirements for the convergence of the iterative scheme are always met for configurations with parameter profiles that are continuous and piecewise continuously differentiable with respect to x_3. Moreover, Lerch's theorem [Widder (1946)] ensures that the convergence of the Neumann series carries over to the causal time domain solution. Once we know the series solution of the wave vector $\hat{\tilde{w}}_I$, we can apply the composition relation (4.3) to obtain the series solution of the acoustic state vector $\hat{\tilde{b}}_I$.

5. PHYSICAL INTERPRETATION

The results of eqs. (4.9), (4.13), (4.14), and (4.15) indicate that all $\hat{\tilde{w}}_1^{(i)}$ represent waves that travel in the direction of increasing x_3, while all $\hat{\tilde{w}}_2^{(i)}$ represent waves that travel in the direction of decreasing x_3. According to eq. (4.13), the direct or zero order waves $\hat{\tilde{w}}_1^{(0)}$ and $\hat{\tilde{w}}_2^{(0)}$ are directly generated by the source. Next, each higher order iterate is generated from the reflection of its previous one traveling in the opposite direction.

6. INVERSE TRANSFORMATIONS

After the transform domain solution has been found, we return to the space-time domain using the modified Cagniard method of inversion [De Hoop (1960, 1988)]. This will be illustrated by considering the zero order pressure wave travelling upwards from a volume injection type source to a receiver on a more elevated level, i.e., $p^{(0)-}$. Subsequently, we will shortly describe the inverse transformation process for a first order wave. Upon using eqs. (4.3), (4.9), (4.11) and (4.13), and applying the inverse Fourier transformation (3.3) to $\hat{\tilde{p}}^{(0)-}$, we obtain an expression for $\hat{p}^{(0)-}$ at the receiver level x_3^R. This can formally be written as

$$\hat{p}^{(0)-} = s^2 \hat{Q}^S(s)\, \hat{G}^{(0)-}, \tag{6.1}$$

in which the Green's function $\hat{G}^{(0)-}$ thus introduced contains s only in its exponential part of the representation

$$\hat{G}^{(0)-} = \frac{-1}{8\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Pi \exp[-s(i\alpha_1 x_1 + i\alpha_2 x_2 + \textstyle\int_{x_3^R}^{x_3^S} \gamma \, d\zeta)] \, d\alpha_1 \, d\alpha_2, \tag{6.2}$$

where

$$\Pi = Y^{-1/2}(x_3^S)\,Y^{-1/2}(x_3^R) \tag{6.3}$$

represents the coupling of the wave with the source and the receiver. Since the configuration shows rotational symmetry with respect to the x_3-axis, we can without loss of generality take $x_1 = r > 0$, $x_2 = 0$. Replacing $i\alpha_1$ by p and α_2 by q, eq. (6.2) becomes

$$\hat{G}^{(0)-} = \frac{i}{8\pi^2}\int_{-\infty}^{\infty}\int_{-i\infty}^{i\infty} \bar{\Pi}\exp[-s(pr + \int_{x_3^R}^{x_3^S}\bar{\gamma}\,d\zeta)]\,dp\,dq. \tag{6.4}$$

Here, p is the horizontal slowness. Quantities in which α_1, α_2 are replaced by p, q are indicated by an overbar; the vertical propagation coefficient or vertical slowness follows from eq. (3.5) as

$$\bar{\gamma}(x_3) = \left[c^{-2}(x_3) - p^2 + q^2\right]^{1/2}. \tag{6.5}$$

As a next step we deform the path of integration from the imaginary p-axis into the so-called modified Cagniard contour, defined by

$$\tau = pr + \int_{x_3^R}^{x_3^S}\bar{\gamma}(\zeta)\,d\zeta, \tag{6.6}$$

where p is such that τ is always real. In order to do so we continue the integrand analytically into the complex p-plane. We define $\mathrm{Re}\,(\bar{\gamma}) \geq 0$ in order to keep the square root in eq. (6.5) single valued. The singularities of the integrand in eq. (6.4) are the branch points associated with $\bar{\gamma}(x_3^S)$ and $\bar{\gamma}(x_3^R)$ in $\bar{\Pi}$ and associated with the integral of $\bar{\gamma}(\zeta)$ in the argument of the exponential function. All branch points will be provided with branch cuts along the positive and negative real p-axis, respectively, from the relevant branch point to infinity. In view of Jordan's lemma we are, since $r > 0$, only allowed to deform our path of integration into the right half of the p-plane. Applying Cauchy's theorem and Jordan's lemma, we find that the integration along the imaginary p-axis can be replaced by an integration along the complex part of the modified Cagniard contour. The contour is symmetrical with respect to the real p-axis and the integrand satisfies Schwarz' reflection principle; further the integrand is an even function of q. Progressing along the contour away form the point p_0 where it meets the real p-axis, τ monotonically increases from $T(q)$ to infinity. This means that we can straightaway replace the integration over the complex contour by an integration over the real parameter τ, which gives

$$\hat{G}^{(0)-} = \frac{-1}{2\pi^2}\int_0^\infty \int_{T(q)}^\infty \mathrm{Im}\,(\bar{\Pi}\,\partial_\tau p)\exp(-s\tau)\,d\tau\,dq. \tag{6.7}$$

Interchanging the order of integrations in eq. (6.7) leads to

$$\hat{G}^{(0)-} = \frac{-1}{2\pi^2}\int_{T_{arr}}^\infty \int_0^{Q(\tau)} \mathrm{Im}\,(\bar{\Pi}\,\partial_\tau p)\exp(-s\tau)\,dq\,d\tau, \tag{6.8}$$

where $q = Q(\tau)$ is the (unique) inverse of $\tau = T(q)$ and $T_{arr} = T(0)$ is the arrival time of the wave. By inspection we find, on account of Lerch's theorem [Widder (1946)],

$$G^{(0)-} = \frac{-1}{2\pi^2}H(t - T_{arr})\int_0^{Q(t)} \mathrm{Im}\,(\bar{\Pi}\,\partial_t p)\;dq, \tag{6.9}$$

which is the desired space-time domain Green's funtion. The integral in eq. (6.9) can easily be computed using a Gaussian quadrature rule. Referring to eq. (6.1) and the theory of the Laplace transformation, the space-time domain acoustic pressure $p^{(0)-}$ due to a source of volume injection becomes

$$p^{(0)-} = \partial_t^2\left[Q^S(t) *_t G^{(0)-}(t)\right], \tag{6.10}$$

where $*_t$ indicates a convolution with respect to time.

In the continuously layered case eq. (6.6) can give rise to two different types of contours [Verweij & De Hoop (1990)], the actual type being dependent upon the sign of the quantity

$$\partial_p\tau|_{p_\ell} = r - p_\ell\int_{x_3^R}^{x_3^S}\left[c^{-2}(\zeta) - p_\ell^2 + q^2\right]^{-1/2} d\zeta. \tag{6.11}$$

Here, p_ℓ is the leftmost branch point in the complex p-plane, i.e., $p_\ell = p_\ell(q) = (c_{max}^{-2} + q^2)^{1/2}$ with $c_{max} = \max_{\zeta\in[x_3^R;x_3^S]}\{c(\zeta)\}$. If $\partial_p\tau|_{p_\ell} < 0$, the countour crosses the real p-axis vertically in a point p_0 to the left of p_ℓ, in which case there is no interference with the branch cuts, see Fig. 1(a). If $\partial_p\tau|_{p_\ell} > 0$, *a case which is impossible for piecewise homogeneous configurations*, the complex part of the countour meets the real axis horizontally at $p_0 = p_\ell$ and the modified Cagniard contour must be supplemented by a small circle with radius $\delta > 0$ around the leftmost branch point, see Fig. 1(b). In the limit $\delta \to 0$, this circle turns out to give a vanishing contribution to the total integral.

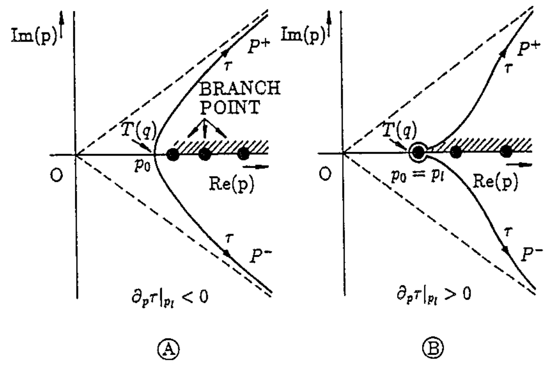

Figure 1: Possible locations of the Cagniard contour and the branch points in the right half of the complex p-plane. For case (b) the detour around the branch point has already been made.

Now that we have demonstrated the general applicabilty of the modified Cagniard method for the inversion of the zero order components, we can use the same method for the inversion of the first order components. Compared to the zero order case there are two additional complications. First of all, we have to perform an integration with respect to depth (again we can apply a Gaussian quadrature rule here), since the first order components are a continuous summation of partial reflections. These partial reflections, being dependent upon the reflection level, must first be transformed back to the space-time domain, so the inversion process becomes dependent upon depth as well. Another complication is the possibility that the leftmost branch point is also a pole of the inhomogeneity function $\chi(x_3)$ [see eq. (4.6): the first term contains $\gamma^2(x_3)$ in its denominator]. If this takes place *and* if we have a contour as in Fig. 1(b), the detour around this leftmost singularity gives a nonzero residue contribution to the first order Green's function.

7. NUMERICAL RESULTS

Finally, we will present some numerical results for a first order upward traveling pressure wave $p^{(1)-}$, caused by the continuous reflection of a downgoing zero order wave generated by a source of volume injection. This type of reflection has its application in, e.g., seismic research using the fluid model of the earth. For a configuration with parameter profiles as given in Fig. 2, the Green's function has been shown in Fig. 3 for some values of the horizontal offset

r. The sharp peak in the Green's function for $r = 5000$ m has a logarithmic nature and can be associated with a caustic in the corresponding ray theory. Using a zero order Blackman pulse with unit amplitude and a duration of 0.1 s as the source signature, the acoustic pressure at the receiver will be as shown in Fig. 4.

ACKNOWLEDGMENT

The research reported in this paper has been financially supported through Research Grants from the Stichting Fund for Science, Technology and Research (a companion organization to the Schlumberger Foundation in the U.S.A.), from Schlumberger-Doll Research, Ridgefield, CT, U.S.A., from Etudes et Productions Schlumberger, Clamart, France, from Schlumberger Cambridge Research Limited, Cambridge, England, and from the Royal/Dutch Shell Exploration and Production Laboratories, Rijswijk, The Netherlands. This support is gratefully acknowledged.

REFERENCES

[1] Cagniard, L., Réflection et réfraction des ondes seismiques progressives, Gauthier-Villars, Paris, 1939 (in French; translated and revised version: Flinn, E. A., and Dix, C. H., Reflection and refraction of progressive seismic waves, McGraw-Hill, New York, 1962).

[2] Chapman, C. H., Generalized ray theory for an inhomogeneous medium, Geophys. J. R. Astr. Soc., vol. 36, 1974, pp. 673-704.

[3] Chapman, C. H., Exact and approximate generalized ray theory in vertically inhomogeneous media, Geophys. J. R. Astr. Soc., vol. 46, 1976, pp. 201-233.

[4] Chapman, C. H., Long-period corrections to body waves: Theory, Geophys. J. R. Astr. Soc., vol. 64, 1981, pp. 321-372.

[5] Helmberger, D. V., The crust-mantle transition in the Bering Sea, Bull. Seismol. Soc. Am., vol. 58, 1968, pp. 179-214.

[6] De Hoop, A. T., A modification of Cagniard's method for solving seismic pulse problems, Appl. Sci. Res., vol. B8, 1960, pp. 349-356.

[7] De Hoop, A. T., Acoustic radiation from impulsive sources in a layered fluid, Nieuw Archief voor Wiskunde, vol. 6, 1988, pp. 111-129.

[8] De Hoop, A. T., Acoustic radiation from an impulsive point source in a continuously layered fluid – An analysis based upon the Cagniard method, J. Acoust. Soc. Am., vol. 88,. 1990, pp. 2376-2388.

[9] Verweij, M. D. and De Hoop, A. T., Determination of seismic wave fields in arbitrarily continuously layered media using the modified Cagniard method, Geophys. J. Int., vol. 103, 1990, pp. 731-754.

[10] Widder, D. V., The Laplace Transform, Princeton University Press, Princeton, New Jersey, 1946.

[11] Wiggins, R. A. and Helmberger, D. V., Synthetic seismogram computation by expansion in generalized rays, Geophys. J. R. Astr. Soc., vol. 37, 1974, pp. 73-90.

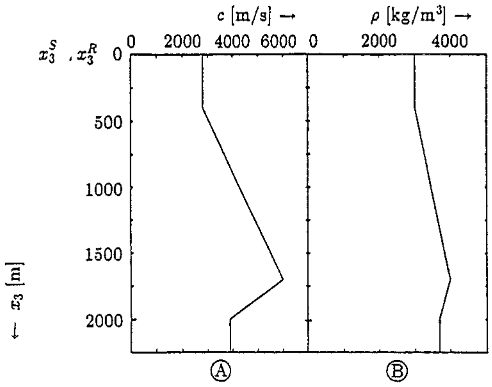

Figure 2: Parameter profiles used for the numerical example; (a) wave speed profile; (b) mass density profile. Here, $x_3^S = x_3^R = 0$ m indicate the source and receiver levels.

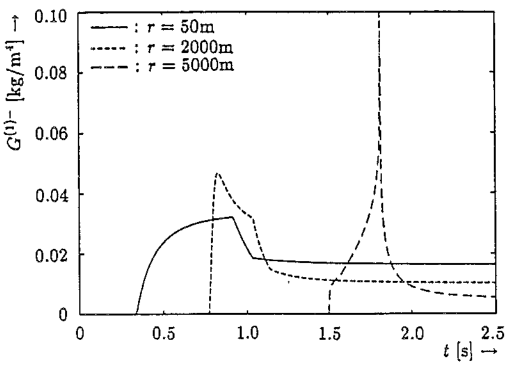

Figure 3: The Green's function $G^{(1)-}$ for various values of the horizontal offsets r.

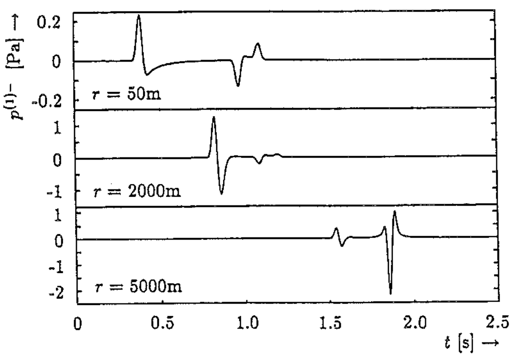

Figure 4: The acoustic pressure $p^{(1)-}$ for various values of the horizontal offsets r.