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Transient electromagnetic vs. seismic prospecting—a correspondence principle[1]

Adrianus T. de Hoop[2]

Abstract

A correspondence principle is derived that relates the Green's functions (point-receiver responses to point-source excitations) for 2D transient diffusive electromagnetic fields with electric field in the vertical plane to 2D seismic waves (in the acoustic approximation) with particle velocity in the vertical plane in arbitrarily inhomogeneous media. The constituent medium parameters in the two cases are related via two global proportionality constants. The kernels in the integral operators that express the diffusion phenomenon in terms of the wave phenomenon are of a smoothing nature. The fact that they are explicitly known can be of importance to the inverse operation. The correspondence principle is the fundamental tool in comparing the spatial resolving powers in the two methods of geophysical prospecting.

Introduction

The study of correspondences that exist between seismic methods of geophysical prospecting (i.e. prospecting via the physics of a transient wave propagation phenomenon) and transient electromagnetic methods (i.e. for data acquisition in the time windows long after the wavefront has passed, prospecting via the physics of a transient diffusion phenomenon) is a subject of interest for several reasons. Firstly, there is the aspect of the amount of spatial resolution that can be obtained from processing the data in the two cases. Conclusions on this aspect can be drawn from the correspondence principle derived in this paper, since the principle establishes a quantitative relationship between the Green's functions (point-receiver responses to point-source excitations) in the two cases, for general inhomogeneous configurations where the spatial distributions of the constituent medium parameters are the same.

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[2] Laboratory of Electromagnetic Research, Faculty of Electrical Engineering, Delft University of Technology, PO Box 5031, 2600 GA Delft, The Netherlands.

Secondly, the way in which the correspondence principle is derived reveals the underlying structure of the formulation of the physical problem in the two cases. This structure may be used for the development of the data processing software in which the high degree of structural compatibility in the two cases is incorporated.

In 3D data acquisition and interpretation there is, in general, no strict component-by-component correspondence between seismic wave phenomena (either in the acoustic approximation or in the full elastodynamic theory) and transient electromagnetic diffusion phenomena. A component-by-component correspondence does, however, exist between 2D seismic wave phenomena in the acoustic approximation and 2D transient electromagnetic diffusion phenomena. The correspondence principle will be derived for this case. The relationships between the Green's functions in the two cases are indicative of the general behaviour when going from a wave phenomenon to a diffusion phenomenon. The present analysis can be applied in the 2D case, where sources with different components can generate the field and different field components can be measured with the receivers employed. The correspondences between the associated Green's function components yield more information than an analysis based solely on the scalar wave equation vs. the scalar diffusion equation.

The mathematical derivation of the correspondence principle is based on the Schouten–Van der Pol theorem (Schouten 1934, 1961; Van der Pol 1934, 1960; Van der Pol and Bremmer 1950) which relates two (space-)time functions whose time Laplace transforms are related by replacing the time Laplace transform parameter s by a suitable function of s. Recent applications of this theorem to relate the Green's functions for wave propagation in a medium with relaxation to the Green's functions for wave propagation in a corresponding lossless medium have been described by De Hoop (1993, 1995).

Let $G = G(\mathbf{r}, \mathbf{r}', t)$ denote any of the Green's functions, i.e. the causal response at position $\mathbf{r}$ and time t due to a point-source excitation at position $\mathbf{r}'$ and time $t = 0$. Then $G = G(\mathbf{r}, \mathbf{r}', t)$ vanishes for $t < 0$ and its time Laplace transform $\hat{G} = \hat{G}(\mathbf{r}, \mathbf{r}', s)$ is given by

$$\hat{G}(\mathbf{r}, \mathbf{r}', s) = \int_{t=0}^{\infty} \exp(-st) G(\mathbf{r}, \mathbf{r}', t)\, \mathrm{d}t \tag{1}$$

for any complex s in the right half $\mathrm{Re}(s) > 0$ of the complex s-plane. (In fact, for the causal time functions under consideration, it is, according to Lerch's theorem (Widder 1946), sufficient to specify $\hat{G}$ at the equidistant set of points $\{s = s_0 + nh; s_0 > 0, h > 0, n = 0, 1, 2, \ldots\}$ on the positive real s-axis in order to determine G uniquely for $t > 0$.) The specific case required for the present application is the outcome of the theorem when s is replaced by $s^{1/2}$. This will be discussed below. Work related to the present correspondence theorem has been presented by Lee, Liu and Morrison (1989) and Gershenson (1993).

The two-dimensional seismic wave propagation problem (Acoustic approximation)

In order to establish the desired correspondence principle, it is essential that the governing basic field equations are written as a system of simultaneous first-order partial differential equations. Let the 2D seismic wave propagation take place in the vertical (x, z)-plane, where x and z are the horizontal and vertical space coordinates, respectively. Then the governing acoustic wave equations are

$$\partial_x v_x + \partial_z v_z + \kappa \partial_t p = q, \tag{2}$$

$$\partial_x p + \rho \partial_t v_x = f_x, \tag{3}$$

$$\partial_z p + \rho \partial_t v_z = f_z, \tag{4}$$

where p is the acoustic pressure (in Pa), $v_{x,z}$ is the particle velocity (in m/s), q is the volume source density of injection rate (in s^{-1}), $f_{x,z}$ is the volume source density of force (in N/m^3), ρ is the volume density of mass (in kg/m^3), κ is the compressibility (in Pa^{-1}) and ∂ denotes differentiation. The medium can be arbitrarily inhomogeneous. The medium's acoustic constituent parameters $\rho = \rho(x, z)$ and $\kappa = \kappa(x, z)$ are assumed to be piecewise continuous functions of position. Across jumps in ρ and/or κ, p and the normal component of $v_{x,z}$ are continuous, while $p = 0$ at the free surface (which need not be horizontal). By arranging the field quantities in the 1D array

$$[F] = [p, v_x, v_z] \tag{5}$$

and the source quantities in the 1D array

$$[Q] = [q, f_x, f_z], \tag{6}$$

the solution to the system of equations (2)–(4) can be expressed in terms of the 2D, square array of acoustic Green's functions $[G]$ as the time convolution

$$[F](\mathbf{r}, t) = \int_{t'=0}^{\infty} \mathrm{d}t' \int_{\mathcal{D}} [G](\mathbf{r}, \mathbf{r}', t')[Q](\mathbf{r}', t - t')\, \mathrm{d}V(\mathbf{r}'), \tag{7}$$

where $\mathcal{D}$ is the (bounded) domain in which the source distributions are active.

For the correspondence principle, the time Laplace transform counterparts of (2)–(4) and (7) are needed. These are given by

$$\partial_x \hat{v}_x + \partial_z \hat{v}_z + s\kappa \hat{p} = \hat{q}, \tag{8}$$

$$\partial_x \hat{p} + s\rho \hat{v}_x = \hat{f}_x, \tag{9}$$

$$\partial_z \hat{p} + s\rho \hat{v}_z = \hat{f}_z \tag{10}$$

and

$$[\hat{F}](\mathbf{r}, s) = \int_{\mathcal{D}} [\hat{G}](\mathbf{r}, \mathbf{r}', s)[\hat{Q}](\mathbf{r}', s)\, \mathrm{d}V(\mathbf{r}'), \tag{11}$$

respectively.

The two-dimensional transient electromagnetic diffusion problem (Electric field in the vertical plane)

The 2D transient electromagnetic diffusion problem for which the correspondence principle can be established, is the one in which the electric field is in the vertical plane. The governing electromagnetic diffusion equations are then given by

$$-\partial_x E_z + \partial_z E_x + \mu\partial_t H_y = -K_y, \tag{12}$$

$$\partial_x H_y - \sigma E_z = \mathcal{J}_z, \tag{13}$$

$$\partial_z H_y + \sigma E_x = -\mathcal{J}_x, \tag{14}$$

where $E_{x,z}$ is the electric field strength (in V/m), H_y is the magnetic field strength (in A/m), $\mathcal{J}_{x,z}$ is the volume source density of electric current (in A/m^2), K_y is the volume source density of magnetic current (in V/m^2), σ is the (electric) conductivity (in S/m), μ is the (magnetic) permeability (in H/m).

The medium can be arbitrarily inhomogeneous. The medium's electromagnetic constituent parameters $\sigma = \sigma(x, z)$ and $\mu = \mu(x, z)$ are assumed to be piecewise continuous functions of position. Across jumps in σ and/or μ, H_y and the tangential component of $E_{x,z}$ are continuous, while $H_y = 0$ at the free surface (which need not be horizontal). By arranging the field quantities in the 1D array

$$[\Phi] = [H_y, E_x, E_z] \tag{15}$$

and the source quantities in the 1D array

$$[\Psi] = [K_y, \mathcal{J}_x, \mathcal{J}_z], \tag{16}$$

the solution to the system of equations (12)–(14) can be expressed in terms of the 2D, square array of electromagnetic Green's functions $[\Gamma]$ as the time convolution

$$[\Phi](\mathbf{r}, t) = \int_{t'=0}^{\infty} \mathrm{d}t' \int_{\mathcal{D}} [\Gamma](\mathbf{r}, \mathbf{r}', t')[\Psi](\mathbf{r}', t - t')\,\mathrm{d}V(\mathbf{r}'), \tag{17}$$

where $\mathcal{D}$ is the (bounded) domain where the sources are active.

For the correspondence principle, the time Laplace transform domain counterparts of (12)–(14) and (17) are needed. These are given by

$$-\partial_x \hat{E}_z + \partial_z \hat{E}_x + s\mu\hat{H}_y = -\hat{K}_y, \tag{18}$$

$$\partial_x \hat{H}_y - \sigma\hat{E}_z = \hat{\mathcal{J}}_z, \tag{19}$$

$$\partial_z \hat{H}_y + \sigma\hat{E}_x = -\hat{\mathcal{J}}_x \tag{20}$$

and

$$[\hat{\Phi}](\mathbf{r}, s) = \int_{\mathcal{D}} [\hat{\Gamma}](\mathbf{r}, \mathbf{r}', s)[\hat{\Psi}](\mathbf{r}', s)\,\mathrm{d}V(\mathbf{r}'), \tag{21}$$

respectively.

The correspondence principle in the time Laplace transform domain

For the correspondence principle to hold, it is now assumed that

$$\sigma(x,z) = \alpha\rho(x,z) \tag{22}$$

and

$$\mu(x,z) = \beta\kappa(x,z), \tag{23}$$

where α and β are global, positive constants of the proper physical dimensions. Using (22) and (23), and by introducing the pseudo-field functions

$$\tilde{p} = (\beta s)^{1/2}\hat{H}_y, \tag{24}$$

$$\tilde{v}_x = -\alpha^{1/2}\hat{E}_z, \tag{25}$$

$$\tilde{v}_z = \alpha^{1/2}\hat{E}_x, \tag{26}$$

the pseudo-source functions

$$\tilde{q} = -\alpha^{1/2}\hat{K}_y, \tag{27}$$

$$\tilde{f}_x = (\beta s)^{1/2}\hat{\mathcal{J}}_z, \tag{28}$$

$$\tilde{f}_z = -(\beta s)^{1/2}\hat{\mathcal{J}}_x, \tag{29}$$

and the pseudo-time Laplace transform parameter

$$\tilde{s} = (\alpha\beta s)^{1/2}, \tag{30}$$

(18)–(20) can be rewritten as

$$\partial_x\tilde{v}_x + \partial_z\tilde{v}_z + \tilde{s}\kappa\tilde{p} = \tilde{q}, \tag{31}$$

$$\partial_x\tilde{p} + \tilde{s}\rho\tilde{v}_x = \tilde{f}_x, \tag{32}$$

$$\partial_z\tilde{p} + \tilde{s}\rho\tilde{v}_z = \tilde{f}_z. \tag{33}$$

The definitions (24)–(29) can be used to obtain the equation, similarly to (11),

$$[\tilde{F}](\mathbf{r},s) = \int_{\mathcal{D}} [\tilde{G}](\mathbf{r},\mathbf{r}',s)[\tilde{Q}](\mathbf{r}',s)\,\mathrm{d}V(\mathbf{r}'), \tag{34}$$

and comparison of (31)–(33) with (8)–(10) then leads to the conclusion that

$$[\tilde{G}](\mathbf{r},\mathbf{r}',s) = [\hat{G}](\mathbf{r},\mathbf{r}',\tilde{s}). \tag{35}$$

From (21), (34) and (35) in conjunction with (24)–(29), the different elements of $[\hat{\Gamma}]$ can now be related to corresponding elements of $[\hat{G}]$. Using superscripts to denote to which received field quantity and to which excitation source quantity a particular Green's function element refers, it is found that

$$\hat{\Gamma}^{HK}(\mathbf{r},\mathbf{r}',s) = -\frac{\alpha}{\tilde{s}}\hat{G}^{pq}(\mathbf{r},\mathbf{r}',\tilde{s}), \tag{36}$$

$$\hat{\Gamma}_z^{HJ}(\mathbf{r},\mathbf{r}',s) = \hat{G}_x^{pf}(\mathbf{r},\mathbf{r}',\tilde{s}), \tag{37}$$

$$\hat{\Gamma}_x^{HJ}(\mathbf{r},\mathbf{r}',s) = -\hat{G}_z^{pf}(\mathbf{r},\mathbf{r}',\tilde{s}), \tag{38}$$

$$\hat{\Gamma}_z^{EK}(\mathbf{r},\mathbf{r}',s) = \hat{G}_x^{vq}(\mathbf{r},\mathbf{r}',\tilde{s}), \tag{39}$$

$$\hat{\Gamma}_x^{EK}(\mathbf{r},\mathbf{r}',s) = -\hat{G}_z^{vq}(\mathbf{r},\mathbf{r}',\tilde{s}), \tag{40}$$

$$\hat{\Gamma}_{z,z}^{EJ}(\mathbf{r},\mathbf{r}',s) = -\frac{\tilde{s}}{\alpha}\,\hat{G}_{x,x}^{vf}(\mathbf{r},\mathbf{r}',\tilde{s}), \tag{41}$$

$$\hat{\Gamma}_{z,x}^{EJ}(\mathbf{r},\mathbf{r}',s) = \frac{\tilde{s}}{\alpha}\,\hat{G}_{x,z}^{vf}(\mathbf{r},\mathbf{r}',\tilde{s}), \tag{42}$$

$$\hat{\Gamma}_{x,z}^{EJ}(\mathbf{r},\mathbf{r}',s) = \frac{\tilde{s}}{\alpha}\,\hat{G}_{z,x}^{vf}(\mathbf{r},\mathbf{r}',\tilde{s}), \tag{43}$$

$$\hat{\Gamma}_{x,x}^{EJ}(\mathbf{r},\mathbf{r}',s) = -\frac{\tilde{s}}{\alpha}\,\hat{G}_{z,z}^{vf}(\mathbf{r},\mathbf{r}',\tilde{s}). \tag{44}$$

Equations (36)–(44) serve as the basis for the construction of the time-domain equivalents of the elements of $[\hat{\Gamma}]$.

The correspondence principle in the space–time domain

The time-domain counterparts of the elements of $[\hat{\Gamma}]$ follow from application of the Schouten–Van der Pol theorem in the theory of the time Laplace transformation to the case in which the Laplace transform parameter s is replaced by $\tilde{s} = (\alpha\beta s)^{1/2}$. For this case, the results are derived in the Appendix, where the time-domain equivalents of $[\hat{G}](\mathbf{r},\mathbf{r}',\tilde{s}), \tilde{s}^{-1}[\hat{G}](\mathbf{r},\mathbf{r}',\tilde{s})$ and $\tilde{s}[\hat{G}](\mathbf{r},\mathbf{r}',\tilde{s})$ are determined. Using these results, it is found that

$$\Gamma^{HK}(\mathbf{r},\mathbf{r}',t) = -\alpha\left[\int_{\tau=0}^{\infty} W_0(t,\tau;\alpha,\beta)G^{pq}(\mathbf{r},\mathbf{r}',\tau)\,\mathrm{d}\tau\right]\mathrm{H}(t), \tag{45}$$

$$\Gamma_z^{HJ}(\mathbf{r},\mathbf{r}',t) = \left[\int_{\tau=0}^{\infty} W_1(t,\tau;\alpha,\beta)G_x^{pf}(\mathbf{r},\mathbf{r}',\tau)\,\mathrm{d}\tau\right]\mathrm{H}(t), \tag{46}$$

$$\Gamma_x^{HJ}(\mathbf{r},\mathbf{r}',t) = -\left[\int_{\tau=0}^{\infty} W_1(t,\tau;\alpha,\beta)G_z^{pf}(\mathbf{r},\mathbf{r}',\tau)\,\mathrm{d}\tau\right]\mathrm{H}(t), \tag{47}$$

$$\Gamma_z^{EK}(\mathbf{r},\mathbf{r}',t) = \left[\int_{\tau=0}^{\infty} W_1(t,\tau;\alpha,\beta)G_x^{vq}(\mathbf{r},\mathbf{r}',\tau)\,\mathrm{d}\tau\right]\mathrm{H}(t), \tag{48}$$

$$\Gamma_x^{EK}(\mathbf{r},\mathbf{r}',t) = -\left[\int_{\tau=0}^{\infty} W_1(t,\tau;\alpha,\beta)G_z^{vq}(\mathbf{r},\mathbf{r}',\tau)\,\mathrm{d}\tau\right]\mathrm{H}(t), \tag{49}$$

$$\Gamma_{z,z}^{EJ}(\mathbf{r},\mathbf{r}',t) = -\frac{1}{\alpha}\left[\int_{\tau=0}^{\infty} W_2(t,\tau;\alpha,\beta)G_{x,x}^{vf}(\mathbf{r},\mathbf{r}',\tau)\,\mathrm{d}\tau\right]\mathrm{H}(t), \tag{50}$$

$$\Gamma_{z,x}^{EJ}(\mathbf{r},\mathbf{r}',t) = \frac{1}{\alpha}\left[\int_{\tau=0}^{\infty} W_2(t,\tau;\alpha,\beta)G_{x,z}^{vf}(\mathbf{r},\mathbf{r}',\tau)\,\mathrm{d}\tau\right]\mathrm{H}(t), \tag{51}$$

$$\Gamma^{EJ}_{x,z}(\mathbf{r},\mathbf{r}',t) = \frac{1}{\alpha}\left[\int_{\tau=0}^{\infty} W_2(t,\tau;\alpha,\beta) G^{vf}_{z,x}(\mathbf{r},\mathbf{r}',\tau)\,\mathrm{d}\tau\right]\mathrm{H}(t), \tag{52}$$

$$\Gamma^{EJ}_{x,x}(\mathbf{r},\mathbf{r}',t) = -\frac{1}{\alpha}\left[\int_{\tau=0}^{\infty} W_2(t,\tau;\alpha,\beta) G^{vf}_{z,x}(\mathbf{r},\mathbf{r}',\tau)\,\mathrm{d}\tau\right]\mathrm{H}(t), \tag{53}$$

where

$$W_0 = \left(\frac{1}{\alpha\beta\pi t}\right)^{1/2} \exp\left(-\frac{\alpha\beta\tau^2}{4t}\right)\mathrm{H}(t), \tag{54}$$

$$W_1 = \frac{1}{2}\left(\frac{\alpha\beta}{\pi}\right)^{1/2} \frac{\tau}{t^{3/2}} \exp\left(-\frac{\alpha\beta\tau^2}{4t}\right)\mathrm{H}(t), \tag{55}$$

$$W_2 = \frac{1}{2}\left(\frac{\alpha\beta}{\pi}\right)^{1/2} \frac{1}{t^{3/2}}\left(\frac{\alpha\beta\tau^2}{2t} - 1\right) \exp\left(-\frac{\alpha\beta\tau^2}{4t}\right)\mathrm{H}(t), \tag{56}$$

and $\mathrm{H}(t)$ denotes the Heaviside unit step function. Further,

$$W_1 = -\frac{\partial W_0(t,\tau;\alpha,\beta)}{\partial\tau} \tag{57}$$

and

$$W_2 = -\frac{\partial W_1(t,\tau;\alpha,\beta)}{\partial\tau}. \tag{58}$$

Since the kernel functions given by (54)–(56) are smooth functions of their arguments and the integration operation is a smoothing operation, the Green's functions pertaining to the diffusive case are always smoothed versions of those pertaining to the wave propagation case. As a consequence, the operation of going from the wave propagation case to the diffusive case is always a stable one. To perform the operation in reverse, i.e. going from the diffusive case to the wave propagation one, the integral equations (45)–(53) have to be solved. Due to the nature of the relevant kernels, this operation tends to be unstable.

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Appendix

The Schouten–Van der Pol theorem for the replacement of s by $(\alpha\beta s)^{1/2}$

Since the Schouten–Van der Pol theorem applies to the time behaviour only, the spatial arguments in the functions involved will be omitted. Let $g = g(t)$ be a known, causal function of time t with support $\{t \in \mathcal{R}, t > 0\}$, and let

$$\hat{g}(s) = \int_{\tau=0}^{\infty} \exp(-s\tau) g(\tau)\, \mathrm{d}\tau \tag{A1}$$

be its Laplace transform. Further, let

$$\hat{\gamma}(s; \alpha, \beta) = \hat{g}[(\alpha\beta s)^{1/2}], \tag{A2}$$

then

$$\hat{\gamma}(s; \alpha, \beta) = \int_{\tau=0}^{\infty} \exp[-(\alpha\beta s)^{1/2}\tau] g(\tau)\, \mathrm{d}\tau. \tag{A3}$$

In order to derive the time-domain counterpart $\gamma = \gamma(t; \alpha, \beta)$ of $\hat{\gamma} = \hat{\gamma}(s; \alpha, \beta)$, it is observed that (cf. Abramowitz and Stegun 1964)

$$\exp[-(\alpha\beta s)^{1/2}\tau] = \int_{t=0}^{\infty} \exp(-st) w(t, \tau; \alpha, \beta)\, \mathrm{d}t, \tag{A4}$$

in which

$$w(t, \tau; \alpha, \beta) = \frac{1}{2}\left(\frac{\alpha\beta}{\pi}\right)^{1/2} \frac{\tau}{t^{3/2}} \exp\left(-\frac{\alpha\beta\tau^2}{4t}\right) \mathrm{H}(t), \tag{A5}$$

where $\mathrm{H}(t)$ denotes the Heaviside unit step function. Substituting (A4) in (A3), interchanging the order of integration and applying Lerch's theorem on the uniqueness of the one-sided Laplace transformation (Widder 1946), it follows that

$$\gamma(t; \alpha, \beta) = \left[\int_{\tau=0}^{\infty} w(t, \tau; \alpha, \beta) g(\tau)\, \mathrm{d}\tau\right] \mathrm{H}(t). \tag{A6}$$

For the analysis, the time-domain counterparts of $(\alpha\beta s)^{1/2}\hat{\gamma}(s; \alpha, \beta)$ and $(\alpha\beta s)^{-1/2}\hat{\gamma}(s; \alpha, \beta)$ are also needed. These can be obtained by observing that

$$(\alpha\beta s)^{1/2} \exp[-(\alpha\beta s)^{1/2}\tau] = -\frac{\partial}{\partial\tau}(\exp[-(\alpha\beta s)^{1/2}\tau]) \tag{A7}$$

and

$$(\alpha\beta s)^{-1/2}\hat{\gamma}(s; \alpha, \beta) = \int_{\tau'=\tau}^{\infty} \exp[-(\alpha\beta s)^{1/2}\tau']\, \mathrm{d}\tau'. \tag{A8}$$

Using these relationships, (A6) and (A7) lead to

$$(\alpha\beta s)^{1/2}\hat{\gamma}(s; \alpha, \beta) \Rightarrow -\frac{\partial w(t, \tau; \alpha, \beta)}{\partial\tau}$$
$$= \frac{1}{2}\left(\frac{\alpha\beta}{\pi}\right)^{1/2} \frac{1}{t^{3/2}} \left(\frac{\alpha\beta\tau^2}{2t} - 1\right) \exp\left(-\frac{\alpha\beta\tau^2}{4t}\right) \mathrm{H}(t), \tag{A9}$$

while (A6) and (A8) lead to

$$(\alpha\beta s)^{-1/2}\hat{\gamma}(s;\alpha,\beta) \Rightarrow \int_{\tau'=\tau}^{\infty} w(t,\tau';\alpha,\beta)\,\mathrm{d}\tau'$$

$$= \left(\frac{1}{\alpha\beta\pi t}\right)^{1/2} \exp\left(-\frac{\alpha\beta\tau^2}{4t}\right)\mathrm{H}(t). \qquad \text{(A10)}$$

Equations (A6), (A9) and (A10) are used in the main text.

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