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# A general correspondence principle for time-domain electromagnetic wave and diffusion fields

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## SUMMARY

A general correspondence principle is presented that relates any time-domain electromagnetic diffusion field to an electromagnetic wavefield in a 'corresponding' configuration. The principle applies to arbitrarily inhomogeneous and anisotropic media and arbitrary transmitters and receivers. For the correspondence between the two types of electromagnetic fields to hold, the electric conductivity in the diffusive case and the permittivity in the wavefield case should have the same spatial variation, while the permeability distributions in space in the two cases are to be identical. Essential steps in the derivation of the correspondence principle are the use of the time Laplace transformation of causal signals, taken at real, positive values of the transform parameter, the Schouten–Van der Pol theorem in the theory of the Laplace transformation, and the reliance upon Lerch's theorem of the uniqueness of the interrelation between causal field quantities and their time-Laplace-transform representations at real, positive values of the transform parameter. Correspondence is then established between the tensorial Green's functions in the two cases, where the Green's functions are the point-receiver responses (either electric or magnetic field) to point-transmitter excitations (either electric- or magnetic-current source).

Through the correspondence principle, all transient electromagnetic wavefields (where losses are neglected) have as a counterpart a transient diffusive electromagnetic field (where the electric displacement current is neglected). The interrelation yields the tool to compare quantitatively the potentialities of the two types of fields in transient electromagnetic geophysical prospecting.

Finally, a general medium-parameter scaling law for time-domain electromagnetic wavefields is presented.

**Key words:** correspondence principle, electromagnetic diffusion, electromagnetic methods.

## 1 INTRODUCTION

Recent discussions on the application of transient electromagnetic prospecting methods have stipulated the importance of a correspondence principle that relates transient diffusive electromagnetic fields in an electrically conducting medium to the electromagnetic wavefield in a 'corresponding' dielectric medium (see Lee, Liu & Morrison 1989; Gershenson 1993). To establish such a relationship, Lee *et al.* (1989) introduced the '$q$-domain method', which involves an excursion to imaginary values of the angular frequency in the frequency-domain counterpart of the second-order vector diffusion equation for the electric field strength, upon which an equivalent vector wave equation is obtained. For the same purpose, Gershenson (1993) carried out a similar analysis based on the time-Laplace-transform domain counterparts of the second-order vector differential equations for the electric field strength and the magnetic flux density. For the scalar wave and diffusion equations, Filatov (1984) carried out a correspondence analysis with the aid of a Mellin transform with the square of the time coordinate as a variable.

In the present paper, a correspondence theorem is derived that is based on the first-order coupled Maxwell equations as they apply to arbitrarily inhomogeneous and anisotropic media, with arbitrary source distributions of the electric- and/or magnetic-current types. In the wave-propagation case, the medium is assumed to be lossless and its electric properties are characterized by its (tensorial) permittivity. In the diffusive

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case, the electric displacement current is neglected and the medium's electric properties are characterized by its (tensorial) conductivity. The magnetic properties of the media in the two cases are characterized by their (tensorial) permeabilities. For the correspondence to hold, the electric properties of the media in the two cases should have the same spatial distributions. The same applies to the magnetic properties in the two cases. The spatial distribution of the magnetic properties may, however, differ from the spatial distribution of the electric properties.

Essential steps in establishing the correspondence principle are: the use of the time Laplace transformation for causal signals, taken at real, positive values of the transform parameter; the application of the Schouten–Van der Pol theorem in the theory of the Laplace transformation (Schouten 1934, 1961; Van der Pol 1934, 1960; Van der Pol & Bremmer 1950); and the reliance upon Lerch's theorem (Widder 1946) of the uniqueness of the interrelation between causal source and field quantities and their time-Laplace-transform representations for real, positive values of the transform parameter. The correspondence theorem is derived for the four types of Green's functions occurring in the two cases, viz the causal point-receiver responses (either electric or magnetic field) to point-transmitter excitations (of either the electric- or the magnetic-current type). Through the principle, the Green's functions for the diffusive case are expressed in terms of their wave-propagation counterparts. The kernel functions that perform the interrelation are determined explicitly. The correspondence principle yields the tool to compare quantitatively the potentialities of the two types of fields in transient electromagnetic geophysical prospecting.

In the analysis, $\mathbf{r}$ is the position vector in an orthogonal, Cartesian reference frame, $t$ is the time, $\nabla$ is the spatial vectorial differentiation operator and $\partial_t$ denotes differentiation with respect to time. The field quantities are the electric field strength $\mathbf{E} = \mathbf{E}(\mathbf{r}, t)$ and the magnetic field strength $\mathbf{H} = \mathbf{H}(\mathbf{r}, t)$. The source quantities are the volume source density of electric current $\mathbf{J} = \mathbf{J}(\mathbf{r}, t)$ and the volume source density of magnetic current $\mathbf{K} = \mathbf{K}(\mathbf{r}, t)$. The field quantities are taken to be causally related to their excitation by the sources. The medium properties are characterized by the conductivity $\boldsymbol{\sigma} = \boldsymbol{\sigma}(\mathbf{r})$, the permittivity $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}(\mathbf{r})$, and the permeability $\boldsymbol{\mu} = \boldsymbol{\mu}(\mathbf{r})$. These are assumed to be piecewise continuous tensor functions of position of rank two for anisotropic media and piecewise continuous scalar functions of position for isotropic media.

All source and field quantities are assumed to be bounded functions of time, and vanish prior to the instant, taken as $t=0$, at which the sources are switched on. Then, their one-sided Laplace transforms are given by

$$\hat{F}(\mathbf{r}, s) = \int_{t=0}^{\infty} \exp(-st) F(\mathbf{r}, t)\, dt \qquad \text{for } s \in \mathscr{R}, s > 0, \tag{1}$$

where $F$ stands for any of the source or field quantities. In this context, Lerch's theorem (Widder 1946) states that the interrelation between the set of values $\{\hat{F}(\mathbf{r}, s_n)\}$ for $s_n = s_0 + nh$, with $s_0 \in \mathscr{R}$, $s_0 > 0$, $h \in \mathscr{R}$, $h > 0$ and $n = 0, 1, 2, \ldots$, (i.e. at a set of equidistant points on the positive, real $s$-axis) and $F(\mathbf{r}, t)$ for $t > 0$ is unique. In this respect it is mentioned that a numerical implementation based on this theorem has, in the realm of transient electromagnetic prospecting, recently been carried out by Lee *et al.* (1994). Upon transforming the electromagnetic field equations, the Laplace transform of the time derivative of the field quantities is also needed; for this, the rule that, assuming zero initial values, the time derivative $\partial_t F(\mathbf{r}, t)$ of $F$ transforms into $s\hat{F}(\mathbf{r}, s)$ applies.

## 2 FORMULATION OF THE ELECTROMAGNETIC-WAVE-PROPAGATION PROBLEM

The electromagnetic-wave-propagation problem is governed by the electromagnetic field equations for a lossless medium:

$$\nabla \times \mathbf{H} - \boldsymbol{\varepsilon} \cdot \partial_t \mathbf{E} = \mathbf{J}, \tag{2}$$

$$\nabla \times \mathbf{E} + \boldsymbol{\mu} \cdot \partial_t \mathbf{H} = -\mathbf{K}. \tag{3}$$

Let $\mathscr{D}$ be the spatial support of the source distributions; then, on account of the superposition principle and the time invariance of the medium, the generated wavefield at any receiver point $\mathbf{r}$ can be expressed in terms of the generating source distributions at any transmitting point $\mathbf{r}'$ through the time convolutions

$$\mathbf{E}(\mathbf{r}, t) = \int_{t'=0}^{\infty} dt' \int_{\mathbf{r}' \in \mathscr{D}} \mathscr{G}^{E,J}(\mathbf{r}, \mathbf{r}', t') \cdot \mathbf{J}(\mathbf{r}', t - t')\, dV + \int_{t'=0}^{\infty} dt' \int_{\mathbf{r}' \in \mathscr{D}} \mathscr{G}^{E,K}(\mathbf{r}, \mathbf{r}', t') \cdot \mathbf{K}(\mathbf{r}', t - t')\, dV, \tag{4}$$

$$\mathbf{H}(\mathbf{r}, t) = \int_{t'=0}^{\infty} dt' \int_{\mathbf{r}' \in \mathscr{D}} \mathscr{G}^{H,J}(\mathbf{r}, \mathbf{r}', t') \cdot \mathbf{J}(\mathbf{r}', t - t')\, dV + \int_{t'=0}^{\infty} dt' \int_{\mathbf{r}' \in \mathscr{D}} \mathscr{G}^{H,K}(\mathbf{r}, \mathbf{r}', t') \cdot \mathbf{K}(\mathbf{r}', t - t')\, dV, \tag{5}$$

in which the wavefield Green's tensor functions of rank two, $\mathscr{G} = \mathscr{G}(\mathbf{r}, \mathbf{r}', t)$, relate a certain point-transmitter excitation at $\mathbf{r}'$ and $t = 0$ to a certain point-receiver response at $\mathbf{r}$, and $t > 0$.

### 2.1 Time-Laplace-transform relations

The time-Laplace-transform counterparts of eqs (2) and (3) are

$$\nabla \times \hat{\mathbf{H}} - s\boldsymbol{\varepsilon} \cdot \hat{\mathbf{E}} = \hat{\mathbf{J}}, \tag{6}$$

$$\nabla \times \hat{\mathbf{E}} + s\boldsymbol{\mu} \cdot \hat{\mathbf{H}} = -\hat{\mathbf{K}}. \tag{7}$$

Using the property that the Laplace transform of the convolution of two functions is the product of their Laplace transforms, the time-Laplace-transform counterparts of eqs (4) and (5) are

$$\hat{\mathbf{E}}(\mathbf{r}, s) = \int_{\mathbf{r}' \in \mathscr{D}} \hat{\mathscr{G}}^{E,J}(\mathbf{r}, \mathbf{r}', s) \cdot \hat{\mathbf{J}}(\mathbf{r}', s)\, dV + \int_{\mathbf{r}' \in \mathscr{D}} \hat{\mathscr{G}}^{E,K}(\mathbf{r}, \mathbf{r}', s) \cdot \hat{\mathbf{K}}(\mathbf{r}', s)\, dV, \tag{8}$$

$$\hat{\mathbf{H}}(\mathbf{r}, s) = \int_{\mathbf{r}' \in \mathscr{D}} \hat{\mathscr{G}}^{H,J}(\mathbf{r}, \mathbf{r}', s) \cdot \hat{\mathbf{J}}(\mathbf{r}', s)\, dV + \int_{\mathbf{r}' \in \mathscr{D}} \hat{\mathscr{G}}^{H,K}(\mathbf{r}, \mathbf{r}', s) \cdot \hat{\mathbf{K}}(\mathbf{r}', s)\, dV. \tag{9}$$

## 3 FORMULATION OF THE ELECTROMAGNETIC-DIFFUSION PROBLEM

The electromagnetic-diffusion problem is governed by the electromagnetic field equations in which the electric displacement current is neglected, viz

$$\nabla \times \mathbf{H} - \boldsymbol{\sigma} \cdot \mathbf{E} = \mathbf{J}, \tag{10}$$

$$\nabla \times \mathbf{E} + \boldsymbol{\mu} \cdot \partial_t \mathbf{H} = -\mathbf{K}. \tag{11}$$

If $\mathcal{D}$ is the spatial support of the source distributions, then, on account of the superposition principle and the time invariance of the medium, the generated diffusive field at any receiver point $\mathbf{r}$ can be expressed in terms of the generating source distributions at any transmitting point $\mathbf{r}'$ through the time convolutions

$$\mathbf{E}(\mathbf{r}, t) = \int_{t'=0}^{\infty} dt' \int_{\mathbf{r}' \in \mathcal{D}} \boldsymbol{\Gamma}^{E,J}(\mathbf{r}, \mathbf{r}', t') \cdot \mathbf{J}(\mathbf{r}', t - t')\, dV + \int_{t'=0}^{\infty} dt' \int_{\mathbf{r}' \in \mathcal{D}} \boldsymbol{\Gamma}^{E,K}(\mathbf{r}, \mathbf{r}', t') \cdot \mathbf{K}(\mathbf{r}', t - t')\, dV, \tag{12}$$

$$\mathbf{H}(\mathbf{r}, t) = \int_{t'=0}^{\infty} dt' \int_{\mathbf{r}' \in \mathcal{D}} \boldsymbol{\Gamma}^{H,J}(\mathbf{r}, \mathbf{r}', t') \cdot \mathbf{J}(\mathbf{r}', t - t')\, dV + \int_{t'=0}^{\infty} dt' \int_{\mathbf{r}' \in \mathcal{D}} \boldsymbol{\Gamma}^{H,K}(\mathbf{r}, \mathbf{r}', t') \cdot \mathbf{K}(\mathbf{r}', t - t')\, dV, \tag{13}$$

in which the diffusive-field Green's tensor functions of rank two, $\boldsymbol{\Gamma} = \boldsymbol{\Gamma}(\mathbf{r}, \mathbf{r}', t)$, relate a certain point-transmitter excitation at $\mathbf{r}'$ and $t = 0$ to a certain point-receiver response at $\mathbf{r}$ and $t > 0$.

### 3.1 Time-Laplace-transform relations

The time-Laplace-transform counterparts of eqs (10) and (11) are

$$\nabla \times \hat{\mathbf{H}} - \boldsymbol{\sigma} \cdot \hat{\mathbf{E}} = \hat{\mathbf{J}}, \tag{14}$$

$$\nabla \times \hat{\mathbf{E}} + s\boldsymbol{\mu} \cdot \hat{\mathbf{H}} = -\hat{\mathbf{K}}. \tag{15}$$

Using the property that the Laplace transform of the convolution of two functions is the product of their Laplace transforms, the time-Laplace-transform counterparts of eqs (12) and (13) are

$$\hat{\mathbf{E}}(\mathbf{r}, s) = \int_{\mathbf{r}' \in \mathcal{D}} \hat{\boldsymbol{\Gamma}}^{E,J}(\mathbf{r}, \mathbf{r}', s) \cdot \hat{\mathbf{J}}(\mathbf{r}', s)\, dV + \int_{\mathbf{r}' \in \mathcal{D}} \hat{\boldsymbol{\Gamma}}^{E,K}(\mathbf{r}, \mathbf{r}', s) \cdot \hat{\mathbf{K}}(\mathbf{r}', s) \cdot \hat{\mathbf{K}}(\mathbf{r}', s)\, dV, \tag{16}$$

$$\hat{\mathbf{H}}(\mathbf{r}, s) = \int_{\mathbf{r}' \in \mathcal{D}} \hat{\boldsymbol{\Gamma}}^{H,J}(\mathbf{r}, \mathbf{r}', s) \cdot \hat{\mathbf{J}}(\mathbf{r}', s)\, dV + \int_{\mathbf{r}' \in \mathcal{D}} \hat{\boldsymbol{\Gamma}}^{H,K}(\mathbf{r}, \mathbf{r}', s) \cdot \hat{\mathbf{K}}(\mathbf{r}', s) \cdot \hat{\mathbf{K}}(\mathbf{r}', s)\, dV. \tag{17}$$

## 4 THE CORRESPONDENCE PRINCIPLE

For the correspondence principle to hold, the permittivity in the wave-propagation case and the conductivity in the diffusive case are related by $\boldsymbol{\sigma}(\mathbf{r}) = \alpha\boldsymbol{\varepsilon}(\mathbf{r})$, where $\alpha$ is an arbitrary positive constant, with the dimensions of reciprocal time, while the permeabilities in the two cases are the same. The first step in the derivation of the correspondence principle is to substitute these relations into eqs (14) and (15), multiply eq. (14) by $(s/\alpha)^{1/2}$, and rewrite the resulting equations as

$$\nabla \times [(s/\alpha)^{1/2}\hat{\mathbf{H}}] - (\alpha s)^{1/2}\boldsymbol{\varepsilon} \cdot \hat{\mathbf{E}} = (s/\alpha)^{1/2}\hat{\mathbf{J}}, \tag{18}$$

$$\nabla \times \hat{\mathbf{E}} + (\alpha s)^{1/2}\boldsymbol{\mu} \cdot [(s/\alpha)^{1/2}\hat{\mathbf{H}}] = -\hat{\mathbf{K}}. \tag{19}$$

Comparing eqs (18)–(19) with eqs (6)–(7), it is observed that the diffusive equations in their rewritten form arise from the wave-propagation equations upon replacing $\hat{\mathbf{H}}$ in the latter by $(s/\alpha)^{1/2}\hat{\mathbf{H}}$, $\hat{\mathbf{J}}$ by $(s/\alpha)^{1/2}\hat{\mathbf{J}}$ and $s$ by $(\alpha s)^{1/2}$, while leaving $\hat{\mathbf{E}}$, $\boldsymbol{\varepsilon}$, $\boldsymbol{\mu}$ and $\hat{\mathbf{K}}$ as they are. Using, next, eqs (8)–(9) and (16)–(17), together with the uniqueness of the solutions of the time-Laplace-transform electromagnetic equations in the two cases, it follows by inspection that

$$\hat{\boldsymbol{\Gamma}}^{E,J}(\mathbf{r}, \mathbf{r}', s) = (s/\alpha)^{1/2}\hat{\mathcal{G}}^{E,J}[\mathbf{r}, \mathbf{r}', (\alpha s)^{1/2}], \tag{20}$$

$$\hat{\boldsymbol{\Gamma}}^{H,J}(\mathbf{r}, \mathbf{r}', s) = \hat{\mathcal{G}}^{H,J}[\mathbf{r}, \mathbf{r}', (\alpha s)^{1/2}], \tag{21}$$

$$\hat{\boldsymbol{\Gamma}}^{E,K}(\mathbf{r}, \mathbf{r}', s) = \hat{\mathcal{G}}^{E,K}[\mathbf{r}, \mathbf{r}', (\alpha s)^{1/2}], \tag{22}$$

$$\hat{\boldsymbol{\Gamma}}^{H,K}(\mathbf{r}, \mathbf{r}', s) = (s/\alpha)^{-1/2}\hat{\mathcal{G}}^{H,K}[\mathbf{r}, \mathbf{r}', (\alpha s)^{1/2}]. \tag{23}$$

To obtain the time-domain expressions for each $\boldsymbol{\Gamma}$, the Schouten–Van der Pol theorem in the theory of Laplace transformation is applied. This theorem relates time-domain results that are associated with the replacement of the Laplace-transform parameter $s$ by a function of $s$, subject to some restrictions. For the present case, the result for the replacement of $s$ by $(\alpha s)^{1/2}$ is needed. Using eqs (A6), (A9) and (A10) from Appendix A, it is found that

$$\boldsymbol{\Gamma}^{E,H|J,K}(\mathbf{r}, \mathbf{r}', t) = \left[\int_{\tau=0}^{\infty} W^{E,H|J,K}(t, \tau, \alpha)\mathcal{G}^{E,H|J,K}(\mathbf{r}, \mathbf{r}', \tau)\, d\tau\right] H(t), \tag{24}$$

where the intervening kernel functions $W^{E,H|J,K}$ are given by

$$W^{E,J} = \frac{1}{2}\left(\frac{1}{\alpha\pi}\right)^{1/2} \frac{1}{t^{3/2}}\left(\frac{\alpha\tau^2}{2t} - 1\right)\exp\left(-\frac{\alpha\tau^2}{4t}\right)H(t), \tag{25}$$

$$W^{H,J} = \frac{1}{2}\left(\frac{\alpha}{\pi}\right)^{1/2} \frac{\tau}{t^{3/2}}\exp\left(-\frac{\alpha\tau^2}{4t}\right)H(t), \tag{26}$$

$$W^{E,K} = \frac{1}{2}\left(\frac{\alpha}{\pi}\right)^{1/2} \frac{\tau}{t^{3/2}}\exp\left(-\frac{\alpha\tau^2}{4t}\right)H(t), \tag{27}$$

$$W^{H,K} = \left(\frac{\alpha}{\pi t}\right)^{1/2} \exp\left(-\frac{\alpha\tau^2}{4t}\right)H(t), \tag{28}$$

where the Heaviside unit step function $H$ has been included for reasons of clarity, since the kernel functions are singular at $t = 0$. Note that $W^{H,J} = W^{E,K}$, as it should be on account of reciprocity. Furthermore,

$$W^{E,J} = -\frac{1}{\alpha}\frac{\partial W^{H,J}(t, \tau, \alpha)}{\partial \tau} \tag{29}$$

and

$$W^{E,K} = -\frac{1}{\alpha}\frac{\partial W^{H,K}(t, \tau, \alpha)}{\partial \tau}. \tag{30}$$

Since the kernel functions given by eqs (25)–(28) are smooth functions of their arguments and the integration operation is a smoothing operation, the Green's functions pertaining to the diffusive case are always smoothed versions of the ones pertaining to the wave-propagation case. As a consequence, the operation of going from the wave-propagation case to the diffusive case is always a stable one. To perform the operation from the diffusive case to the wave-propagation case, the integral equation (24), with the kernels given by eqs (25)–(28), have to be solved.

## 5 MEDIUM-PARAMETER SCALING LAW FOR THE ELECTROMAGNETIC-WAVE PROBLEM

If the distribution of the permeability in the diffusive problem is, by a constant scale factor, off from the one in the wave-propagation problem, a scaling up to the value pertaining to the diffusive problem has to be carried out prior to the change from permittivity to conductivity for the correspondence principle. The relevant scaling falls within the wider class of a global scaling of both permittivity and permeability, as will be shown in this section.

Consider the electromagnetic-wave-propagation problem that is governed by the electromagnetic field equations for a lossless medium:

$$\nabla \times \mathbf{H} - \alpha \boldsymbol{\varepsilon} \cdot \partial_t \mathbf{E} = \mathbf{J}, \tag{31}$$

$$\nabla \times \mathbf{E} + \beta \boldsymbol{\mu} \cdot \partial_t \mathbf{H} = -\mathbf{K}, \tag{32}$$

in which $\alpha$ and $\beta$ are arbitrary positive scalar constants, and $\{\mathbf{E}, \mathbf{H}\} = \{\mathbf{E}, \mathbf{H}\}(\mathbf{r}, t; \alpha, \beta)$ denotes their solution. Let $\mathscr{D}$ be the spatial support of the source distributions; then, on account of the superposition principle and the time invariance of the medium, the generated wavefield at any receiver point $\mathbf{r}$ can be expressed in terms of the generating source distributions at any transmitting point $\mathbf{r}'$ through the time convolutions

$$\mathbf{E}(\mathbf{r}, t; \alpha, \beta) = \int_{t'=0}^{\infty} dt' \int_{\mathbf{r}' \in \mathscr{D}} \mathscr{G}^{E,J}(\mathbf{r}, \mathbf{r}', t'; \alpha, \beta) \cdot \mathbf{J}(\mathbf{r}', t - t')\, dV + \int_{t'=0}^{\infty} dt' \int_{\mathbf{r}' \in \mathscr{D}} \mathscr{G}^{E,K}(\mathbf{r}, \mathbf{r}', t'; \alpha, \beta) \cdot \mathbf{K}(\mathbf{r}', t - t')\, dV, \tag{33}$$

$$\mathbf{H}(\mathbf{r}, t; \alpha, \beta) = \int_{t'=0}^{\infty} dt' \int_{\mathbf{r}' \in \mathscr{D}} \mathscr{G}^{H,J}(\mathbf{r}, \mathbf{r}', t'; \alpha, \beta) \cdot \mathbf{J}(\mathbf{r}', t - t')\, dV + \int_{t'=0}^{\infty} dt' \int_{\mathbf{r}' \in \mathscr{D}} \mathscr{G}^{H,K}(\mathbf{r}, \mathbf{r}', t'; \alpha, \beta) \cdot \mathbf{K}(\mathbf{r}', t - t')\, dV, \tag{34}$$

in which $\mathscr{G} = \mathscr{G}(\mathbf{r}, \mathbf{r}', t; \alpha, \beta)$ are the relevant Green's tensor functions of rank two. To derive the scaling law, eq. (31) is multiplied by $\beta^{1/2}$ and eq. (32) by $\alpha^{1/2}$, after which the resulting equations are rewritten as

$$\nabla \times (\beta^{1/2}\mathbf{H}) - (\beta\alpha)^{1/2} \boldsymbol{\varepsilon} \cdot \partial_t (\alpha^{1/2}\mathbf{E}) = \beta^{1/2}\mathbf{J}, \tag{35}$$

$$\nabla \times (\alpha^{1/2}\mathbf{E}) + (\alpha\beta)^{1/2} \boldsymbol{\mu} \cdot \partial_t (\beta^{1/2}\mathbf{H}) = -\alpha^{1/2}\mathbf{K}. \tag{36}$$

These equations arise from their counterparts for the values $\alpha = 1$ and $\beta = 1$ on replacing $\mathbf{E}$ by $\alpha^{1/2}\mathbf{E}$, $\mathbf{H}$ by $\beta^{1/2}\mathbf{H}$, $\mathbf{J}$ by $\beta^{1/2}\mathbf{J}$, $\mathbf{K}$ by $\alpha^{1/2}\mathbf{K}$ and $t$ by $(\alpha\beta)^{-1/2}t$, while leaving $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$ as they are. In view of the uniqueness of the solutions to the electromagnetic-wavefield equations, it then follows by inspection that

$$\mathscr{G}^{E,J}(\mathbf{r}, \mathbf{r}', t; \alpha, \beta) = (\beta/\alpha)^{1/2} \mathscr{G}^{E,J}[\mathbf{r}, \mathbf{r}', (\alpha\beta)^{-1/2}t; 1, 1], \tag{37}$$

$$\mathscr{G}^{H,J}(\mathbf{r}, \mathbf{r}', t; \alpha, \beta) = \mathscr{G}^{H,J}[\mathbf{r}, \mathbf{r}', (\alpha\beta)^{-1/2}t; 1, 1], \tag{38}$$

$$\mathscr{G}^{E,K}(\mathbf{r}, \mathbf{r}', t; \alpha, \beta) = \mathscr{G}^{E,K}[\mathbf{r}, \mathbf{r}', (\alpha\beta)^{-1/2}t; 1, 1], \tag{39}$$

$$\mathscr{G}^{H,K}(\mathbf{r}, \mathbf{r}', t; \alpha, \beta) = (\alpha/\beta)^{1/2} \mathscr{G}^{H,K}[\mathbf{r}, \mathbf{r}', (\alpha\beta)^{-1/2}t; 1, 1]. \tag{40}$$

Eqs (37)–(40) are the desired scaling laws.

## 6 CONCLUSIONS

A correspondence theorem has been derived for time-domain electromagnetic fields in arbitrarily inhomogeneous and anisotropic media. It relates the wavefields that are present in a lossless medium, where the permittivity and the permeability are the constitutive parameters, with the diffusive fields that are present in a conductive medium, where the electric displacement current is neglected and the conductivity and the permeability are the constitutive parameters. The relationship is expressed through the time-domain Green's tensors for the two cases, which yield the point-receiver response (electric or magnetic field) due to a point-transmitter excitation (electric- or magnetic-current dipole source). The theorem enables one to compare quantitatively the potentialities of the two types of fields in view of their application in transient electromagnetic geophysical prospecting.

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## APPENDIX A: THE SCHOUTEN–VAN DER POL THEOREM FOR THE REPLACEMENT OF $s$ BY $(\alpha s)^{1/2}$

Since the Schouten–Van der Pol theorem applies to time behaviour only, the spatial arguments in the functions involved will be omitted in this Appendix. Let $g = g(t)$ be a known, causal function of time $t$ with support $\{t \in \mathscr{R}, t > 0\}$, and let

$$\hat{g}(s) = \int_{\tau=0}^{\infty} \exp(-s\tau) g(\tau)\, d\tau \tag{A1}$$

be its Laplace transform. Further, let

$$\hat{\gamma}(s; \alpha) = \hat{g}[(\alpha s)^{1/2}]; \tag{A2}$$

then,

$$\hat{\gamma}(s; \alpha) = \int_{\tau=0}^{\infty} \exp[-(\alpha s)^{1/2}\tau] g(\tau)\, d\tau. \tag{A3}$$

To arrive at the time-domain counterpart $\gamma = \gamma(t; \alpha)$ of $\hat{\gamma} = \hat{\gamma}(s; \alpha)$, it is observed that

$$\exp[-(\alpha s)^{1/2}\tau] = \int_{t=0}^{\infty} \exp(-st) w(t, \tau; \alpha)\, dt \tag{A4}$$

(*cf.* Abramowitz & Stegun 1965), in which

$$w(t, \tau; \alpha) = \frac{1}{2}\left(\frac{\alpha}{\pi}\right)^{1/2} \frac{\tau}{t^{3/2}} \exp\left(-\frac{\alpha\tau^2}{4t}\right) H(t), \tag{A5}$$

where the Heaviside unit step function $H$ has been included for reasons of clarity since the relevant kernel function is singular at $t = 0$. Substituting eq. (A4) into eq. (A3), interchanging the order of integration and using Lerch's theorem on the uniqueness of the one-sided Laplace transformation (Widder 1946), it follows that

$$\gamma(t; \alpha) = \left[\int_{\tau=0}^{\infty} w(t, \tau; \alpha) g(\tau)\, d\tau\right] H(t). \tag{A6}$$

In the analysis the time-domain counterparts of $(s/\alpha)^{1/2}\hat{\gamma}(s; \alpha)$ and $(s/\alpha)^{-1/2}\hat{\gamma}(s; \alpha)$ are also needed. To obtain these, it is observed that

$$(s/\alpha)^{1/2} \exp[-(\alpha s)^{1/2}\tau] = -\frac{1}{\alpha}\frac{\partial}{\partial\tau}(\exp[-(\alpha s)^{1/2}\tau]) \tag{A7}$$

and

$$(s/\alpha)^{-1/2} \exp[-(\alpha s)^{1/2}\tau] = \alpha \int_{\tau'=\tau}^{\infty} \exp[-(\alpha s)^{1/2}\tau']\, d\tau'. \tag{A8}$$

Using these relations, eqs (A6) and (A7) result in

$$\begin{aligned}(s/\alpha)^{1/2} \exp[-(\alpha s)^{1/2}\tau] &\Rightarrow -\frac{1}{\alpha}\frac{\partial w(t, \tau; \alpha)}{\partial\tau} \\ &= \frac{1}{2}\left(\frac{1}{\alpha\pi}\right)^{1/2} \frac{1}{t^{3/2}}\left(\frac{\alpha\tau^2}{2t} - 1\right) \\ &\quad \times \exp\left(-\frac{\alpha\tau^2}{4t}\right) H(t),\end{aligned} \tag{A9}$$

while eqs (A6) and (A8) result in

$$\begin{aligned}(s/\alpha)^{-1/2} \exp[-(\alpha s)^{1/2}\tau] &\Rightarrow \alpha \int_{\tau'=\tau}^{\infty} w(t, \tau'; \alpha)\, d\tau' \\ &= \left(\frac{\alpha}{\pi t}\right)^{1/2} \exp\left(-\frac{\alpha\tau^2}{4t}\right) H(t).\end{aligned} \tag{A10}$$

Eqs (A6), (A9) and (A10) are used in the main text.