Electromagnetic theory – Structure of the physics of the omnipervious

by

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Synopsis:

- Introduction
- Historical highlights
- Maxwell fields (tensorial structure)
- Maxwell equations and field compatibility relations (topological structure)
- EM field strengths and flux densities
- Maxwell fields in vacuum and matter
- Subsequent parts of the theory

Electromagnetic (EM) fields are **omnipervious**:

- **permeate into all matter**
- **permeate into empty space (vacuum)**

EM fields behave as **waves** traveling at a **finite wavespeed**

- **Electromagnetic wavespeed in vacuum:**
  - $c_0 = 299792458$ **m/s (exactly, SI)**

EM fields carry

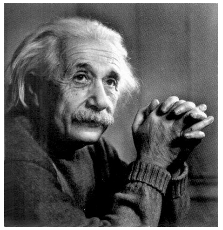
- **energy & information**

Principal actors in EM theory:

J. C. Maxwell
(1831 – 1879)

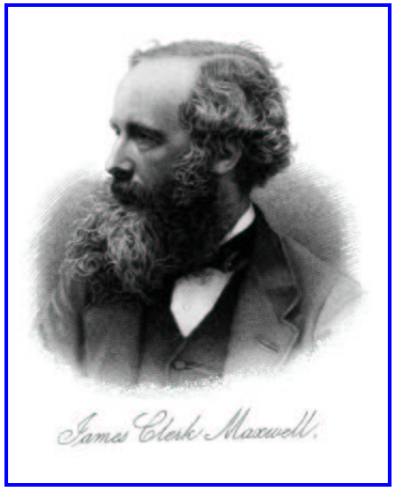
O. Heaviside
(1850 – 1925)

H. A. Lorentz
(1853 – 1928)



A. Einstein
(1879 – 1955)

- **Maxwell, James Clerk (1831–1879)**
- **Heaviside, Oliver (1850–1925)**
- **Lorentz, Hendrik Antoon (1853–1928)**
- **Einstein, Albert (1879–1955)**



J. C. Maxwell
(1831 – 1879)

Maxwell, J. C., *A Treatise on Electricity and Magnetism*, Oxford, Clarendon Press, Vols. I & II, 1873.

⟹

- Unifies the description of electric and magnetic phenomena (on a laboratory scale) into a single, ElectroMagnetic Theory
- Concludes, on theoretical grounds, that 'Light' must be an electromagnetic (wave) phenomenon

James Clerk Maxwell (1831–1879)

O. Heaviside
(1850 – 1925)

Heaviside, O., *Electromagnetic Theory*, New York, Van Nostrand & Co, Vols. I, II & III, 1893.

⟹

- Brings to attention the parallel structure of Maxwell's equations and the 'telegraphist's equations' of electric transmission line theory
- Casts Maxwell's equations into their 'Engineering form'
- Invents his 'Operational Calculus'

Oliver Heaviside (1850–1925)

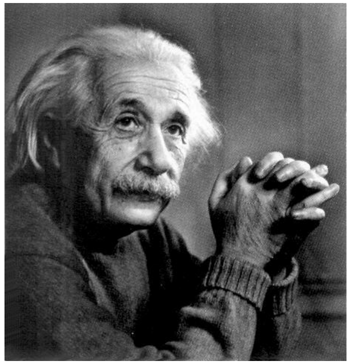
H. A. Lorentz
(1853 – 1928)

Lorentz, H. A., *The Theory of Electrons and its Applications to the Phenomena of Light and Radiant Heat*, Leipzig, Teubner, 1909.

⟹

- Extrapolates Maxwell's equations to moving matter ('Lorentz transformation')
- Extrapolates the concept of EM field action to the atomic scale
- **BUT:** The procedure fails to explain quantum phenomena

Hendrik Antoon Lorentz (1853–1928)



A. Einstein
(1879 – 1955)

Einstein, A., *The Meaning of Relativity*, Princeton University Press, 5th. ed., 2004.

⟹

- 'Any physical field quantity = (Geometrical) tensor quantity'
- Extrapolates the concept of EM field action to the cosmological scale
- **BUT:** The procedure fails to explain gravity phenomena

Albert Einstein (1879–1955)

- **EM fields are 'states' in space-time:** $\mathbb{R}^3 \times \mathbb{R}$
- **Every observer decomposes space-time into (his/her) SPACE** $(\mathbb{R}^3)$ $\times$ **(his/her) TIME** $(\mathbb{R})$
- **Every observer employs**
  - **(his/her)** $\{x_1, x_2, x_3\} \in \mathbb{R}^3 =$ Cartesian position coordinates
  - **(his/her)** $t \in \mathbb{R} =$ time coordinate
- **Observers at rest with respect to each other employ the same space** $\times$ **time decomposition**
- **Observers moving with respect to each other employ each their own space** $\times$ **time decomposition**

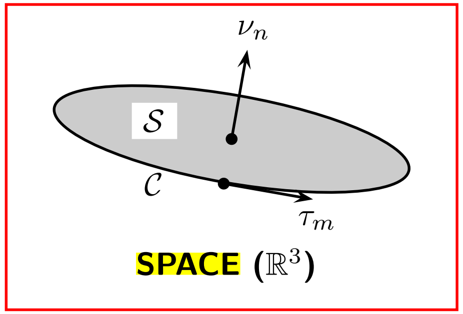
EINSTEIN:

- **All observable quantities in physics are TENSORS (TENSOR = array of N^k quantities, N = dimension of space, k = rank of the tensor; $k = 0 \Longrightarrow$ scalar, $k = 1 \Longrightarrow$ vector)**
- **All laws of physics are tensorial relations between tensorial quantities**

TENSORS:

- **Notationally, tensors are SUBSCRIPTED VARIABLES with number of subscripts = rank of tensor**
- **In a product of tensors a REPEATED SUBSCRIPT indicates SUMMATION over the range of that subscript (EINSTEIN summation convention); e.g.: $a_i b_{i,j} \Longleftrightarrow \sum_{i=1}^{N} a_i b_{i,j}$ for $j = 1, \ldots, N$**

EM field topology in space-time



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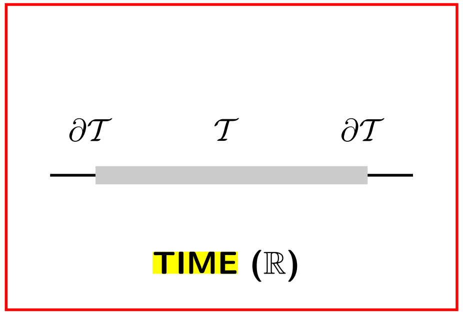


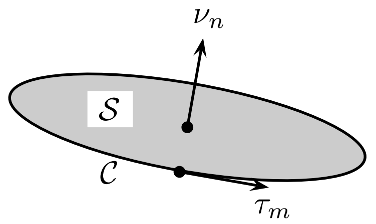
$\epsilon_{n,p,m}$ = Levi-Civita tensor

- $\mathcal{C} \subset \mathbb{R}^3$: Bounded, oriented, closed curve with piecewise continuously turning unit vector along its tangent τ_m
- $\mathcal{S} \subset \mathbb{R}^3$: Bounded, oriented, surface with piecewise continuously turning unit vector along its normal ν_n and $\mathcal{C}$ as its boundary curve

Orientation $\Longrightarrow$ • $\epsilon_{n,p,m}\nu_n\xi_p\tau_m = \begin{vmatrix} \nu_1 & \nu_2 & \nu_3 \\ \xi_1 & \xi_2 & \xi_3 \\ \tau_1 & \tau_2 & \tau_3 \end{vmatrix} > 0, \ \xi_p = x_p^{\mathcal{C}} - x_p^{\mathcal{S}}$ **(right-handed)**

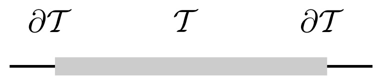
- $\mathcal{T} \subset \mathbb{R}$: Time interval with boundary $\partial\mathcal{T} \subset \mathbb{R}^0$

EM field topology in space-time



SPACE ($\mathbb{R}^3$)

$\times$



TIME ($\mathbb{R}$)

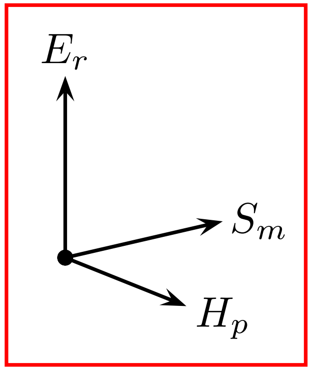
$\epsilon_{n,p,m}$ = Levi-Civita tensor

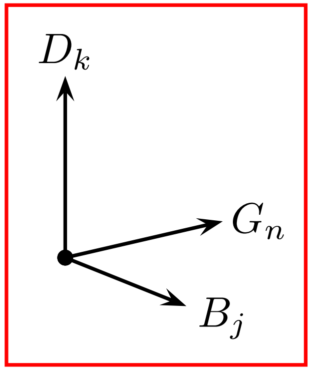
- $\oint_{\mathcal{C}} [\ldots]\tau_m \mathrm{d}s =$ **circulation of $[\ldots]$ along $\mathcal{C}$**
- $\int_{\mathcal{S}} [\ldots]\nu_m \mathrm{d}A =$ **flux of $[\ldots]$ across $\mathcal{S}$**
- $\int_{\mathcal{T}} \partial_t [\ldots]\mathrm{d}t = [\ldots]\big|_{\partial\mathcal{T}}$

$\Longrightarrow$ **MAXWELL'S EQUATIONS mutually couple EM field circulations and flux densities**

EM field topology in space-time (contd.)

EM field quantities

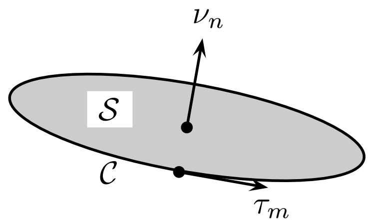




- E_r = electric field strength (V/m)
- H_p = magnetic field strength (A/m)
- D_k = electric flux density (C/m^2)
- B_j = magnetic flux density (T)
- $S_m = \epsilon_{m,r,p} E_r H_p$ = area density of EM power flow (W/m^2)
- $G_n = \epsilon_{n,k,j} D_k B_j$ = volume density of EM momentum (J·s/m^4)

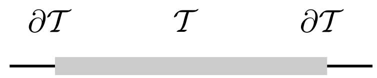
EM field strengths,flux densities, power flow and momentum

Global EM field relations for EM field topology and time interval $\mathcal{T} \subset \mathbb{R}$ with boundary points $\partial\mathcal{T}$:



SPACE ($\mathbb{R}^3$)

$\times$



TIME ($\mathbb{R}$)

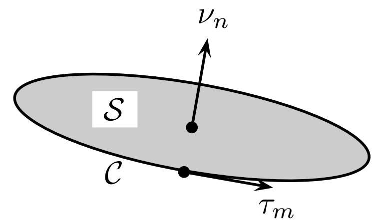
$\epsilon_{n,p,m} =$ Levi-Civita tensor

- $\int_{\mathcal{T}}$ [circulation of **magnetic field** along $\mathcal{C}$] $\mathrm{d}t =$ [**electric flux** across $\mathcal{S}$]$|_{\partial\mathcal{T}}$
- $\int_{\mathcal{T}}$ [circulation of **electric field** along $\mathcal{C}$] $\mathrm{d}t = -$ [**magnetic flux** across $\mathcal{S}$]$|_{\partial\mathcal{T}}$

Global EM field compatibility relations:

- outward electric flux across closed surface $= 0$
- outward magnetic flux across closed surface $= 0$

Global EM field relations for EM field topology and time interval $\mathcal{T} \subset \mathbb{R}$ with boundary points $\partial\mathcal{T}$:



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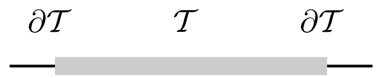

$\epsilon_{n,p,m}$ = Levi-Civita tensor

- $\int_{\mathcal{T}} \mathrm{d}t \oint_{\mathcal{C}} H_p \tau_p \mathrm{d}\lambda = \int_{\mathcal{S}} D_i \nu_i \mathrm{d}A \Big|_{\partial\mathcal{T}}$
- $\int_{\mathcal{T}} \mathrm{d}t \oint_{\mathcal{C}} E_r \tau_r \mathrm{d}\lambda = -\int_{\mathcal{S}} B_j \nu_j \mathrm{d}A \Big|_{\partial\mathcal{T}}$

Global EM compatibility relations:

- $\oint_{\text{closed surface}} D_i \nu_i \mathrm{d}A = 0$
- $\oint_{\text{closed surface}} B_j \nu_j \mathrm{d}A = 0$
- ν_j = unit vector along outward normal

Global EM field and compatibility relations

The 2 EM field equations yield $2 \times 3 = 6$ linearly independent relations, with $4 \times 3 = 12$ unknowns $\Longrightarrow$

NUMBER OF UNKNOWNS = 2 * NUMBER OF EQUATIONS

$\Longrightarrow$ to be supplemented by **CONSTITUTIVE RELATIONS**

Local EM constitutive relations in vacuum (SI):

- $D_i(x_m,t) = \epsilon_0 E_i(x_m,t)$
- $B_j(x_m,t) = \mu_0 H_j(x_m,t)$
- $\mu_0 = 4\pi * 10^{-7}$ H/m (permeability of vacuum)
- $\epsilon_0 = 1/\mu_0 c_0^2$ F/m (permittivity of vacuum)
- $c_0 = 299792458$ m/s (EM wavespeed in vacuum)

$$\boldsymbol{\epsilon_0 \mu_0 c_0^2 = 1} \quad \textbf{(Maxwell)}$$

LORENTZ's theory of electrons (matter = contrast with vacuum):

- $D_i = \epsilon_0 E_i + P_i^{\rm ind} + P_i^{\rm ext}$
- $B_j = \mu_0 H_j + M_j^{\rm ind} + M_j^{\rm ext}$
- $P_i^{\rm ind}$ = (field-dependent) induced electric polarization (C/m^2)
  = atomic mechanical response to electric field excitation
  (electric force on electric charge)
- $P_i^{\rm ext}$ = (field-independent) external electric polarization (C/m^2)
- $M_j^{\rm ind}$ = (field-dependent) induced magnetic polarization (T)
  = atomic mechanical response to magnetic field excitation
  (magnetic force on orbital electric charge and torque on magnetic spin)
- $M_j^{\rm ext}$ = (field-independent) external magnetic polarization (T)

Macroscopic EM constitutive relations in matter (passive, field-dependent part):

- **Volume averaging** of the **(local, causal)** response of (classical or quantum) atomic models over $\mathcal{D}_{\in}(x_m)$ ('continuum hypothesis')
- $\mathcal{D}_{\in}(x_m)$ = representative elementary domain (x_m = barycenter of $\mathcal{D}_{\in}$)

$\Longrightarrow$

General:

- $\boxed{\{E_r, H_p\}}\,(x_m, t')|_{t' \in (-\infty < t' \leq t)} \longmapsto \boxed{\{P_i^{\text{ind}}, M_j^{\text{ind}}\}}\,(x_m, t)$

Most materials:

- $\boxed{E_r}(x_m, t')|_{t' \in (-\infty < t' \leq t)} \longmapsto \boxed{P_i^{\text{ind}}}(x_m, t)$
- $\boxed{H_p}(x_m, t')|_{t' \in (-\infty < t' \leq t)} \longmapsto \boxed{M_j^{\text{ind}}}(x_m, t)$

Subsequent parts of EM theory:

- **EM transfer of energy**
- **EM transfer of momentum**
- **Interaction of two different EM 'states': Reciprocity (Lorentz, 1896)**
- **Propagation of EM waves**
- **Radiation and Scattering of EM waves**
- **ElectroMagnetic Interference (EMI)**
- **ElectroMagnetic Compatibility (EMC)**
- **Computational Electromagnetics**