ElectroMagnetic (EM) field preliminaries to the observation and interpretation of Cosmic Microwave Background (CMB) radiation

by

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Synopsis:

- Introduction CMB radiation
- Maxwell - Lorentz ElectroMagnetic field equations for CMB radiation
- EM field and source quantities in CMB radiation
- EM moving particle source quantities in CMB radiation
- Configuration for analyzing CMB radiation
- Far-source approximation in CMB radiation
- Spectral representation of the CMB radiation field
- Observation with the Cosmic Background Imager (CBI)
- State of polarization of the observed field

- **CMB radiation** (equivalent black-body radiation temperature $T = 2.725$ K, spectral intensity $I(\nu)$ peaked at $\nu = 160.2$ GHz) is conjectured to be[1]
  - **ElectroMagnetic (EM) radiation** generated by accelerated motion of electrically charged and/or electrically polarized and/or (spin)magnetized particles (Lorentz theory of electrons, 1905), present in ranges of the universe with very low number density of particles

- **Radiation damping** is ascribed to **Thompson scattering:** EM field incident on particle $\longrightarrow$ EM force on particle $\longrightarrow$ acceleration of particle $\longrightarrow$ (Thompson scattered) EM field radiated by particle:
  - **$\boldsymbol{E}$-mode damping:** influence of electric force on particle predominates
  - **$\boldsymbol{B}$-mode damping:** influence of magnetic force (Lorentz force) on particle predominates (usually negligibly small)

- **CMB radiation** is observed and interpreted in **4D space-time** ($\mathbb{R}^3 \times \mathbb{R}$) with
  - **3D Euclidean space** $\mathbb{R}^3$
    - orthonormal Cartesian reference frame $\{\mathcal{O}, \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3\}$
    - position vector $\boldsymbol{x} = x_1\boldsymbol{i}_1 + x_2\boldsymbol{i}_2 + x_3\boldsymbol{i}_3 \in \mathbb{R}^3$
  - **1D time** $t \in \mathbb{R}$
- **Notation:**
  - $\boldsymbol{\partial_m}$ = differentiation with respect to $\boldsymbol{x_m}$ $\boldsymbol{(m = 1, 2, 3)}$
  - $\boldsymbol{\partial_t}$ = differentiation with respect to $\boldsymbol{t}$

- **Maxwell - Lorentz EM field equations for CMB radiation:**

- $$\begin{bmatrix} 0 & -\partial_3 & \partial_2 \\ \partial_3 & 0 & -\partial_1 \\ -\partial_2 & \partial_1 & 0 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} - \epsilon_0 \partial_t \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} + \partial_t \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

- $$\begin{bmatrix} 0 & -\partial_3 & \partial_2 \\ \partial_3 & 0 & -\partial_1 \\ -\partial_2 & \partial_1 & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} + \mu_0 \partial_t \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = -\mu_0 \partial_t \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}$$

- $\{\boldsymbol{E}, \boldsymbol{H}\}$ = **radiated EM field**
- $\{\boldsymbol{J}, \boldsymbol{P}, \boldsymbol{M}\}$ = **moving sources generating the field**
- μ_0 = (magnetic) permeability of vacuum [$\mu_0 = 4\pi * 10^{-7}$ H/m [(SI)]
- ϵ_0 = (electric) permittivity of vacuum [$\epsilon_0 = (\mu_0 c_0^2)^{-1}$ F/m (SI)]
- c_0 = electromagnetic wave speed in vacuum [$c_0 = 299792458$ m/s (SI)]

Maxwell (1873) - Lorentz (1905) ElectroMagnetic field equations for CMB radiation

- **EM field quantities in CMB radiation:**
  - $\boldsymbol{E}(\boldsymbol{x}, t) =$ electric field strength (V/m)
  - $\boldsymbol{H}(\boldsymbol{x}, t) =$ magnetic field strength (A/m)
- **EM source quantities in CMB radiation:**
  - $\boldsymbol{J}(\boldsymbol{x}, t) =$ volume density of electric convection current ($\mathrm{A/m^2}$)
  - $\boldsymbol{P}(\boldsymbol{x}, t) =$ volume density of electric polarization ($\mathrm{C/m^2}$)
  - $\boldsymbol{M}(\boldsymbol{x}, t) =$ volume density of (spin) magnetization (A/m)

- **EM moving particle source quantities in CMB radiation:**[2]
  - $\boldsymbol{x}_0(t) =$ position of particle at time t
  - $\boldsymbol{v}_0(t) =$ velocity of particle at time t $[\boldsymbol{v}_0(t) = \mathrm{d}_t\boldsymbol{x}_0(t)]$
  - $\delta^3(\boldsymbol{x}) =$ 3D Dirac delta distribution operative at $\boldsymbol{x} = \mathbf{0}$
    - $\delta^3(\boldsymbol{x}) = 0$ for $\boldsymbol{x} \neq \mathbf{0}$
    - $\int_{\boldsymbol{x}\in\mathbb{R}^3} \delta^3[\boldsymbol{x} - \boldsymbol{x}_0(t)] f(\boldsymbol{x}, t)\mathrm{d}V = f[\boldsymbol{x}_0(t), t]$ ("sifting property")
  - $\boldsymbol{J}(\boldsymbol{x}, t) = q(t)\boldsymbol{v}_0(t)\delta^3[\boldsymbol{x} - \boldsymbol{x}_0(t)]$
    - $q(t) =$ electric charge (C)
  - $\boldsymbol{P}(\boldsymbol{x}, t) = \boldsymbol{p}(t)\delta^3[\boldsymbol{x} - \boldsymbol{x}_0(t)]$
    - $\boldsymbol{p}(t) =$ electric dipole moment (C$\cdot$ m)
  - $\boldsymbol{M}(\boldsymbol{x}, t) = \boldsymbol{m}(t)\delta^3[\boldsymbol{x} - \boldsymbol{x}_0(t)]$
    - $\boldsymbol{m}(t) =$ magnetic spin moment (A$\cdot$ m^2)

- **Note that**

- $$\begin{bmatrix}\partial_1 & \partial_2 & \partial_3\end{bmatrix}\begin{bmatrix}0 & \partial_3 & -\partial_2\\ -\partial_3 & 0 & \partial_1\\ \partial_2 & -\partial_1 & 0\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\end{bmatrix}$$ **implies:**

- **EM compatibility relations**

- $$\begin{bmatrix}\partial_1 & \partial_2 & \partial_3\end{bmatrix}\begin{bmatrix}-\epsilon_0\partial_t E_1\\ -\epsilon_0\partial_t E_2\\ -\epsilon_0\partial_t E_3\end{bmatrix} = \begin{bmatrix}\partial_1 & \partial_2 & \partial_3\end{bmatrix}\begin{bmatrix}J_1 + \partial_t P_1\\ J_2 + \partial_t P_2\\ J_3 + \partial_t P_3\end{bmatrix}$$

- $$\begin{bmatrix}\partial_1 & \partial_2 & \partial_3\end{bmatrix}\begin{bmatrix}-\mu_0\partial_t H_1\\ -\mu_0\partial_t H_2\\ -\mu_0\partial_t H_3\end{bmatrix} = \begin{bmatrix}\partial_1 & \partial_2 & \partial_3\end{bmatrix}\begin{bmatrix}-\mu_0\partial_t M_1\\ -\mu_0\partial_t M_2\\ -\mu_0\partial_t M_3\end{bmatrix}$$

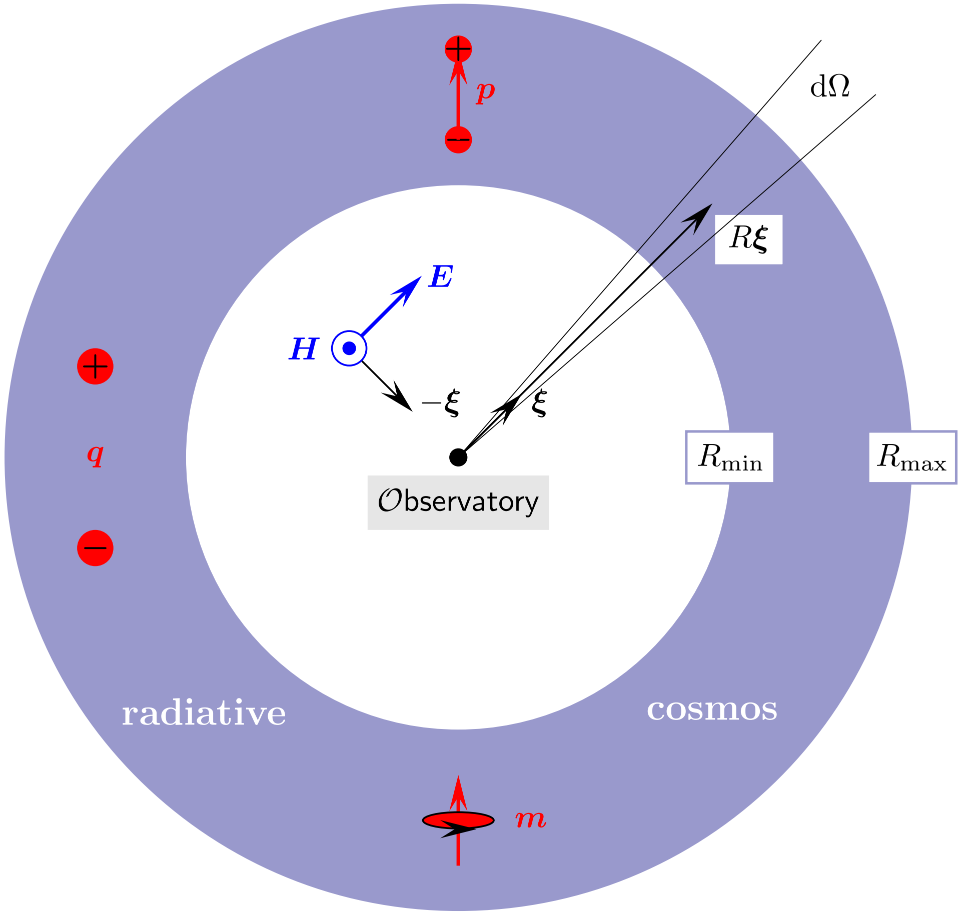

Fig. 1. Radiative cosmos with moving electric charges, electric dipoles and magnetic (spin) dipoles, and observation coordinates for CMB radiation.

Configuration for analyzing CMB radiation

- **Maxwell field calculation/computation in operator form:**
  - **G**: $\{\boldsymbol{J},\boldsymbol{P},\boldsymbol{M}\}(\boldsymbol{x}',t') \stackrel{\mathbf{G}}{\Longrightarrow} \{\boldsymbol{E},\boldsymbol{H}\}(\boldsymbol{x},t)$

$$\begin{aligned}\{\boldsymbol{J},\boldsymbol{P},\boldsymbol{M}\}(\boldsymbol{x}',t') &= \text{sources}\\ \{\boldsymbol{E},\boldsymbol{H}\}(\boldsymbol{x},t) &= \text{field}\\ \mathbf{G}(\boldsymbol{x},t;\boldsymbol{x}',t') &= \text{Maxwell field propagator}\end{aligned}$$

- **Far-source approximation:** • $c_0/|\boldsymbol{x}'-\boldsymbol{x}| \ll \partial_t \Longrightarrow \partial_m \simeq (\xi_m/c_0)\partial_t$
  - $\mathbf{G}^{\text{far}}$: $\{\boldsymbol{J},\boldsymbol{P},\boldsymbol{M}\}(\boldsymbol{x}',t') \stackrel{\mathbf{G}^{\text{far}}}{\Longrightarrow} \{\boldsymbol{E},\boldsymbol{H}\}^{\text{far}}(\boldsymbol{x},t)$

$$\begin{aligned}\{\boldsymbol{J},\boldsymbol{P},\boldsymbol{M}\}(\boldsymbol{x}',t') &= \text{sources}\\ \{\boldsymbol{E},\boldsymbol{H}\}^{\text{far}}(\boldsymbol{x},t) &= \text{field in far-source approximation}\\ \mathbf{G}^{\text{far}}(\boldsymbol{x},t;\boldsymbol{x}',t') &= \text{far-source Maxwell field propagator}\\ \xi_m &= \frac{\boldsymbol{x}'-\boldsymbol{x}}{|\boldsymbol{x}'-\boldsymbol{x}|} \rightarrow \frac{\boldsymbol{x}'}{|\boldsymbol{x}'|} \text{ as } |\boldsymbol{x}'| \rightarrow \infty\end{aligned}$$

• The CMB observed field (far-source approximation):

• $\{E_m, H_m\}(\boldsymbol{x}, t) = \dfrac{1}{4\pi}\displaystyle\int_{\boldsymbol{\xi}\in\Omega} \{E_m^\infty, H_m^\infty\}(\boldsymbol{\xi}, t)\mathrm{d}\Omega \quad (m = 1, 2, 3)$

• $$\begin{bmatrix} E_1^\infty \\ E_2^\infty \\ E_3^\infty \end{bmatrix} = \mu_0 \begin{bmatrix} \xi_1\xi_1 - 1 & \xi_1\xi_2 & \xi_1\xi_3 \\ \xi_2\xi_1 & \xi_2\xi_2 - 1 & \xi_2\xi_3 \\ \xi_3\xi_1 & \xi_3\xi_2 & \xi_3\xi_3 - 1 \end{bmatrix} \begin{bmatrix} \partial_t A_1 \\ \partial_t A_2 \\ \partial_t A_3 \end{bmatrix} + \frac{1}{c_0} \begin{bmatrix} 0 & -\xi_3 & \xi_2 \\ \xi_3 & 0 & -\xi_1 \\ -\xi_2 & \xi_1 & 0 \end{bmatrix} \begin{bmatrix} \partial_t F_1 \\ \partial_t F_2 \\ \partial_t F_3 \end{bmatrix}$$

• $$\begin{bmatrix} H_1^\infty \\ H_2^\infty \\ H_3^\infty \end{bmatrix} = \epsilon_0 \begin{bmatrix} \xi_1\xi_1 - 1 & \xi_1\xi_2 & \xi_1\xi_3 \\ \xi_2\xi_1 & \xi_2\xi_2 - 1 & \xi_2\xi_3 \\ \xi_3\xi_1 & \xi_3\xi_2 & \xi_3\xi_3 - 1 \end{bmatrix} \begin{bmatrix} \partial_t F_1 \\ \partial_t F_2 \\ \partial_t F_3 \end{bmatrix} - \frac{1}{c_0} \begin{bmatrix} 0 & -\xi_3 & \xi_2 \\ \xi_3 & 0 & -\xi_1 \\ -\xi_2 & \xi_1 & 0 \end{bmatrix} \begin{bmatrix} \partial_t A_1 \\ \partial_t A_2 \\ \partial_t A_3 \end{bmatrix}$$

• $A_m(\boldsymbol{\xi}, t) = \displaystyle\int_{R=R_{\min}}^{R_{\max}} [J_m(R\boldsymbol{\xi}, t - R/c_0) + \partial_t P_m(R\boldsymbol{\xi}, t - R/c_0)] R\mathrm{d}R \quad (m = 1, 2, 3)$

• $F_m(\boldsymbol{\xi}, t) = \displaystyle\int_{R=R_{\min}}^{R_{\max}} \partial_t M_m(R\boldsymbol{\xi}, t - R/c_0) R\mathrm{d}R \quad (m = 1, 2, 3)$

The CMB observed field (far-source approximation)

- **Far-source relationships EM field equations $[\partial_m \simeq (\xi_m/c_0)\partial_t]$:**

  - $$\begin{bmatrix} 0 & -\xi_3 & \xi_2 \\ \xi_3 & 0 & -\xi_1 \\ -\xi_2 & \xi_1 & 0 \end{bmatrix} \begin{bmatrix} H_1^\infty \\ H_2^\infty \\ H_3^\infty \end{bmatrix} - c_0\epsilon_0 \begin{bmatrix} E_1^\infty \\ E_2^\infty \\ E_3^\infty \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
  - $$\begin{bmatrix} 0 & -\xi_3 & \xi_2 \\ \xi_3 & 0 & -\xi_1 \\ -\xi_2 & \xi_1 & 0 \end{bmatrix} \begin{bmatrix} E_1^\infty \\ E_2^\infty \\ E_3^\infty \end{bmatrix} + c_0\mu_0 \begin{bmatrix} H_1^\infty \\ H_2^\infty \\ H_3^\infty \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- **Far-source relationships EM compatibility relations $[\partial_m \simeq (\xi_m/c_0)\partial_t]$:**

  - $$[\xi_1 \;\; \xi_2 \;\; \xi_3] \begin{bmatrix} E_1^\infty \\ E_2^\infty \\ E_3^\infty \end{bmatrix} = 0$$
  - $$[\xi_1 \;\; \xi_2 \;\; \xi_3] \begin{bmatrix} H_1^\infty \\ H_2^\infty \\ H_3^\infty \end{bmatrix} = 0$$

- **Spectral representation of electric and magnetic field strength (Fourier representation):**

  - $$\{\boldsymbol{E},\boldsymbol{H}\}(\boldsymbol{x},t) = \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} \exp(\mathrm{j}\omega t)\{\hat{\boldsymbol{E}},\hat{\boldsymbol{H}}\}(\boldsymbol{x},\mathrm{j}\omega)\mathrm{d}\omega$$
    ($\omega =$ angular frequency)
    $$= \int_{\nu=-\infty}^{\infty} \exp(\mathrm{j}2\pi\nu t)\{\hat{\boldsymbol{E}},\hat{\boldsymbol{H}}\}(\boldsymbol{x},\mathrm{j}2\pi\nu)\mathrm{d}\nu$$
    ($\nu =$ frequency $= \omega/2\pi$)

- **Fourier theorem:**

$$\{\hat{\boldsymbol{E}},\hat{\boldsymbol{H}}\}(\boldsymbol{x},\mathrm{j}\omega) = \int_{t=-\infty}^{\infty} \exp(-\mathrm{j}\omega t)\{\boldsymbol{E},\boldsymbol{H}\}(\boldsymbol{x},t)\mathrm{d}t$$

- **Since the field vectors are real-valued, also:**

$$\{\boldsymbol{E},\boldsymbol{H}\}(\boldsymbol{x},t) = \frac{1}{\pi}\int_{\boldsymbol{\omega}=\mathbf{0}}^{\boldsymbol{\infty}} \mathrm{Re}\left[\exp(\mathrm{j}\omega t)\{\hat{\boldsymbol{E}},\hat{\boldsymbol{H}}\}(\boldsymbol{x},\mathrm{j}\omega)\right]\mathrm{d}\omega$$

- **For any set of components of the electric and the magnetic field strength (Parseval's theorem):**
  - $$\int_{t=-\infty}^{\infty} E(\boldsymbol{x},t)H(\boldsymbol{x},t)\mathrm{d}t = \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} \hat{E}(\boldsymbol{x},\mathrm{j}\omega)\hat{H}(\boldsymbol{x},-\mathrm{j}\omega)\mathrm{d}\omega$$
    $$= \int_{\nu=-\infty}^{\infty} \hat{E}(\boldsymbol{x},\mathrm{j}2\pi\nu)\hat{H}(\boldsymbol{x},-\mathrm{j}2\pi\nu)\mathrm{d}\nu$$
    - $\hat{E}(\boldsymbol{x},\mathrm{j}\omega)\hat{H}(\boldsymbol{x},-\mathrm{j}\omega) =$ **spectral power density**

- **Since the field vectors are real-valued, also:**

$$\int_{t=-\infty}^{\infty} E(\boldsymbol{x},t)H(\boldsymbol{x},t)\mathrm{d}t = \frac{1}{\pi}\int_{\boldsymbol{\omega}=0}^{\boldsymbol{\infty}} \mathrm{Re}\left[\hat{E}(\boldsymbol{x},\mathrm{j}\omega)\hat{H}(\boldsymbol{x},-\mathrm{j}\omega)\right]\mathrm{d}\omega$$

Notation: Vectors are denoted by bold-face symbols, $\cdot$ denotes inner product,

- $|\boldsymbol{E}|^2 = \boldsymbol{E}\cdot\boldsymbol{E}$

- **Each spectral field component:**

  - $\boldsymbol{E}(\boldsymbol{x},t) = \mathrm{Re}[\hat{\boldsymbol{E}}(\boldsymbol{x},\mathrm{j}\omega)]\cos(\omega t) - \mathrm{Im}[\hat{\boldsymbol{E}}(\boldsymbol{x},\mathrm{j}\omega)]\sin(\omega t)$

  describes at position $\boldsymbol{x}$ in the plane through $\mathrm{Re}[\hat{\boldsymbol{E}}(\boldsymbol{x},\mathrm{j}\omega)]$ and $\mathrm{Im}[\hat{\boldsymbol{E}}(\boldsymbol{x},\mathrm{j}\omega)]$ a **closed curve** in the course of time over any cycle of duration $2\pi/\omega$.

- **In:**

  - $\boldsymbol{E}(\boldsymbol{x},t) = \boldsymbol{E}_X\cos(\omega t+\psi) + \boldsymbol{E}_Y\sin(\omega t+\psi)$

  $\{\boldsymbol{E}_X,\boldsymbol{E}_Y\}$ can be constructed such that

  - $\boldsymbol{E}_X\cdot\boldsymbol{E}_Y = 0 \quad (\boldsymbol{E}_X \perp \boldsymbol{E}_Y)$

- **Since:**
  - $\boldsymbol{E}_X = \mathrm{Re}(\hat{\boldsymbol{E}})\cos(\psi) + \mathrm{Im}(\hat{\boldsymbol{E}})\sin(\psi) \quad (\omega t = 0)$
  - $\boldsymbol{E}_Y = \mathrm{Re}(\hat{\boldsymbol{E}})\sin(\psi) - \mathrm{Im}(\hat{\boldsymbol{E}})\cos(\psi) \quad (\omega t = \pi/2)$
- $\tan(2\ \psi)$ is found as
  - $\tan(2\psi) = \dfrac{2\ \mathrm{Re}(\hat{\boldsymbol{E}})\cdot\mathrm{Im}(\hat{\boldsymbol{E}})}{|\mathrm{Re}(\hat{\boldsymbol{E}}|)|^2 - |\mathrm{Im}(\hat{\boldsymbol{E}})|^2}$
- **while:**
  - $\boldsymbol{E}\cdot\boldsymbol{E}_X = |\boldsymbol{E}_X|^2\cos(\omega t + \psi)$
  - $\boldsymbol{E}\cdot\boldsymbol{E}_Y = |\boldsymbol{E}_Y|^2\sin(\omega t + \psi)$
- $\Longrightarrow$
  - $\left(\dfrac{\boldsymbol{E}\cdot\boldsymbol{E}_X}{|\boldsymbol{E}_X|^2}\right)^2 + \left(\dfrac{\boldsymbol{E}\cdot\boldsymbol{E}_Y}{|\boldsymbol{E}_Y|^2}\right)^2 = 1 \quad \Longleftarrow$ **ellipse if** $|\boldsymbol{E}_X|^2 \neq |\boldsymbol{E}_Y|^2$

State of polarization (Spectral field component)(Contd.)

- **Semi major axis of ellipse (if** • $|\boldsymbol{E}_X|^2 \neq |\boldsymbol{E}_Y|^2$**) :**

  - $$|\boldsymbol{E}_X|^2 = \frac{1}{2}\left(|\mathrm{Re}(\hat{\boldsymbol{E}})|^2 + |\mathrm{Im}(\hat{\boldsymbol{E}})|^2\right) + \left[\frac{1}{4}\left(|\mathrm{Re}(\hat{\boldsymbol{E}})|^2 - |\mathrm{Im}(\hat{\boldsymbol{E}})|^2\right)^2 + \left(\mathrm{Re}(\hat{\boldsymbol{E}})\cdot\mathrm{Im}(\hat{\boldsymbol{E}})\right)^2\right]^{1/2}$$

- **Semi minor axis of ellipse (if** • $|\boldsymbol{E}_X|^2 \neq |\boldsymbol{E}_Y|^2$**) :**

  - $$|\boldsymbol{E}_Y|^2 = \frac{1}{2}\left(|\mathrm{Re}(\hat{\boldsymbol{E}})|^2 + |\mathrm{Im}(\hat{\boldsymbol{E}})|^2\right) + \left[\frac{1}{4}\left(|\mathrm{Re}(\hat{\boldsymbol{E}})|^2 - |\mathrm{Im}(\hat{\boldsymbol{E}})|^2\right)^2 - \left(\mathrm{Re}(\hat{\boldsymbol{E}})\cdot\mathrm{Im}(\hat{\boldsymbol{E}})\right)^2\right]^{1/2}$$

If: • $|\boldsymbol{E}_X|^2 = |\boldsymbol{E}_Y|^2$ **ellipse** $\Longrightarrow$ **circle**

If: • $\boldsymbol{E}_X \parallel \boldsymbol{E}_Y$ **ellipse** $\Longrightarrow$ **line**

[1] http//:en.wikipedia.org/wiki/ Cosmic_microwave_background_radiation

[2] De Hoop, A. T., Fields and waves excited by point sources in motion – The general 3D time-domain Doppler effect, *Wave Motion, 43, 2005, pp. 116-122*

[3] De Hoop, A. T., *Theorie van het elektromagnetische veld*, Delftse Universitaire Pers, 1975