ElectroMagnetic field theory - A modern tensor/array approach

by

Adrianus T. de Hoop

Delft University of Technology

Laboratory of Electromagnetic Research

Faculty of Electrical Engineering, Mathematics and Computer Science

Mekelweg 4 • 2628 CD Delft • the Netherlands

T: +31 15 2785203 / +31 15 2781034

E: a.t.dehoop@tudelft.nl **W:** www.atdehoop.com

Presentation given at:

Delft University of Technology, Delft, the Netherlands, 2011 April 19

Title and affiliation

SYNOPSIS:

- Introduction: Structure of the EM field equations
- Selected corollaries:
  - Radiation from sources
  - Field/source reciprocity
  - Interaction Maxwell fields $\Longleftrightarrow$ Kirchhoff circuits
- Applied EM topics in Electronics and IT:
  - Signal integrity in inter-/intra-system pulsed-field radiative signal transfer
  - Pulsed-field EM Interference (EMI) analysis
  - Time-domain (TD) regulatory EM Compatibility (EMC) specifications (TD emission limits, TD immunity limits)

PREREQUISITES:

- time differentiation ∂_t
- spatial differentiation $\partial_{\boldsymbol{x}}$
- time convolution $\overset{(t)}{*}$
- spatial convolution $\overset{(\boldsymbol{x})}{*}$
- ~~VECTOR CALCULUS~~ $(\boldsymbol{E}, \boldsymbol{H}, \cdot, \times, \boldsymbol{\nabla})$
- ~~RIGHT-HANDEDNESS~~ in orientation of spatial reference frame

NOTE:

PHYSICS $\in$ SPACE-TIME $\Longrightarrow$

The concepts of 'frequency' and 'wavenumber' are mathematical artifacts

INTRODUCTION: STRUCTURE OF THE EM FIELD EQUATIONS

ElectroMagnetic fields: $\Longrightarrow$

- physical (wave) phenomena in (affine) $(N+1)$-spacetime $\mathbb{R}^N \times \mathbb{R}$ with
  - (Observer's) position $\{x_1, \ldots, x_N\} \in \mathbb{R}^N$
  - (Observer's) time registration $t \in \mathbb{R}$
- mathematically represented by multicomponent quantities with N^p $(p = 0, 1, 2, \ldots)$ elements (**Einstein**) that are
  - arranged as (arithmetic/computational) arrays of size N and dimension p

OR

  - interpreted as (geometrical) tensors over $\mathbb{R}^N$ and rank p
- Everyday experience $N = 3$
- Cosmological theories $N > 3$

• EINSTEIN: PHYSICAL WAVEFIELD HAS N^p ELEMENTS

NOTATION	$p=0$	$p=1$	$p=2$	
Array notation ($N=3$)	$[a]$	$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$	$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Tensor subscript notation	a	a_i	$a_{i,j}$	$\delta_{m,n}$

- $\boldsymbol{x} \to x_n$: position vector (tensor of rank 1) • $|\boldsymbol{x}| = (x_n x_n)^{1/2} > 0$ • t : time
- $\partial_m = \partial/\partial x_m$ $(m = 1, \ldots, N)$ • $\partial_t = \partial/\partial t$ (reserved symbol)
- $\partial_m x_n = \delta_{m,n}$: Kronecker (unit) tensor (tensor of rank 2)

• SUMMATION CONVENTION (repeated subscripts):

- $\delta_{p,n}\delta_{n,q} = \sum_{n=1}^{N} \delta_{p,n}\delta_{n,q} = \delta_{p,q}$ • $\delta_{m,m} = \sum_{m=1}^{N} \delta_{m,m} = N$

Tensor/array EM theory; Einstein subscript notation and summation convention

	Intensive field quantity 'field strength'	Extensive field quantity 'flux density'	Source quantity 'volume density of impressed current'
Electric	E_k	D_k	$-J_k$
Magnetic	$[H_{i,j}]^-$	$[B_{i,j}]^-$	$-[K_{i,j}]^-$
	'PRODUCT' ⇓	'PRODUCT' ⇓	
	$S_m = [H_{m,k}]^- E_k$ area density of EM power flow	$G_i = [B_{i,j}]^- D_j$ volume density of EM momentum	⟸ Transport quantities ⟸

- $[H_{i,j}]^- \stackrel{\text{def}}{=} (H_{i,j} - H_{j,i})/2 = -[H_{j,i}]^-$ (anti-symmetric, rank 2)

EM field and source quantities

- **WAVEFIELDS:**
  - Changes in space ($\partial_m = \partial/\partial x_m$) **counterbalance**
  - Changes in time ($\partial_t = \partial/\partial t$)

EM field equations (Maxwell)

- $\partial_m [H_{m,k}]^- + \partial_t D_k = -J_k$
- $[\partial_i E_j]^- + \partial_t [B_{i,j}]^- = -[K_{i,j}]^-$

- **FIELD/SOURCE COMPATIBILITY RELATIONS**

(= existence conditions for solution of field equations):

- $\partial_k \partial_m [H_{m,k}]^- = 0$ $\Longrightarrow$ • $\partial_t(\partial_k D_k) = -\partial_k J_k$
- $[\partial_k [\partial_i E_j]^-]^{\circlearrowright} \stackrel{\text{def}}{=} \partial_k [\partial_i E_j]^- + \partial_i [\partial_j E_k]^- + \partial_j [\partial_k E_i]^- = 0 \ (i \neq j \neq k)$ $\Longrightarrow$
- $\partial_t [\partial_k [B_{i,j}]^-]^{\circlearrowright} = -[\partial_k [K_{i,j}]^-]^{\circlearrowright}$ $(i \neq j \neq k)$ $\Longrightarrow$ $N \geq 3$

EM field equations and field/source compatibility relations

∂_t^{-1} = time integration

• VOLUME DENSITY OF ELECTRIC CHARGE:

• $\rho \stackrel{\text{def}}{=} -\partial_t^{-1}\partial_k J_k$ (scalar)

$\Longrightarrow$ • $\partial_k D_k = \rho$

• VOLUME DENSITY OF MAGNETIC CHARGE:

• $[\sigma_{k,i,j}]^{\circlearrowleft} \stackrel{\text{def}}{=} -\partial_t^{-1}[\partial_k[K_{i,j}]^{-}]^{\circlearrowleft} \quad (i \neq j \neq k)$

[completely antisymmetric tensor of rank 3, no scalar! (Dirac, 1931)]

$\Longrightarrow$ • $[\partial_k[B_{i,j}]^{-}]^{\circlearrowleft} = [\sigma_{k,i,j}]^{\circlearrowleft} \quad (i \neq j \neq k)$

Volume densities of electric and magnetic charge

CONSTITUTIVE RELATIONS (standard form):

- $D_k(\boldsymbol{x},t) = \epsilon_{k,r}(\boldsymbol{x},t) \overset{(t)}{*} E_r(\boldsymbol{x},t)$ ($\epsilon_{k,r}$ = electric permittivity)
- $[B_{i,j}]^-(\boldsymbol{x},t) = \mu^{-;-}_{i,j,p,q}(\boldsymbol{x},t) \overset{(t)}{*} [H_{p,q}]^-(\boldsymbol{x},t)$ ($\mu^{-;-}_{i,j,p,q}$ = magnetic permeability)

PROPERTIES:

- linear | • time invariant | • locally reacting in space
- causal in time $\Longrightarrow$ • $\{\epsilon_{k,r}, \mu^{-;-}_{i,j,p,q}\}(\boldsymbol{x},t) = 0$ for $t < 0$
- $\{\epsilon_{k,r}, \mu^{-;-}_{i,j,p,q}\}(\boldsymbol{x},t) = \{\epsilon_{k,r}, \mu^{-;-}_{i,j,p,q}\}(t) \Longrightarrow$ medium HOMOGENEOUS
- $\{\epsilon_{k,r}, \mu^{-;-}_{i,j,p,q}\}(\boldsymbol{x},t) = \{\epsilon_{k,r}(\boldsymbol{x}), \mu^{-;-}_{i,j,p,q}(\boldsymbol{x})\}\,\delta(t) \Longrightarrow$ medium INSTANTANEOUSLY REACTING
- $\epsilon_{k,r}(\boldsymbol{x},t) = \epsilon(\boldsymbol{x},t)\,\delta_{k,r}$ | • $\mu^{-;-}_{i,j,p,q}(\boldsymbol{x},t) = \mu(\boldsymbol{x},t)^-\,\delta_{i,p}\delta_{j,q} \Longrightarrow$ medium ISOTROPIC $\Longrightarrow$ • $D_k = \epsilon \overset{(t)}{*} E_k$ | $[B_{i,j}]^- = \mu^- \overset{(t)}{*} [H_{i,j}]^-$

• (PASSIVE) INTERFACE BOUNDARY CONDITIONS

at jump discontinuity in constitutive properties:

- Σ: passive interface between two different media

NO JUMPS ALLOWED in certain FIELD QUANTITIES:

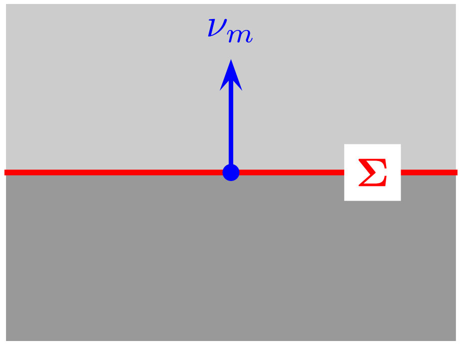
WHICH ONES?

- **DECOMPOSITION OF** ∂_m **about** $x \in \Sigma$**:**

- $\partial_m = \underbrace{\nu_m(\nu_n\partial_n)}_{\text{normal to }\Sigma} + \underbrace{[\partial_m - \nu_m(\nu_n\partial_n)]}_{\text{tangential to }\Sigma}$

$(\nu_n\partial_n)$[jump] $\Longrightarrow$ Dirac delta function

(would violate passivity)



CONTINUITY CONDITIONS:

- $\nu_m[H_{m,k}]^-\big|_-^+ = 0$ • $[\nu_i E_j]^-\big|_-^+ = 0$

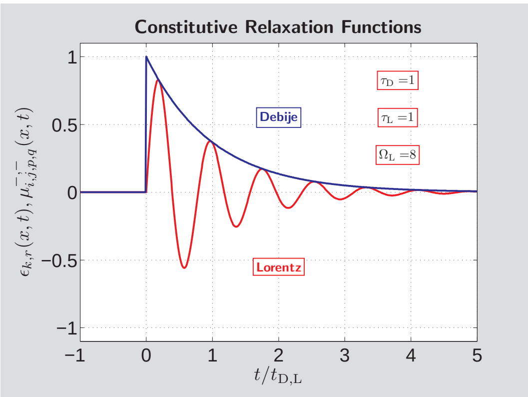
• CAUSALITY (UNIQUENESS OF TIME EVOLUTION = UNIQUENESS OF INITIAL-VALUE PROBLEM): (De Hoop, 2003)

• **EM field**$(\boldsymbol{x}, t_0)$ for $\boldsymbol{x} \in \mathbb{R}^3$ $\overset{\text{Field equations}}{\Longrightarrow \Longrightarrow}$

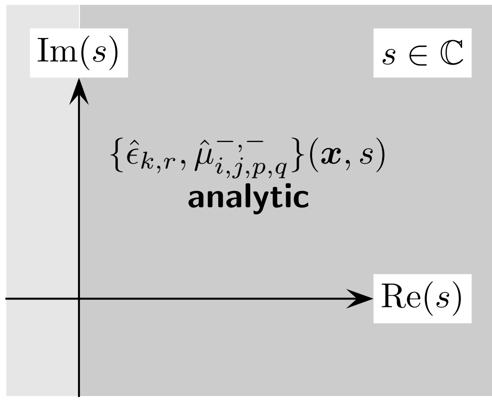
EM field$(\boldsymbol{x}, t)$ for $\boldsymbol{x} \in \mathbb{R}^3, t > t_0$ (uniquely)

• PROOF VIA TIME LAPLACE TRANSFORMATION:

• $\{\hat{\epsilon}_{k,r}, \hat{\mu}^{-;-}_{i,j,p,q}\}(\boldsymbol{x}, s) = \int_{t=0}^{\infty} \exp(-st) \{\epsilon_{k,r}, \mu^{-;-}_{i,j,p,q}\}(\boldsymbol{x}, t)\mathrm{d}t$ for $s \in \mathbb{C}, \mathrm{Re}(s) > 0$



time Laplace transformation $\Longrightarrow$

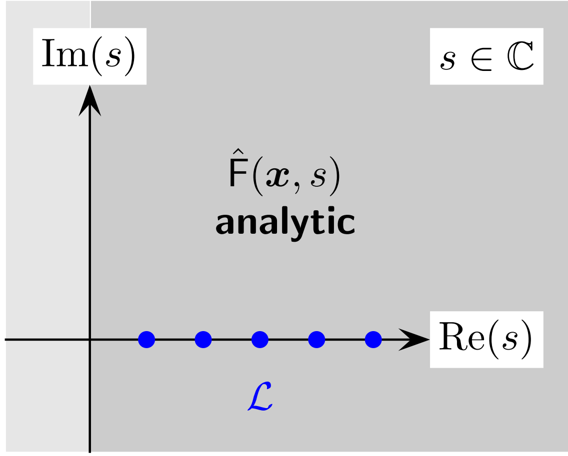


$\delta(t) \Rightarrow 1$

$H(t) \Rightarrow \frac{1}{s}$

Causality, uniqueness of initial-value problem

• CONDITIONS FOR UNIQUENESS OF INITIAL-VALUE PROBLEM



- **Based on Lerch's uniqueness theorem:**
- $\{\hat{\mathsf{F}}(\boldsymbol{x}, s)|_{s\in\mathcal{L}}\} \overset{\text{1-to-1}}{\longmapsto} \mathsf{F}(\boldsymbol{x}, t)H(t)$

  (Widder, The Laplace Transform, 1946)

- $\mathcal{L} = \{s \in \mathbb{C}; \mathrm{Im}(s) = 0, \mathrm{Re}(s) = \boldsymbol{s_0 + nh}, s_0 > 0, h > 0, n = 0, 1, 2, \ldots\}$

  (Lerch sequence)

$\Longrightarrow$ • $\{\hat{E}_k(\boldsymbol{x}, s)\hat{\epsilon}_{k,r}(\boldsymbol{x}, s)\hat{E}_r(\boldsymbol{x}, s)|_{s\in\mathcal{L}}\} > 0$ for $\hat{E}_r(\boldsymbol{x}, s) \neq 0$

$\Longrightarrow$ • $\{[\hat{H}_{i,j}]^-(\boldsymbol{x}, s)\hat{\mu}^{-,-}_{i,j,p,q}(\boldsymbol{x}, s)[\hat{H}_{p,q}]^-(\boldsymbol{x}, s)|_{s\in\mathcal{L}}\} > 0$ for $[\hat{H}_{p,q}]^-(\boldsymbol{x}, s) \neq 0$

• TENSOR/ARRAY EM THEORY:

- Leads to (considerable) simplifications
- Involves only elementary mathematics ($\partial_t\partial_x = \partial_x\partial_t, \overset{(t)}{*}, \overset{(\boldsymbol{x})}{*}$)
- Makes standard ~~VECTOR CALCULUS~~ superfluous
- Does away with the condition of ~~RIGHT-HANDEDNESS~~ in orientation of spatial reference frame
- Considers SpaceTime as an 'affine' space (not a (Lorentz) 'metric' space)
- Reformulates the notion of the '~~MAGNETIC MONOPOLE~~' ($=$ tensor of rank 3)
- Has implications for (Dirac) 'string theory' in quantum electrodynamics
- Generalizes to $(N + 1)$-spacetime (relativity, cosmology) (De Hoop, 2011)
- Directly matches array handling in computational coding

Tensor/array EM theory: CONCLUSION

• (More or less) COMPLETE EM COURSE:

Part	Degree of difficulty
• Field equations	*
• Radiation from sources	**
• Field/source/contrast-in-medium reciprocity relations	*
• Energy balance	*
• Balance of field momentum ('radiation pressure')	***

in 4 PAGES ! ⟹

De Hoop, Conference Proceedings URSI EMT-S Berlin 2010

• ALSO:

- Lorentz covariance (special relativity)

END
INTRODUCTION

RADIATION FROM SOURCES IN UNBOUNDED HOMOGENEOUS, ISOTROPIC MEDIA

Prerequisites (differentiation, convolution, Dirac pulse)

- $\partial_t\,[\,\partial_m\,F(\cdot)] = \partial_m\,[\,\partial_t\,F(\cdot)]$
- $F(\cdot) \overset{(t)}{*} G(\cdot) = G(\cdot) \overset{(t)}{*} F(\cdot)$ | • $F(\cdot) \overset{(\boldsymbol{x})}{*} G(\cdot) = G(\cdot) \overset{(\boldsymbol{x})}{*} F(\cdot)$
- $\overset{(\boldsymbol{x},t)}{*} = \overset{(\boldsymbol{x})}{*}\overset{(t)}{*} = \overset{(t)}{*}\overset{(\boldsymbol{x})}{*}$
- $\partial_t\,[F(\cdot) \overset{(\boldsymbol{x},t)}{*} G(\cdot)] = [\,\partial_t\,F(\cdot)] \overset{(\boldsymbol{x},t)}{*} G(\cdot) = F(\cdot) \overset{(\boldsymbol{x},t)}{*} [\,\partial_t\,G(\cdot)]$
- $\partial_m\,[F(\cdot) \overset{(\boldsymbol{x},t)}{*} G(\cdot)] = [\,\partial_m\,F(\cdot)] \overset{(\boldsymbol{x},t)}{*} G(\cdot) = F(\cdot) \overset{(\boldsymbol{x},t)}{*} [\,\partial_m\,G(\cdot)]$
- $\delta(t) \overset{(t)}{*} F(\cdot) = F(\cdot)$ | • $\delta(\boldsymbol{x}) \overset{(\boldsymbol{x})}{*} F(\cdot) = F(\cdot)$

• EM RADIATION FROM SOURCES: $\epsilon > 0,\ \mu^- > 0$

EM field equations at (x,t)	operation
• $\partial_m [H_{m,k}]^- + \epsilon\, \partial_t E_k = -J_k$	$\mu^-\, \partial_t$
• $[\partial_i E_j]^- + \mu^-\, \partial_t\, [H_{i,j}]^- = -[K_{i,j}]^-$	⇑

- Elimination of $[H_{i,j}]^-$ + use of
- $\partial_k E_k = -(1/\epsilon)\partial_t^{-1}(\partial_m J_m)$ (compatibility relation) $\Longrightarrow$

• ELECTRIC-FIELD VECTOR WAVE EQUATION:

• $$(\partial_m \partial_m) E_k - c^{-2} \partial_t^2 E_k = -F_k$$

- $c = 1/(2\mu^-\,\epsilon)^{1/2}$ (EM wavespeed)
- $F_k = -2\,\mu^-\,\partial_t J_k + (1/\epsilon)\partial_t^{-1}\partial_k(\partial_m J_m) + 2\,\partial_m [K_{m,k}]^-$

EM radiation from sources (homogeneous, isotropic medium)

• ELECTRIC-FIELD VECTOR WAVE EQUATION:

$$\bullet\ (\partial_m\partial_m)E_k - c^{-2}\partial_t^2 E_k = -F_k = -\delta(\boldsymbol{x},t) \overset{(\boldsymbol{x},t)}{*} F_k$$

$$\overset{(\boldsymbol{x},t)}{*} = \overset{(\boldsymbol{x})}{*}\overset{(t)}{*}$$

$$\bullet\ F_k = -2\,\mu^{-}\,\partial_t J_k + (1/\epsilon)\partial_t^{-1}\partial_k(\partial_m J_m) + 2\,\partial_m[K_{m,k}]^{-}$$

$\Longrightarrow$ **SOLUTION:**

$$\bullet\ E_k = G \overset{(\boldsymbol{x},t)}{*} F_k$$

$$\bullet\ (\partial_m\partial_m)G - c^{-2}\partial_t^2 G = -\delta(\boldsymbol{x},t)$$

$\bullet\ G = \dfrac{\delta(t - |\boldsymbol{x}|/c)}{4\pi|\boldsymbol{x}|}$ for $\boldsymbol{x} \neq \mathbf{0}$ (3D) (scalar Green's function)

$$\overset{(\boldsymbol{x},t)}{*}\ \partial_m\partial_t = \partial_m\partial_t \overset{(\boldsymbol{x},t)}{*} \quad \Downarrow$$

The electric-field vector wave equation and its solution

• THE RADIATED FIELD: [• $E_k = G \overset{(\boldsymbol{x},t)}{*} F_k \Longrightarrow$]

- $E_k = -2\,\mu^-\,\partial_t\, A_k + (1/\epsilon)\partial_t^{-1}\partial_k\partial_m\, A_m + 2\,\partial_m\,[\Psi_{m,k}]^-$
- $[H_{i,j}]^- = -2\,\epsilon\,\partial_t\,[\Psi_{i,j}]^- + (1/\mu^-)\partial_t^{-1}\partial_m[\partial_m\,[\Psi_{i,j}]^-\,]^{\circlearrowleft} + 2\,[\partial_i\, A_j\,]^-$

• VECTOR POTENTIAL:

- $A_k = G \overset{(\boldsymbol{x},t)}{*} J_k$
- $(\partial_m\partial_m)\, A_k - c^{-2}\partial_t^2\, A_k = -J_k$

• TENSOR POTENTIAL:

- $[\Psi_{i,j}]^- = G \overset{(\boldsymbol{x},t)}{*} [K_{i,j}]^-$
- $(\partial_m\partial_m)\,[\Psi_{i,j}]^- - c^{-2}\partial_t^2\,[\Psi_{i,j}]^- = -[K_{i,j}]^-$

• SCALAR GREEN'S FUNCTION:

- $G = \dfrac{\delta(t-|\boldsymbol{x}|/c)}{4\pi|\boldsymbol{x}|}$ for $\boldsymbol{x} \neq \boldsymbol{0}$ (3D)
- $(\partial_m\partial_m)G - c^{-2}\partial_t^2 G = -\delta(\boldsymbol{x},t)$

N.B. Use is made of the identity • $\partial_m\partial_n[\partial_n[\Psi_{m,k}]^-]^{\circlearrowleft} = 0$

The radiated field (vector and tensor potentials)

• THE RADIATED FIELD:

Sources	Medium	Field	3D
$\begin{bmatrix} J_k \\ [K_{i,j}]^- \end{bmatrix}$	ϵ, μ^- $\Longrightarrow$ $c = 1/(2\,\mu^-\,\epsilon)^{1/2}$	$\begin{bmatrix} E_r \\ [H_{p,q}]^- \end{bmatrix}$	Physics
$\boldsymbol{x} \in \text{supp}(J_k, [K_{i,j}]^-) \subset \mathbb{R}^3$	$\boldsymbol{x} \in \mathbb{R}^3$	$\boldsymbol{x} \in \mathbb{R}^3$	Position
$t \in \mathbb{R}, t > t_0$		$t \in \mathbb{R}, t > t_0 + \lvert\boldsymbol{x}\rvert/c$	Time

• STRUCTURE:

[• $S(t)$ = source signature]

Magnitude	Pulse shape	Directional characteristic
$1/\lvert\boldsymbol{x}\rvert$	$\partial_t S(t)$	'Far-field'
$1/\lvert\boldsymbol{x}\rvert^2$	$S(t)$	
$1/\lvert\boldsymbol{x}\rvert^3$	$\partial_t^{-1} S(t)$	'Near-field'

$\Longrightarrow$ **COMPLICATED MIXTURE !!!**

Structure of the radiated field

• THE RADIATED FIELD (FAR-FIELD APPROXIMATION):

- $\boldsymbol{x}' \in \mathrm{supp}(J_k, [K_{i,j}]^-)$ | • $\boldsymbol{x} \in \mathbb{R}^3$ | • $|\boldsymbol{x}| \to \infty$
- $|\boldsymbol{x}-\boldsymbol{x}'| = |\boldsymbol{x}| - \xi_m x'_m + O(|\boldsymbol{x}|^{-1})$ as $|\boldsymbol{x}| \to \infty$ | • $\xi_m = x_m/|\boldsymbol{x}|$ $(\xi_m\xi_m) = 1$
- $\{A_k, [\Psi_{i,j}]^-, E_r, [H_{p,q}]^-\}(\boldsymbol{x}, t) = \dfrac{\{A_k^\infty, [\Psi_{i,j}^\infty]^-, E_r^\infty, [H_{p,q}^\infty]^-\}(\boldsymbol{\xi}, t - |\boldsymbol{x}|/c)}{4\pi|\boldsymbol{x}|}$

  $\left[1 + O(|\boldsymbol{x}|^{-1})\right]$ as $|\boldsymbol{x}| \to \infty$
- $\partial_m(\cdot) = (\xi_m/c)\partial_t(\cdot)\left[1 + O(|\boldsymbol{x}|^{-1})\right]$ as $|\boldsymbol{x}| \to \infty$
- $A_k^\infty = \displaystyle\int_{\mathrm{supp}(J_k)} J_k(\boldsymbol{x}', t + \xi_m x'_m/c)\mathrm{d}V(\boldsymbol{x}')$
- $[\Psi_{i,j}^\infty]^- = \displaystyle\int_{\mathrm{supp}([K_{i,j}]^-)} [K_{i,j}]^-(\boldsymbol{x}', t + \xi_m x'_m/c)\mathrm{d}V(\boldsymbol{x}')$
- $E_r^\infty = -2\,\mu^-(\delta_{r,k} - \xi_r\xi_k)\partial_t A_r^\infty + 2\,(\xi_m/c)\partial_t[\Psi_{m,r}^\infty]^-$
- $[H_{i,j}^\infty]^- = -2\,\epsilon\,(\partial_t[\Psi_{i,j}^\infty]^- - \xi_m[\xi_m\partial_t[\Psi_{i,j}^\infty]^-]^{\circlearrowright}) + 2\,[(\xi_i/c)\partial_t A_j^\infty]^-$

Far-field approximation

- **THE RADIATED FIELD:**
- **FAR-FIELD APPROXIMATION:**
- $\{E_r, [H_{p,q}]^-\}(\boldsymbol{x}, t) = \dfrac{\{E_r^\infty, [H_{p,q}^\infty]^-\}(\boldsymbol{\xi}, t - |\boldsymbol{x}|/c)}{4\pi|\boldsymbol{x}|} \left[1 + O(|\boldsymbol{x}|^{-1})\right]$ as $|\boldsymbol{x}| \to \infty$

$\Longrightarrow$ **LOCAL PLANE-WAVE AMPLITUDES (DIRECTION OF PROPAGATION ALONG ξ):**

- $(\xi_m/c)\,[H_{m,k}^\infty]^- + \epsilon\, E_k^\infty = 0$
- $[(\xi_i/c)\, E_j^\infty]^- + \mu^-\, [H_{i,j}^\infty]^- = 0$

Far-field approximation, local plane-wave amplitudes

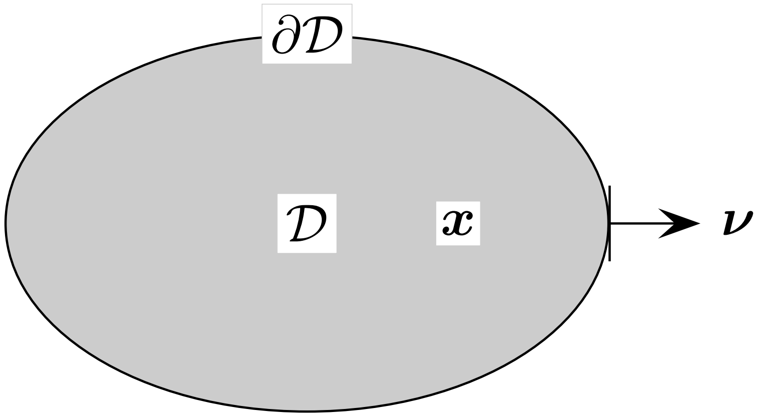
END

RADIATION FROM SOURCES

TIME-DOMAIN FIELD/SOURCE RECIPROCITY (TIME-CONVOLUTION TYPE)

• FIELD RECIPROCITY ('interaction' of two EM 'STATES'):

Reciprocity $(\boldsymbol{x} \in \mathcal{D} \cup \partial\mathcal{D},\ t \in \mathbb{R})$			
State	Field strengths	Flux densities	Sources
A	$E_r^A, [H_{p,q}^A]^-$	$D_k^A, [B_{i,j}^A]^-$	$J_k^A, [K_{i,j}^A]^-$
B	$E_k^B, [H_{i,j}^B]^-$	$D_r^B, [B_{p,q}^B]^-$	$J_r^B, [K_{p,q}^B]^-$



• TIME-CONVOLUTION FIELD RECIPROCITY:

State	EM field equations ($\boldsymbol{x} \in \mathcal{D} \cup \partial\mathcal{D},\ t \in \mathbb{R}$)	Operation
A	• $\partial_m [H^A_{m,k}]^-(t) + \partial_t D^A_k(t) = -J^A_k(t)$	$\overset{(t)}{*}\, E^B_k(t)$
B	• $[\partial_p E^B_q(t)]^- + \partial_t [B^B_{p,q}]^-(t) = -[K^B_{p,q}]^-(t)$	$\overset{(t)}{*}\, [H^A_{p,q}]^-(t)$
	• $-\ A \rightleftharpoons B, k \rightleftharpoons r, \{p,q\} \rightleftharpoons \{i,j\}$	

• LOCAL:

• ∂_m [Electric-field/Magnetic-field interaction term] +
∂_t [Contrast-in-media interaction terms] =
Sources/Fields (electric/electric, magnetic/magnetic) interaction terms

Time-convolution EM field/source reciprocity (local)

• TIME-CONVOLUTION FIELD RECIPROCITY (CONTINUED):

Electric-field/Magnetic-field interaction term:

- $[H^A_{m,k}]^-(t) \overset{(t)}{*} E^B_k(t) - [H^B_{m,r}]^-(t) \overset{(t)}{*} E^A_r(t)$

Contrast-in-media interaction terms:

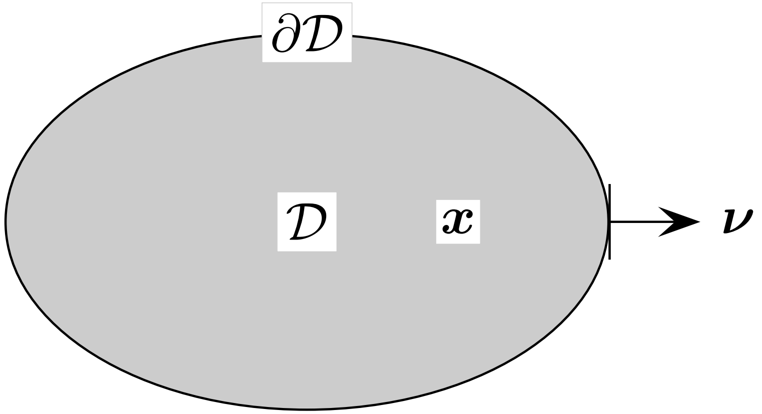
- $E^B_k(t) \overset{(t)}{*} D^A_k(t) - E^A_r(t) \overset{(t)}{*} D^B_r(t) +$
  $[B^B_{i,j}]^-(t) \overset{(t)}{*} [H^A_{i,j}]^-(t) - [B^A_{p,q}]^-(t) \overset{(t)}{*} [H^B_{p,q}]^-(t) =$
  $E^B_k(t) \overset{(t)}{*} \boxed{[\epsilon^A_{k,r}(t) - \epsilon^B_{r,k}(t)]} \overset{(t)}{*} E^A_r(t)$
  $[H^A_{i,j}]^-(t) \overset{(t)}{*} \boxed{[\mu^{-,-\ B}_{i,j,p,q}(t) - \mu^{-,-\ A}_{p,q,i,j}(t)]} \overset{(t)}{*} [H^B_{p,q}]^-(t)$

Sources/Fields (electric/electric, magnetic/magnetic) interaction terms:

- $-J^A_k(t) \overset{(t)}{*} E^B_k(t) + J^B_r(t) \overset{(t)}{*} E^A_r(t) -$
  $[K^B_{i,j}]^-(t) \overset{(t)}{*} [H^A_{i,j}]^-(t) + [K^A_{p,q}]^-(t) \overset{(t)}{*} [H^B_{p,q}]^-(t)$

Time-convolution EM field/source reciprocity (continued)

• TIME-CONVOLUTION FIELD RECIPROCITY (GLOBAL):



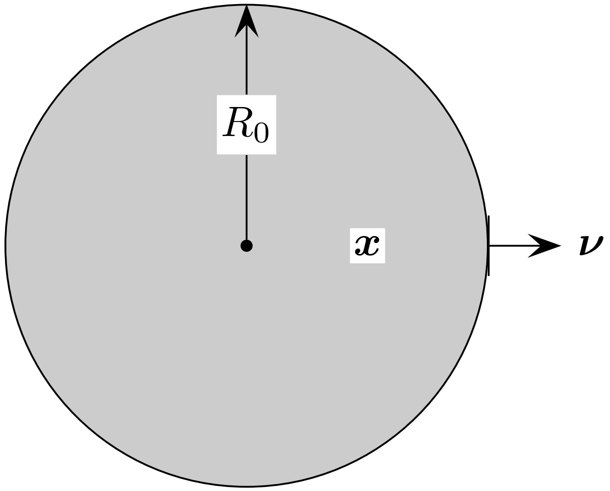
• $\int_{\mathcal{D}} \partial_m [\ldots] \mathrm{d}V \overset{\text{GAUSS}}{===} \int_{\partial\mathcal{D}} \nu_m [\ldots] \mathrm{d}A$

$$\int_{\partial\mathcal{D}} \nu_m \text{ [Electric-field/Magnetic-field interaction term] } \mathrm{d}A \, +$$
$$\int_{\mathcal{D}} \partial_t \text{ [Contrast-in-media interaction terms] } \mathrm{d}V \, =$$
$$\int_{\mathcal{D}} \text{ [Sources/Fields (electric/electric, magnetic/magnetic) interaction terms] } \mathrm{d}V$$

Time-convolution EM field/source reciprocity (global)

- **TIME-CONVOLUTION FIELD RECIPROCITY**

- **(UNBOUNDED DOMAINS):**



- **FAR-FIELD APPROXIMATION**

$$\Longrightarrow \lim_{R_0 \to \infty} \int_{\{|\boldsymbol{x}|=R_0\}} \nu_m \,\text{[Electric-field/Magnetic-field interaction term]}\, \mathrm{d}A = 0$$

• TIME-CONVOLUTION FIELD RECIPROCITY (APPLICATIONS):

- Analysis of Transmission/Reception properties of antennas
- ElectroMagnetic Compatibility (EMC) disturbance analysis (Emitter = Susceptor)
- Necessary (but not sufficient) check on EM field computational procedures

END

TIME-DOMAIN FIELD/SOURCE

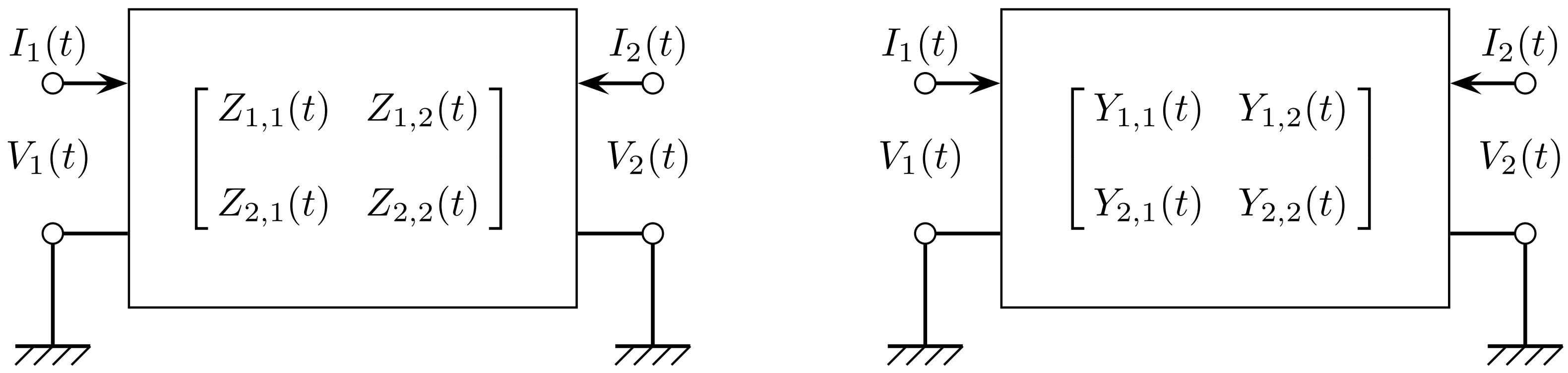
RECIPROCITY

(TIME-CONVOLUTION TYPE)

End time-domain field/source reciprocity (time-convolution type)

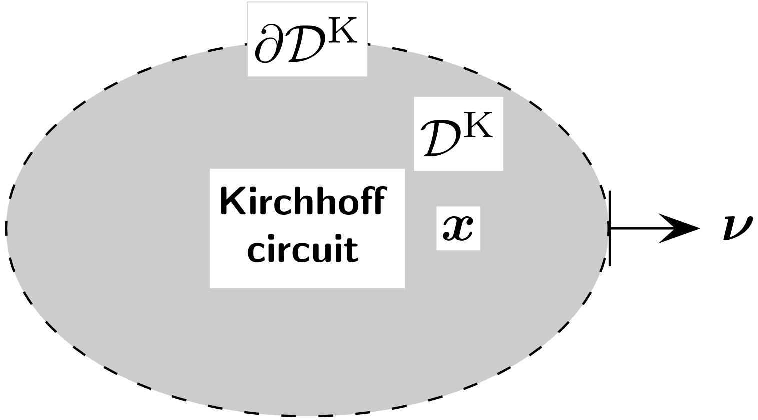
MAXWELL FIELDS / KIRCHHOFF CIRCUITS INTERACTION

• KIRCHHOFF CIRCUITS (LINEAR, TIME-INVARIANT, CAUSAL):



- $[V(t)] = [Z(t)] \overset{(t)}{*} [I(t)]$
- $[I(t)] = [Y(t)] \overset{(t)}{*} [V(t)]$
- $[V(t)] =$ port voltages
- $[I(t)] =$ port electric currents
- $[Z(t)] =$ impedance matrix $\quad [Z(t)] = 0$ for $t < 0$
- $[Y(t)] =$ admittance matrix $\quad [Y(t)] = 0$ for $t < 0$

• MAXWELL FIELDS / KIRCHHOFF CIRCUITS INTERACTION:



- $\mathcal{D}^{\rm K} = \mathrm{supp}($Kirchhoff circuit$)$
- $\mathrm{maxdiam}(\mathcal{D}^{\rm K}) \ll$

(EM wavespeed embedding) ·

(signal pulse time width)

$$\int_{\partial\mathcal{D}^{\rm K}} \nu_m \left[[H^A_{m,k}]^-(t) \overset{(t)}{*} E^B_k(t) - [H^B_{m,r}]^-(t) \overset{(t)}{*} E^A_r(t) \right] \mathrm{d}A = \sum_{p\in\{\text{ports}\}} \left[I^A_p \overset{(t)}{*} V^B_p - I^B_p \overset{(t)}{*} V^A_p \right]$$

END

MAXWELL FIELDS /

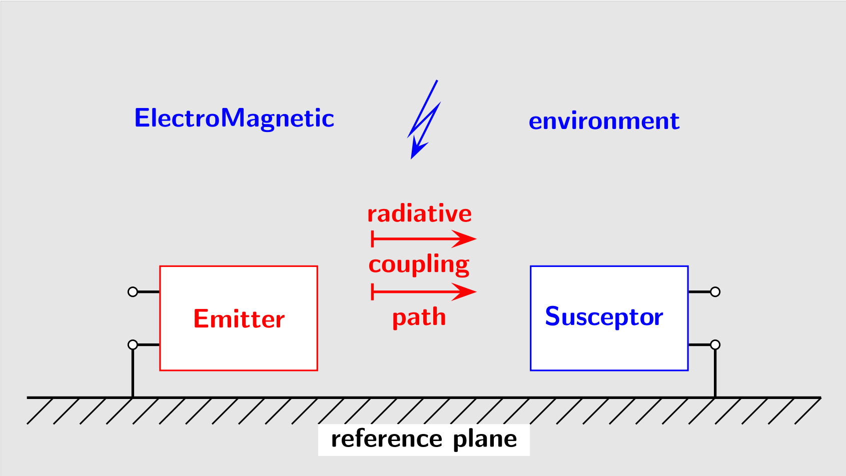
KIRCHHOFF CIRCUITS

INTERACTION

APPLIED EM TOPICS IN ELECTRONICS AND IT

• APPLIED EM TOPICS IN ELECTRONICS AND IT:

- Signal integrity in inter-/intra-system pulsed-field radiative signal transfer
- Pulsed-field EM Interference (EMI) analysis



- Time-domain (TD) regulatory EM Compatibility (EMC) specifications (TD emission limits, TD immunity limits)

END

APPLIED EM TOPICS IN ELECTRONICS AND IT

End applied EM topics in Electronics and IT

THE END

THANK YOU FOR YOUR ATTENTION