

ElectroMagnetic field theory - A modern tensor/array approach

by

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SYNOPSIS:

- Introduction: Structure of the EM field equations
- Selected corollaries:
 - Radiation from sources
 - Field/source reciprocity
 - Interaction Maxwell fields \iff Kirchhoff circuits
- Applied EM topics in Electronics and IT:
 - Signal integrity in inter-/intra-system pulsed-field radiative signal transfer
 - Pulsed-field EM Interference (EMI) analysis
 - Time-domain (TD) regulatory EM Compatibility (EMC) specifications (TD emission limits, TD immunity limits)

PREREQUISITES:

- time differentiation ∂_t
- spatial differentiation $\partial_{\mathbf{x}}$
- time convolution $\overset{(t)}{*}$
- spatial convolution $\overset{(\mathbf{x})}{*}$
- ~~VECTOR CALCULUS~~ (\mathbf{E} , \mathbf{H} , \cdot , \times , ∇)
- ~~RIGHT-HANDEDNESS~~ in orientation of spatial reference frame

NOTE:

PHYSICS \in **SPACE-TIME** \implies

The concepts of 'frequency' and 'wavenumber' are mathematical artifacts

**INTRODUCTION:
STRUCTURE OF THE
EM FIELD EQUATIONS**

ElectroMagnetic fields: \Rightarrow

- physical (wave) phenomena in (affine) $(N + 1)$ -spacetime $\mathbb{R}^N \times \mathbb{R}$ with
 - (Observer's) position $\{x_1, \dots, x_N\} \in \mathbb{R}^N$
 - (Observer's) time registration $t \in \mathbb{R}$
- mathematically represented by multicomponent quantities with N^p ($p = 0, 1, 2, \dots$) elements (**Einstein**) that are
 - arranged as (arithmetic/computational) arrays of size N and dimension p
- OR**
 - interpreted as (geometrical) tensors over \mathbb{R}^N and rank p
- Everyday experience $N = 3$ • Cosmological theories $N > 3$

• EINSTEIN: PHYSICAL WAVEFIELD HAS N^p ELEMENTS

NOTATION	$p = 0$	$p = 1$	$p = 2$	
Array notation ($N = 3$)	$[a]$	$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$	$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Tensor subscript notation	a	a_i	$a_{i,j}$	$\delta_{m,n}$

• $\mathbf{x} \rightarrow x_n$: position vector (tensor of rank 1) • $|\mathbf{x}| = (x_n x_n)^{1/2} > 0$ • t : time

• $\partial_m = \partial/\partial x_m$ ($m = 1, \dots, N$) • $\partial_t = \partial/\partial t$ (reserved symbol)

• $\partial_m x_n = \delta_{m,n}$: Kronecker (unit) tensor (tensor of rank 2)

• **SUMMATION CONVENTION (repeated subscripts):**

• $\delta_{p,n} \delta_{n,q} = \sum_{n=1}^N \delta_{p,n} \delta_{n,q} = \delta_{p,q}$ • $\delta_{m,m} = \sum_{m=1}^N \delta_{m,m} = N$

	Intensive field quantity 'field strength'	Extensive field quantity 'flux density'	Source quantity 'volume density of impressed current'
Electric	E_k	D_k	$-J_k$
Magnetic	$[H_{i,j}]^-$	$[B_{i,j}]^-$	$-[K_{i,j}]^-$
	'PRODUCT' ↓	'PRODUCT' ↓	
	$S_m = [H_{m,k}]^- E_k$ area density of EM power flow	$G_i = [B_{i,j}]^- D_j$ volume density of EM momentum	\leftarrow Transport quantities \leftarrow

- $[H_{i,j}]^- \stackrel{\text{def}}{=} (H_{i,j} - H_{j,i})/2 = -[H_{j,i}]^-$ (anti-symmetric, rank 2)

- **WAVEFIELDS:**
 - Changes in space ($\partial_m = \partial/\partial x_m$) **counterbalance**
 - Changes in time ($\partial_t = \partial/\partial t$)

EM field equations (Maxwell)

- $\partial_m [H_{m,k}]^- + \partial_t D_k = -J_k$
- $[\partial_i E_j]^- + \partial_t [B_{i,j}]^- = -[K_{i,j}]^-$

• FIELD/SOURCE COMPATIBILITY RELATIONS (= existence conditions for solution of field equations):

- $\partial_k \partial_m [H_{m,k}]^- = 0 \implies \partial_t (\partial_k D_k) = -\partial_k J_k$
- $[\partial_k [\partial_i E_j]^-]^{\circlearrowleft} \stackrel{\text{def}}{=} \partial_k [\partial_i E_j]^- + \partial_i [\partial_j E_k]^- + \partial_j [\partial_k E_i]^- = 0 \quad (i \neq j \neq k) \implies$
- $\partial_t [\partial_k [B_{i,j}]^-]^{\circlearrowleft} = -[\partial_k [K_{i,j}]^-]^{\circlearrowleft} \quad (i \neq j \neq k) \implies N \geq 3$

$$\partial_t^{-1} = \text{time integration}$$

- VOLUME DENSITY OF ELECTRIC CHARGE:**

- $\rho \stackrel{\text{def}}{=} -\partial_t^{-1} \partial_k J_k$ (scalar)

$$\implies \bullet \quad \partial_k D_k = \rho$$

- VOLUME DENSITY OF MAGNETIC CHARGE:**

- $[\sigma_{k,i,j}]^{\circlearrowleft} \stackrel{\text{def}}{=} -\partial_t^{-1} [\partial_k [K_{i,j}]^-]^{\circlearrowleft} \quad (i \neq j \neq k)$

[completely antisymmetric tensor of rank 3, no scalar! (Dirac, 1931)]

$$\implies \bullet \quad [\partial_k [B_{i,j}]^-]^{\circlearrowleft} = [\sigma_{k,i,j}]^{\circlearrowleft} \quad (i \neq j \neq k)$$

CONSTITUTIVE RELATIONS (standard form):

- $D_k(\mathbf{x}, t) = \epsilon_{k,r}(\mathbf{x}, t) \overset{(t)}{*} E_r(\mathbf{x}, t)$ ($\epsilon_{k,r}$ = electric permittivity)
- $[B_{i,j}]^-(\mathbf{x}, t) = \mu_{i,j,p,q}^{\bar{,}\bar{}}(\mathbf{x}, t) \overset{(t)}{*} [H_{p,q}]^-(\mathbf{x}, t)$ ($\mu_{i,j,p,q}^{\bar{,}\bar{}}$ = magnetic permeability)

PROPERTIES:

- linear | • time invariant | • locally reacting in space
- causal in time \implies • $\{\epsilon_{k,r}, \mu_{i,j,p,q}^{\bar{,}\bar{}}\}(\mathbf{x}, t) = 0$ for $t < 0$
- $\{\epsilon_{k,r}, \mu_{i,j,p,q}^{\bar{,}\bar{}}\}(\mathbf{x}, t) = \{\epsilon_{k,r}, \mu_{i,j,p,q}^{\bar{,}\bar{}}\}(t) \implies$ medium **HOMOGENEOUS**
- $\{\epsilon_{k,r}, \mu_{i,j,p,q}^{\bar{,}\bar{}}\}(\mathbf{x}, t) = \{\epsilon_{k,r}(\mathbf{x}), \mu_{i,j,p,q}^{\bar{,}\bar{}}(\mathbf{x})\} \delta(t) \implies$
medium **INSTANTANEOUSLY REACTING**
- $\epsilon_{k,r}(\mathbf{x}, t) = \epsilon(\mathbf{x}, t) \delta_{k,r}$ | • $\mu_{i,j,p,q}^{\bar{,}\bar{}}(\mathbf{x}, t) = \mu(\mathbf{x}, t)^- \delta_{i,p} \delta_{j,q} \implies$
medium **ISOTROPIC** \implies • $D_k = \epsilon \overset{(t)}{*} E_k$ | $[B_{i,j}]^- = \mu^- \overset{(t)}{*} [H_{i,j}]^-$

- (PASSIVE) INTERFACE BOUNDARY CONDITIONS

at jump discontinuity in constitutive properties:

- Σ : passive interface between two different media

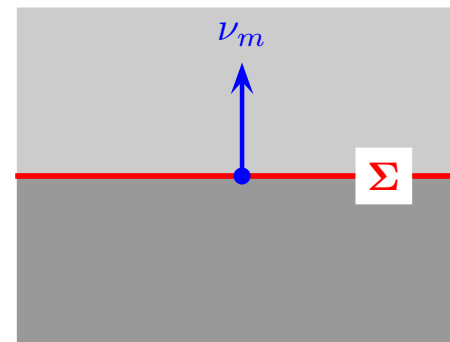
NO JUMPS ALLOWED in certain FIELD QUANTITIES:

WHICH ONES?

- DECOMPOSITION OF ∂_m about $x \in \Sigma$:

$$\bullet \partial_m = \underbrace{\nu_m(\nu_n \partial_n)}_{\text{normal to } \Sigma} + \underbrace{[\partial_m - \nu_m(\nu_n \partial_n)]}_{\text{tangential to } \Sigma}$$

$(\nu_n \partial_n)[\text{jump}] \implies$ Dirac delta function
(would violate passivity)



CONTINUITY CONDITIONS:

$$\bullet \nu_m [H_{m,k}]^- \Big|_-^+ = 0 \quad \bullet [\nu_i E_j]^- \Big|_-^+ = 0$$

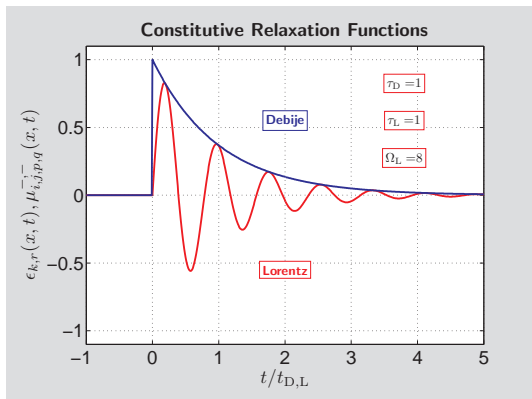
CAUSALITY (UNIQUENESS OF TIME EVOLUTION = UNIQUENESS OF INITIAL-VALUE PROBLEM):

(De Hoop, 2003)

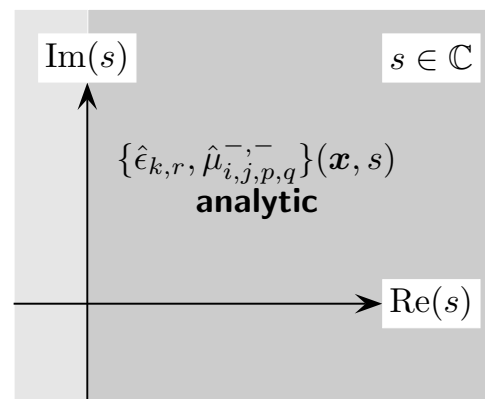
- EM field(\mathbf{x}, t_0) for $\mathbf{x} \in \mathbb{R}^3$ $\xRightarrow{\text{Field equations}} \xRightarrow{\text{Field equations}}$
EM field(\mathbf{x}, t) for $\mathbf{x} \in \mathbb{R}^3, t > t_0$ (uniquely)

PROOF VIA TIME LAPLACE TRANSFORMATION:

- $\{\hat{\epsilon}_{k,r}, \hat{\mu}_{i,j,p,q}^{-,-}\}(\mathbf{x}, s) = \int_{t=0}^{\infty} \exp(-st) \{\epsilon_{k,r}, \mu_{i,j,p,q}^{-,-}\}(\mathbf{x}, t) dt$ for $s \in \mathbb{C}, \text{Re}(s) > 0$



time
Laplace
transformation



$\delta(t) \Rightarrow 1$

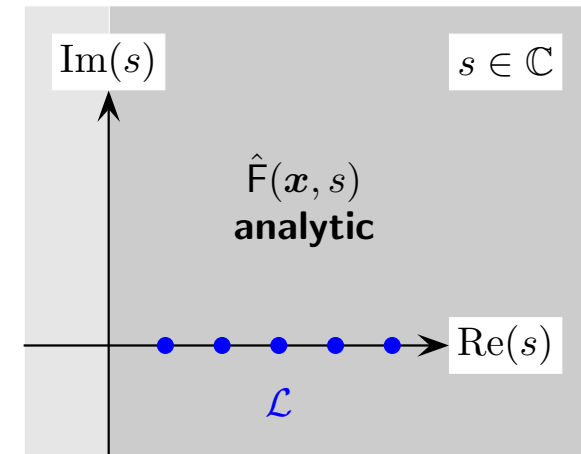
$H(t) \Rightarrow \frac{1}{s}$

• CONDITIONS FOR UNIQUENESS OF INITIAL-VALUE PROBLEM

• Based on Lerch's uniqueness theorem:

$$\bullet \{ \hat{F}(\mathbf{x}, s) |_{s \in \mathcal{L}} \} \xrightarrow{1\text{-to-}1} F(\mathbf{x}, t)H(t)$$

(Widder, The Laplace Transform, 1946)



$$\bullet \mathcal{L} = \{ s \in \mathbb{C}; \text{Im}(s) = 0, \text{Re}(s) = s_0 + nh, s_0 > 0, h > 0, n = 0, 1, 2, \dots \}$$

(Lerch sequence)

$$\implies \bullet \{ \hat{E}_k(\mathbf{x}, s) \hat{\epsilon}_{k,r}(\mathbf{x}, s) \hat{E}_r(\mathbf{x}, s) |_{s \in \mathcal{L}} \} > 0 \text{ for } \hat{E}_r(\mathbf{x}, s) \neq 0$$

$$\implies \bullet \{ [\hat{H}_{i,j}]^-(\mathbf{x}, s) \hat{\mu}_{i,j,p,q}^-, \hat{H}_{p,q}]^-(\mathbf{x}, s) |_{s \in \mathcal{L}} \} > 0 \text{ for } [\hat{H}_{p,q}]^-(\mathbf{x}, s) \neq 0$$

Conditions for uniqueness

- **TENSOR/ARRAY EM THEORY:**

- Leads to (considerable) **simplifications**
- Involves only elementary mathematics ($\partial_t \partial_x = \partial_x \partial_t$, $\overset{(t)}{*}$, $\overset{(x)}{*}$)
- Makes standard ~~VECTOR CALCULUS~~ superfluous
- Does away with the condition of ~~RIGHT-HANDEDNESS~~ in orientation of spatial reference frame
- Considers SpaceTime as an 'affine' space (not a (Lorentz) 'metric' space)
- Reformulates the notion of the ~~MAGNETIC MONOPOLE~~ (= tensor of rank 3)
- Has implications for (Dirac) 'string theory' in quantum electrodynamics
- Generalizes to **(N + 1)-spacetime** (relativity, cosmology) (De Hoop, 2011)
- Directly matches **array handling** in computational coding

• (More or less) COMPLETE EM COURSE:

Part	Degree of difficulty
• Field equations	*
• Radiation from sources	**
• Field/source/contrast-in-medium reciprocity relations	*
• Energy balance	*
• Balance of field momentum ('radiation pressure')	***

in 4 PAGES ! \implies

De Hoop, Conference Proceedings URSI EMT-S Berlin 2010

• ALSO:

- Lorentz covariance (special relativity)

END

INTRODUCTION

RADIATION FROM SOURCES

IN UNBOUNDED

HOMOGENEOUS,

ISOTROPIC MEDIA

Prerequisites (differentiation, convolution, Dirac pulse)

- $\partial_t [\partial_m F(\cdot)] = \partial_m [\partial_t F(\cdot)]$
- $F(\cdot) \overset{(t)}{*} G(\cdot) = G(\cdot) \overset{(t)}{*} F(\cdot)$ | • $F(\cdot) \overset{(\mathbf{x})}{*} G(\cdot) = G(\cdot) \overset{(\mathbf{x})}{*} F(\cdot)$
- $\overset{(\mathbf{x},t)}{*} = \overset{(\mathbf{x})(t)}{*} = \overset{(t)(\mathbf{x})}{*}$
- $\partial_t [F(\cdot) \overset{(\mathbf{x},t)}{*} G(\cdot)] = [\partial_t F(\cdot)] \overset{(\mathbf{x},t)}{*} G(\cdot) = F(\cdot) \overset{(\mathbf{x},t)}{*} [\partial_t G(\cdot)]$
- $\partial_m [F(\cdot) \overset{(\mathbf{x},t)}{*} G(\cdot)] = [\partial_m F(\cdot)] \overset{(\mathbf{x},t)}{*} G(\cdot) = F(\cdot) \overset{(\mathbf{x},t)}{*} [\partial_m G(\cdot)]$
- $\delta(t) \overset{(t)}{*} F(\cdot) = F(\cdot)$ | • $\delta(\mathbf{x}) \overset{(\mathbf{x})}{*} F(\cdot) = F(\cdot)$

- **EM RADIATION FROM SOURCES:** $\epsilon > 0, \mu^- > 0$

EM field equations at (\boldsymbol{x}, t)	operation
• $\partial_m [H_{m,k}]^- + \epsilon \partial_t E_k = -J_k$	$\mu^- \partial_t$
• $[\partial_i E_j]^- + \mu^- \partial_t [H_{i,j}]^- = -[K_{i,j}]^-$	\uparrow

- Elimination of $[H_{i,j}]^-$ + use of

- $\partial_k E_k = -(1/\epsilon) \partial_t^{-1} (\partial_m J_m)$ (compatibility relation) \implies

- **ELECTRIC-FIELD VECTOR WAVE EQUATION:**

$$\bullet (\partial_m \partial_m) E_k - c^{-2} \partial_t^2 E_k = -F_k$$

- $c = 1/(2\mu^- \epsilon)^{1/2}$ (EM wavespeed)
- $F_k = -2\mu^- \partial_t J_k + (1/\epsilon) \partial_t^{-1} \partial_k (\partial_m J_m) + 2\partial_m [K_{m,k}]^-$

• **ELECTRIC-FIELD VECTOR WAVE EQUATION:**

$$\bullet (\partial_m \partial_m) E_k - c^{-2} \partial_t^2 E_k = - F_k = - \delta(\mathbf{x}, t) \underset{*}{*}^{(\mathbf{x}, t)} F_k$$

$$\underset{*}{*}^{(\mathbf{x}, t)} = \underset{*}{*}^{(\mathbf{x})(t)}$$

$$\bullet F_k = -2 \mu^- \partial_t J_k + (1/\epsilon) \partial_t^{-1} \partial_k (\partial_m J_m) + 2 \partial_m [K_{m,k}]^-$$

⇒ **SOLUTION:**

$$\bullet E_k = G \underset{*}{*}^{(\mathbf{x}, t)} F_k$$

$$\bullet (\partial_m \partial_m) G - c^{-2} \partial_t^2 G = - \delta(\mathbf{x}, t)$$

$$\bullet G = \frac{\delta(t - |\mathbf{x}|/c)}{4\pi |\mathbf{x}|} \text{ for } \mathbf{x} \neq \mathbf{0} \quad (3D) \quad (\text{scalar Green's function})$$

$$\underset{*}{*}^{(\mathbf{x}, t)} \partial_m \partial_t = \partial_m \partial_t \underset{*}{*}^{(\mathbf{x}, t)} \quad \Downarrow$$

• **THE RADIATED FIELD:** [• $E_k = G \stackrel{(\mathbf{x},t)}{*} F_k \implies]$

$$\begin{aligned} & \bullet E_k = -2 \mu^- \partial_t A_k + (1/\epsilon) \partial_t^{-1} \partial_k \partial_m A_m + 2 \partial_m [\Psi_{m,k}]^- \\ & \bullet [H_{i,j}]^- = -2 \epsilon \partial_t [\Psi_{i,j}]^- + (1/\mu^-) \partial_t^{-1} \partial_m [\partial_m [\Psi_{i,j}]^-]^\circ + 2 [\partial_i A_j]^- \end{aligned}$$

• **VECTOR POTENTIAL:**

$$\begin{aligned} & \bullet A_k = G \stackrel{(\mathbf{x},t)}{*} J_k \\ & \bullet (\partial_m \partial_m) A_k - c^{-2} \partial_t^2 A_k = -J_k \end{aligned}$$

• **TENSOR POTENTIAL:**

$$\begin{aligned} & \bullet [\Psi_{i,j}]^- = G \stackrel{(\mathbf{x},t)}{*} [K_{i,j}]^- \\ & \bullet (\partial_m \partial_m) [\Psi_{i,j}]^- - c^{-2} \partial_t^2 [\Psi_{i,j}]^- = -[K_{i,j}]^- \end{aligned}$$

• **SCALAR GREEN'S FUNCTION:**

$$\begin{aligned} & \bullet G = \frac{\delta(t - |\mathbf{x}|/c)}{4\pi|\mathbf{x}|} \text{ for } \mathbf{x} \neq \mathbf{0} \text{ (3D)} \\ & \bullet (\partial_m \partial_m) G - c^{-2} \partial_t^2 G = -\delta(\mathbf{x}, t) \end{aligned}$$

N.B. Use is made of the identity • $\partial_m \partial_n [\partial_n [\Psi_{m,k}]^-]^\circ = 0$

• THE RADIATED FIELD:

Sources	Medium	Field	3D
$\begin{bmatrix} J_k \\ [K_{i,j}]^- \end{bmatrix}$	ϵ, μ^- \implies $c = 1/(2 \mu^- \epsilon)^{1/2}$	$\begin{bmatrix} E_r \\ [H_{p,q}]^- \end{bmatrix}$	Physics
$\mathbf{x} \in \text{supp}(J_k, [K_{i,j}]^-) \subset \mathbb{R}^3$	$\mathbf{x} \in \mathbb{R}^3$	$\mathbf{x} \in \mathbb{R}^3$	Position
$t \in \mathbb{R}, t > t_0$		$t \in \mathbb{R}, t > t_0 + \mathbf{x} /c$	Time

• STRUCTURE: [• $S(t)$ = source signature]

Magnitude	Pulse shape	Directional characteristic
$1/ \mathbf{x} $	$\partial_t S(t)$	'Far-field'
$1/ \mathbf{x} ^2$	$S(t)$	
$1/ \mathbf{x} ^3$	$\partial_t^{-1} S(t)$	'Near-field'

\implies **COMPLICATED MIXTURE !!!**

• THE RADIATED FIELD (FAR-FIELD APPROXIMATION):

- $\mathbf{x}' \in \text{supp}(J_k, [K_{i,j}]^-)$ | • $\mathbf{x} \in \mathbb{R}^3$ | • $|\mathbf{x}| \rightarrow \infty$
- $|\mathbf{x} - \mathbf{x}'| = |\mathbf{x}| - \xi_m x'_m + O(|\mathbf{x}|^{-1})$ as $|\mathbf{x}| \rightarrow \infty$ | • $\xi_m = x_m/|\mathbf{x}|$ ($\xi_m \xi_m = 1$)
- $\{A_k, [\Psi_{i,j}]^-, E_r, [H_{p,q}]^-\}(\mathbf{x}, t) = \frac{\{A_k^\infty, [\Psi_{i,j}^\infty]^- , E_r^\infty, [H_{p,q}^\infty]^- \}(\boldsymbol{\xi}, t - |\mathbf{x}|/c)}{4\pi|\mathbf{x}|}$
 $[1 + O(|\mathbf{x}|^{-1})]$ as $|\mathbf{x}| \rightarrow \infty$
- $\partial_m(\cdot) = (\xi_m/c)\partial_t(\cdot) [1 + O(|\mathbf{x}|^{-1})]$ as $|\mathbf{x}| \rightarrow \infty$
- $A_k^\infty = \int_{\text{supp}(J_k)} J_k(\mathbf{x}', t + \xi_m x'_m/c) dV(\mathbf{x}')$
- $[\Psi_{i,j}^\infty]^- = \int_{\text{supp}([K_{i,j}]^-)} [K_{i,j}]^-(\mathbf{x}', t + \xi_m x'_m/c) dV(\mathbf{x}')$
- $E_r^\infty = -2\mu^-(\delta_{r,k} - \xi_r \xi_k)\partial_t A_r^\infty + 2(\xi_m/c)\partial_t [\Psi_{m,r}^\infty]^-$
- $[H_{i,j}^\infty]^- = -2\epsilon(\partial_t [\Psi_{i,j}^\infty]^- - \xi_m [\xi_m \partial_t [\Psi_{i,j}^\infty]^-]^\circ) + 2[(\xi_i/c)\partial_t A_j^\infty]^-$

- THE RADIATED FIELD:

- FAR-FIELD APPROXIMATION:

- $\{E_r, [H_{p,q}]^-\}(\mathbf{x}, t) = \frac{\{E_r^\infty, [H_{p,q}^\infty]^- \}(\boldsymbol{\xi}, t - |\mathbf{x}|/c)}{4\pi|\mathbf{x}|} [1 + O(|\mathbf{x}|^{-1})] \text{ as } |\mathbf{x}| \rightarrow \infty$

⇒ LOCAL PLANE-WAVE AMPLITUDES
(DIRECTION OF PROPAGATION ALONG $\boldsymbol{\xi}$):

- $(\xi_m/c) [H_{m,k}^\infty]^- + \epsilon E_k^\infty = 0$
- $[(\xi_i/c) E_j^\infty]^- + \mu^- [H_{i,j}^\infty]^- = 0$

END

RADIATION FROM SOURCES

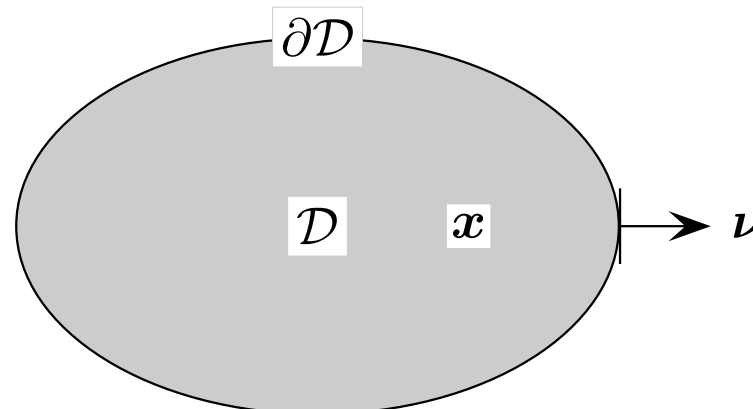
TIME-DOMAIN FIELD/SOURCE

RECIPROACITY

(TIME-CONVOLUTION TYPE)

- FIELD RECIPROCITY ('interaction' of two EM 'STATES'):

Reciprocity ($x \in \mathcal{D} \cup \partial\mathcal{D}, t \in \mathbb{R}$)			
State	Field strengths	Flux densities	Sources
A	$E_r^A, [H_{p,q}^A]^-$	$D_k^A, [B_{i,j}^A]^-$	$J_k^A, [K_{i,j}^A]^-$
B	$E_k^B, [H_{i,j}^B]^-$	$D_r^B, [B_{p,q}^B]^-$	$J_r^B, [K_{p,q}^B]^-$



EM field/source reciprocity

• TIME-CONVOLUTION FIELD RECIPROcity:

State	EM field equations ($x \in \mathcal{D} \cup \partial\mathcal{D}$, $t \in \mathbb{R}$)	Operation
A	• $\partial_m [H_{m,k}^A]^- (t) + \partial_t D_k^A(t) = -J_k^A(t)$	$\begin{matrix} (t) \\ * \\ E_k^B(t) \end{matrix}$
B	• $[\partial_p E_q^B(t)]^- + \partial_t [B_{p,q}^B]^- (t) = -[K_{p,q}^B]^- (t)$	$\begin{matrix} (t) \\ * \\ [H_{p,q}^A]^- (t) \end{matrix}$
	• $- A \rightleftharpoons B, k \rightleftharpoons r, \{p, q\} \rightleftharpoons \{i, j\}$	

• LOCAL:

- ∂_m [Electric-field/Magnetic-field interaction term] +
 ∂_t [Contrast-in-media interaction terms] =
 Sources/Fields (electric/electric, magnetic/magnetic) interaction terms

• TIME-CONVOLUTION FIELD RECIPROcity (CONTINUED):

Electric-field/Magnetic-field interaction term:

- $[H_{m,k}^A]^{-}(t) * E_k^B(t) - [H_{m,r}^B]^{-}(t) * E_r^A(t)$

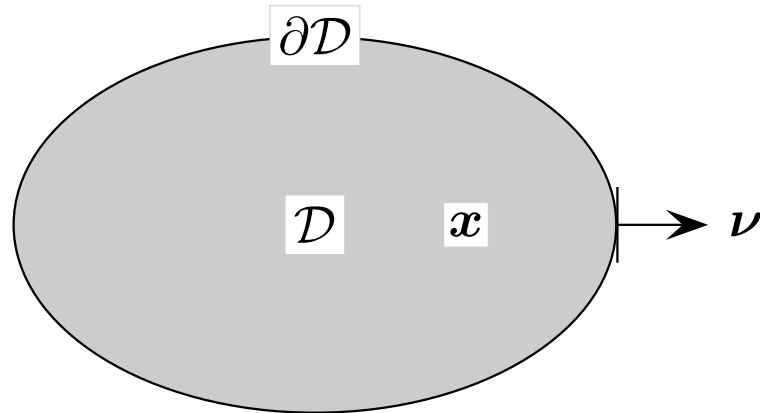
Contrast-in-media interaction terms:

- $E_k^B(t) * D_k^A(t) - E_r^A(t) * D_r^B(t) +$
 $[B_{i,j}^B]^{-}(t) * [H_{i,j}^A]^{-}(t) - [B_{p,q}^A]^{-}(t) * [H_{p,q}^B]^{-}(t) =$
 $E_k^B(t) * [\epsilon_{k,r}^A(t) - \epsilon_{r,k}^B(t)] * E_r^A(t)$
 $[H_{i,j}^A]^{-}(t) * [\mu_{i,j,p,q}^{-,-} B(t) - \mu_{p,q,i,j}^{-,-} A(t)] * [H_{p,q}^B]^{-}(t)$

Sources/Fields (electric/electric, magnetic/magnetic) interaction terms:

- $-J_k^A(t) * E_k^B(t) + J_r^B(t) * E_r^A(t) -$
 $[K_{i,j}^B]^{-}(t) * [H_{i,j}^A]^{-}(t) + [K_{p,q}^A]^{-}(t) * [H_{p,q}^B]^{-}(t)$

• TIME-CONVOLUTION FIELD RECIPROcity (GLOBAL):

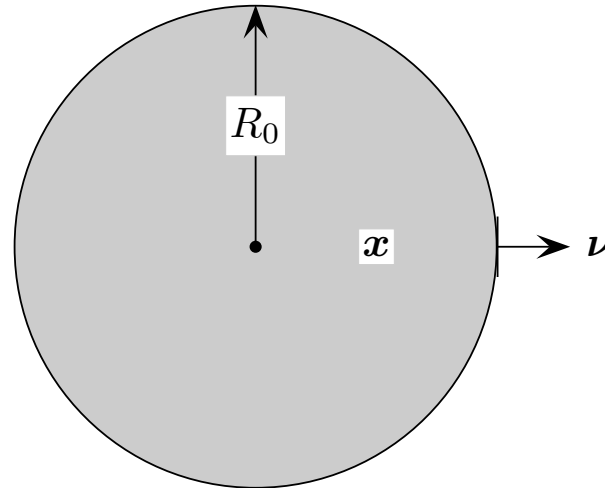


• $\int_D \partial_m [\dots] dV \stackrel{\text{GAUSS}}{=} \int_{\partial D} \nu_m [\dots] dA$

$$\int_{\partial D} \nu_m [\text{Electric-field/Magnetic-field interaction term}] dA + \int_D \partial_t [\text{Contrast-in-media interaction terms}] dV = \int_D [\text{Sources/Fields (electric/electric, magnetic/magnetic) interaction terms}] dV$$

- TIME-CONVOLUTION FIELD RECIPROcity

- (UNBOUNDED DOMAINS):



- FAR-FIELD APPROXIMATION

$$\Rightarrow \lim_{R_0 \rightarrow \infty} \int_{\{|\mathbf{x}|=R_0\}} \nu_m \text{ [Electric-field/Magnetic-field interaction term]} dA = 0$$

- **TIME-CONVOLUTION FIELD RECIPROcity**

- **(APPLICATIONS):**

- Analysis of Transmission/Reception properties of antennas
- ElectroMagnetic Compatibility (EMC) disturbance analysis (Emitter = Susceptor)
- Necessary (but not sufficient) check on EM field computational procedures

END

TIME-DOMAIN FIELD/SOURCE

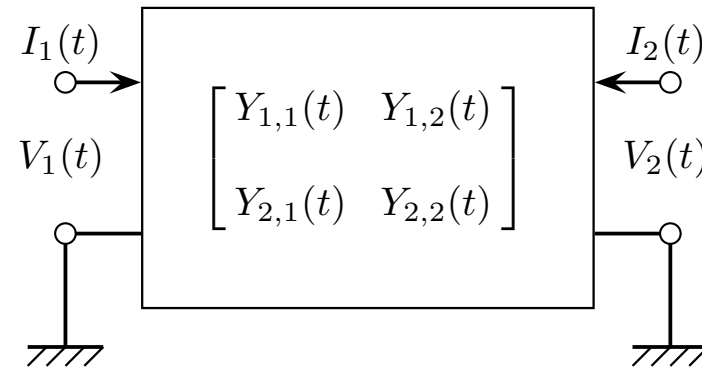
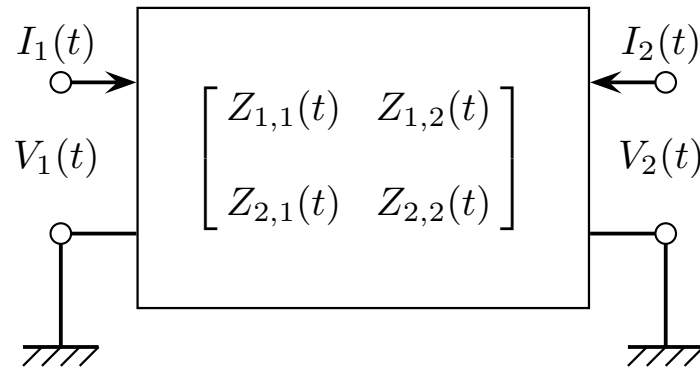
RECIPROACITY

(TIME-CONVOLUTION TYPE)

MAXWELL FIELDS /
KIRCHHOFF CIRCUITS
INTERACTION

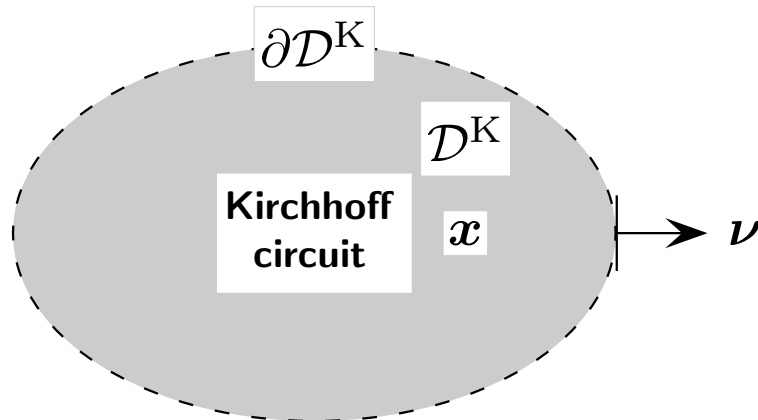
Maxwell fields / Kirchhoff circuits interaction

• KIRCHHOFF CIRCUITS (LINEAR, TIME-INVARIANT, CAUSAL):



- $[V(t)] = [Z(t)]^{(t)} * [I(t)]$
- $[I(t)] = [Y(t)]^{(t)} * [V(t)]$
- $[V(t)] =$ port voltages
- $[I(t)] =$ port electric currents
- $[Z(t)] =$ impedance matrix $[Z(t)] = 0$ for $t < 0$
- $[Y(t)] =$ admittance matrix $[Y(t)] = 0$ for $t < 0$

• MAXWELL FIELDS / KIRCHHOFF CIRCUITS INTERACTION:



- $\mathcal{D}^K = \text{supp}(\text{Kirchhoff circuit})$
- $\text{maxdiam}(\mathcal{D}^K) \ll$
(EM wavespeed embedding) ·
(signal pulse time width)

$$\int_{\partial \mathcal{D}^K} \nu_m \left[[H_{m,k}^A]^{-}(t) \stackrel{(t)}{*} E_k^B(t) - [H_{m,r}^B]^{-}(t) \stackrel{(t)}{*} E_r^A(t) \right] dA =$$

$$\sum_{p \in \{\text{ports}\}} \left[I_p^A \stackrel{(t)}{*} V_p^B - I_p^B \stackrel{(t)}{*} V_p^A \right]$$

END

MAXWELL FIELDS /

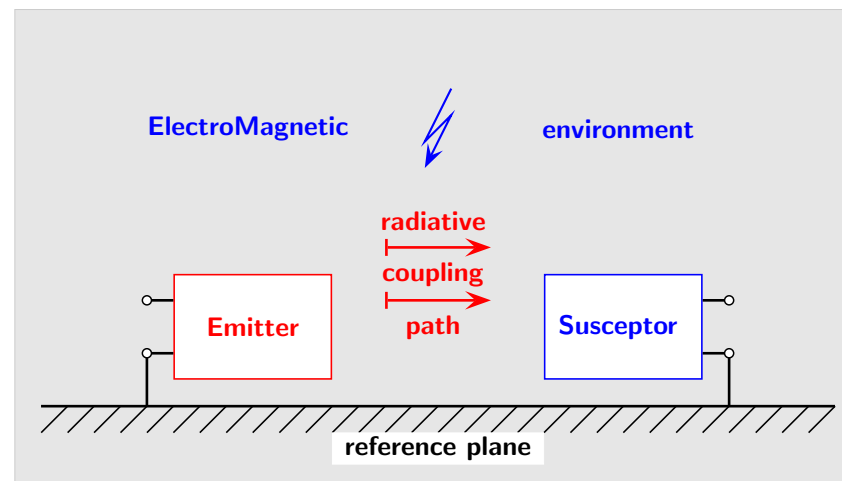
KIRCHHOFF CIRCUITS

INTERACTION

**APPLIED EM TOPICS IN
ELECTRONICS
AND IT**

- **APPLIED EM TOPICS IN ELECTRONICS AND IT:**

- Signal integrity in inter-/intra-system pulsed-field radiative signal transfer
- Pulsed-field EM Interference (EMI) analysis



- Time-domain (TD) regulatory EM Compatibility (EMC) specifications (TD emission limits, TD immunity limits)

END

APPLIED EM TOPICS IN

ELECTRONICS

AND IT

THE END

THANK YOU FOR

YOUR ATTENTION