18

The electromagnetic field equations

The electromagnetic field equations take on their simplest form in a vacuum domain. In such a domain, they follow, in principle, from a series of basic experiments in which an observer handles an electric point charge as a measuring device, and the force exerted on this point charge is measured. Through this force, the electric and the magnetic field strengths, as experienced by the observer who is assumed to be located in a vacuum domain, are introduced. The electromagnetic field equations in matter are next introduced through an axiomatic procedure that leaves intact the structure of the field equations in a vacuum domain in case the point of observation is located in a vacuum domain, even when matter is present elsewhere. Since external sources that generate the electromagnetic field are essentially composed of matter, they are encompassed in the volume densities of electric and magnetic current that describe the electromagnetic action of matter. The induced parts of the latter quantities then describe the passive reaction of a piece of matter to an electromagnetic field.

18.1 Force exerted on an electric point charge

From experience it is known that electrically charged particles exert forces on each other. These forces depend on the relative position and the relative state of motion of the charged particles. We take a single electrically charged particle of known strength and use this as a measuring device for defining, in a vacuum domain, the electric and the magnetic field strengths of an electromagnetic field. It is assumed that an observer who is handling this measuring device can measure the force exerted on it (for example, by counterbalancing the force by an adjustable mechanical one or by following the trajectory of the charged particle). The experiments show the following:

(a) the force F_r (a vectorial quantity known from Newtonian mechanics) is proportional to the strength q of the test point charge (a scalar quantity and a fundamental concept in electromagnetics);

(b) the force F_r contains a term that is independent of the velocity v_m (a kinematical vectorial quantity) of the test point charge with respect to the observer;

(c) the force F_r contains a term that is proportional to the velocity v_m of the test point charge with respect to the observer, which term has an orientation perpendicular to v_m.

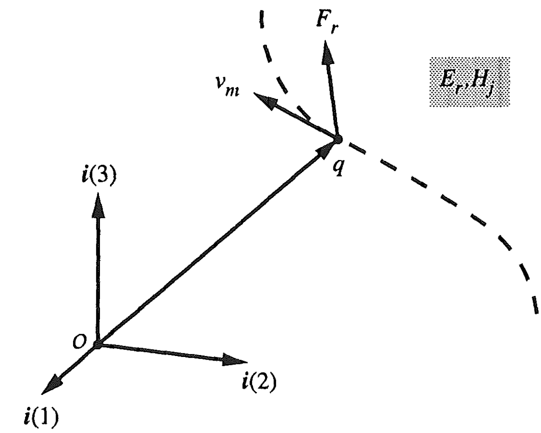

Figure 18.1-1 Force F_r on an electric point charge of strength q and moving with velocity v_m with respect to an observer at rest in the reference frame $\{O;\boldsymbol{i}(1),\boldsymbol{i}(2),\boldsymbol{i}(3)\}$.

In accordance with this, and with the conventions of the International System of Units, we postulate the expression (Figure 18.1-1)

$$F_r = qE_r + q\mu_0\varepsilon_{r,m,j}v_mH_j \,. \tag{18.1-1}$$

In this expression, the use of an orthogonal Cartesian reference frame with origin O and three mutually perpendicular base vectors $\{\boldsymbol{i}(1),\boldsymbol{i}(2),\boldsymbol{i}(3)\}$ of unit length each is assumed. In the indicated order, the base vectors form a right-handed system. In this reference frame $\boldsymbol{x} = \Sigma_{m=1}^{3} x_m\boldsymbol{i}(m)$ is the position vector, t is the time, and

F_r = force (N),

q = electric charge (C),

v_m = velocity of point charge with respect to observer (m/s),

μ_0 = permeability of vacuum ($\mu_0 = 4\pi\times10^{-7}$ H/m),

E_r = electric field strength (V/m),

H_j = magnetic field strength (A/m),

while $\varepsilon_{r,m,j}$ is the three-dimensional completely anti-symmetric unit tensor of rank three (Levi–Civita tensor). The constant μ_0 is fixed by the choice of the employed system of units. (The given value is the one that applies to the International System of Units.) Assuming that the value of q is so small that the reaction of the test point charge on the sources that generate the electromagnetic field can be neglected, Equation (18.1-1) defines the value of the electric field strength $E_r = E_r(\boldsymbol{x},t)$ and the value of the magnetic field strength $H_j = H_j(\boldsymbol{x},t)$ at the position of observation $\boldsymbol{x}$ with respect to a given, fixed, orthogonal Cartesian reference frame and at the time of observation t. Since the observer must be free to position and to move his or her measuring device, the operational definition given in Equation (18.1-1) can only be applied in a vacuum domain and cannot be used in matter.

Note: The assumption that the measuring device has a negligibly small influence on the quantities to be measured, is fundamental to any macroscopic measurement; Heisenberg's uncertainty relation in quantum theory puts a non-zero lower limit on this kind of interaction.

By first choosing, in Equation (18.1-1), $v_m = 0$, one determines by measuring F_r the value of $E_r = E_r(x,t)$. By giving, subsequently, v_m three linearly independent values, one can determine $H_j = H_j(x,t)$ by measuring F_r and using the already known values of E_r (see Exercise 18.1-2). The quantity

$$F_r^{\mathrm{e}} = qE_r \tag{18.1-2}$$

is sometimes denoted as the *electric force*; the quantity

$$F_r^{\mathrm{m}} = q\mu_0 \varepsilon_{r,m,j} v_m H_j \tag{18.1-3}$$

is sometimes denoted as the *magnetic force* or *Lorentz force* (after H.A. Lorentz).

It is emphasised that the thus determined values of E_r and H_j have the meaning of electric and magnetic field strength, respectively, only for the observer who carries out the measurements and interprets them according to Equation (18.1-1). A second observer who is in relative motion with respect to the first one, measures different values of the electric and the magnetic field strengths. In particular, if the second observer moves with the point charge handled by the first, we have, denoting the latter's quantities by primed symbols, $v_m' = 0$, and hence, $F_r' = qE_r'$, i.e. the second observer interprets the electromagnetic field present as a pure electric one. (In accordance with the theory of relativity, we have put $q' = q$.) The interrelations that exist between the values of the electric and the magnetic field strengths of two observers who are in relative motion are investigated in the theory of relativity.

Exercises

Exercise 18.1-1

Show that from Equation (18.1-1) it follows that $v_r(F_r - qE_r) = 0$. (*Hint*: Note that $v_r \varepsilon_{r,m,j} v_m = 0$.) Hence, the magnetic force has no component along the velocity of the point charge; in other words: it is perpendicular to it.

Exercise 18.1-2

Show that from Equation (18.1-1) it follows that $q\mu_0(v_m v_m H_p - v_p v_j H_j) = \varepsilon_{p,r,s}(F_r - qE_r)v_s$. (*Hint*: Use Equation (A.7-51).)

Exercise 18.1-3

Write Equation (18.1-1) out in its components.

Answer:

$$F_1 = qE_1 + q\mu_0(v_2H_3 - v_3H_2)\,, \tag{18.1-4}$$

$$F_2 = qE_2 + q\mu_0(v_3H_1 - v_1H_3)\,, \tag{18.1-5}$$

$$F_3 = qE_3 + q\mu_0(v_1H_2 - v_2H_1) . \tag{18.1-6}$$

Exercise 18.1-4

Show that from the result of Exercise 18.1-3 it follows that

$$F_1v_1 + F_2v_2 + F_3v_3 = q(E_1v_1 + E_2v_2 + E_3v_3) . \tag{18.1-7}$$

18.2 The electromagnetic field equations in vacuum

Using the electric point charge as a measuring device one can investigate the properties of an electromagnetic field in a vacuum domain. In particular, it proves to be fruitful to investigate the interrelation between the changes of the field in space and the changes of the field in time. Quantitatively, this is done by mutually comparing the first-order partial derivatives with respect to the spatial coordinates with the first-order partial derivatives with respect to time. As to the spatial derivatives one can conjecture that they must, in the equations, occur in an "isotropic" manner, since vacuum is, by definition, isotropic in its geometrical structure. Carrying out the analysis, it is found that

$$-\varepsilon_{k,m,p}\partial_m H_p + \varepsilon_0\partial_t E_k = 0 , \tag{18.2-1}$$

$$\varepsilon_{j,n,r}\partial_n E_r + \mu_0\partial_t H_j = 0 . \tag{18.2-2}$$

In Equation (18.2-1),

$$\varepsilon_0 = (\mu_0 c_0^2)^{-1} \tag{18.2-3}$$

denotes the permittivity of vacuum. Substituting the values $\mu_0 = 4\pi \times 10^{-7}$ H/m and $c_0 =$ 299 792 458 m/s (exact value in the International System of Units), one obtains

$$\varepsilon_0 \approx 8.8541878 \times 10^{-12}\text{F/m}. \tag{18.2-4}$$

Equation (18.2-1) is known as *Maxwell's first equation* (in vacuum); Equation (18.2-2) is known as *Maxwell's second equation* (in vacuum).

Compatibility relations

By carrying out on Equation (18.2-1) the differentiation ∂_k and noting that $\varepsilon_{k,m,p}\partial_k\partial_m H_p = 0$, it follows that

$$\partial_t\partial_k E_k = 0 . \tag{18.2-5}$$

Similarly, by carrying out on Equation (18.2-2) the differentiation ∂_j and noting that $\varepsilon_{j,n,r}\partial_j\partial_n E_r = 0$, it follows that

$$\partial_t\partial_j H_j = 0 . \tag{18.2-6}$$

Now, assume that the electromagnetic field that we are probing in a vacuum domain is due to the causal action of sources located somewhere else in space that have been switched on at the instant $t = t_0$ in the finite past. Then, prior to this instant (i.e. prior to the action of the sources), the field vanishes in the vacuum domain, i.e.

$$E_k(\boldsymbol{x},t) = 0 \qquad \text{for } t \leqslant t_0 \tag{18.2-7}$$

and

$$H_j(\boldsymbol{x},t) = 0 \qquad \text{for } t \leqslant t_0 \tag{18.2-8}$$

at all positions $\boldsymbol{x}$ of that domain, while Equations (18.2-5) and (18.2-6) hold for all $t > t_0$. By carrying out the differentiation ∂_k on Equation (18.2-7) it follows that

$$\partial_k E_k(\boldsymbol{x},t) = 0 \qquad \text{for } t \leqslant t_0 \tag{18.2-9}$$

at all points of the domain under consideration. Similarly, by carrying out the differentiation ∂_j on Equation (18.2-8) it follows that

$$\partial_j H_j(\boldsymbol{x},t) = 0 \qquad \text{for } t \leqslant t_0 \tag{18.2-10}$$

at all points of the domain under consideration. Now, integration of Equation (18.2-5) with respect to t leads to

$$0 = \int_{t'=0}^{t} \partial_{t'}(\partial_k E_k)\, \mathrm{d}t' = \partial_k E_k(\boldsymbol{x},t) - \partial_k E_k(\boldsymbol{x},t_0) \qquad \text{for } t \geqslant t_0\,. \tag{18.2-11}$$

Upon combining Equation (18.2-11) with Equation (18.2-9), the final result

$$\partial_k E_k = 0 \qquad \text{for } t \geqslant t_0 \tag{18.2-12}$$

follows. Similarly, integration of Equation (18.2-6) with respect to t leads to

$$0 = \int_{t'=0}^{t} \partial_{t'}(\partial_j H_j)\, \mathrm{d}t' = \partial_j H_j(\boldsymbol{x},t) - \partial_j H_j(\boldsymbol{x},t_0) \qquad \text{for } t \geqslant t_0\,. \tag{18.2-13}$$

Upon combining Equation (18.2-13) with Equation (18.2-10), the final result

$$\partial_j H_j = 0 \qquad \text{for } t \geqslant t_0 \tag{18.2-14}$$

follows. Equations (18.2-12) and (18.2-14) are therefore not independent of Maxwell's equations and are denoted as *compatibility relations*. The compatibility relations hold for causal fields and prove to be in accordance with the basic experiments that have been used to arrive at Equations (18.2-1) and (18.2-2).

Exercises

Exercise 18.2-1

Let $F = F(\boldsymbol{x},t)$ be a tensor function of arbitrary rank, defined over some subdomain $\mathcal{D}$ of $\mathcal{R}^3$ and for all $t \in \mathcal{R}$. Let, further, $\partial_t F(\boldsymbol{x},t) = 0$ for all $\boldsymbol{x} \in \mathcal{D}$ and all $t \in \mathcal{R}$, while $F(\boldsymbol{x},t_0) = 0$ for all $\boldsymbol{x} \in \mathcal{D}$. Show that also $F(\boldsymbol{x},t) = 0$ for all $\boldsymbol{x} \in \mathcal{D}$ and $t > t_0$. (*Hint*: Note that

$$0 = \int_{t'=t_0}^{t} \partial_{t'} F(\boldsymbol{x},t')\, \mathrm{d}t' = F(\boldsymbol{x},t) - F(\boldsymbol{x},t_0)\ .)$$

18.3 The electromagnetic field equations in matter

Methodologically the simplest way to account for the presence of matter is to retain the vectorial nature of the electromagnetic field equations and to introduce in the right-hand sides of Equations (18.2-1) and (18.2-2) vectorial quantities that differ from zero only in a domain occupied by matter (and necessarily reduce to zero in a vacuum domain). Let us write (Figure 18.3-1)

$$-\varepsilon_{k,m,p}\partial_m H_p + \varepsilon_0 \partial_t E_k = -J_k^{\text{mat}}, \tag{18.3-1}$$

$$\varepsilon_{j,n,r}\partial_n E_r + \mu_0 \partial_t H_j = -K_j^{\text{mat}}. \tag{18.3-2}$$

Here,

J_k^{mat} = volume density of material electric current (A/m^2),

K_j^{mat} = volume density of material magnetic current (V/m^2).

As we have said,

$$J_k^{\text{mat}} = 0 \quad \text{and} \quad K_j^{\text{mat}} = 0 \quad \text{in a vacuum domain}. \tag{18.3-3}$$

First of all, we shall distinguish in J_k^{mat} and K_j^{mat} between an *active part* and a *passive part.* The active parts (also denoted as the source parts, or *external parts*) are the ones that describe the action of sources that generate the field; their detailed physical behaviour is either irrelevant to or outside the scope of the analysis at hand. They will be denoted by J_k^{ext} and K_j^{ext}, respectively, and are taken to be field independent. The passive parts (also denoted as the *induced parts*) describe the reaction of matter to the presence of an electromagnetic field and are typically field dependent; they will be denoted by J_k^{ind} and K_j^{ind}, respectively. Accordingly, we have

$$J_k^{\text{mat}} = J_k^{\text{ind}} + J_k^{\text{ext}} \quad \text{and} \quad K_j^{\text{mat}} = K_j^{\text{ind}} + K_j^{\text{ext}}. \tag{18.3-4}$$

Traditionally, the induced parts are written in a somewhat different manner, viz.

$$J_k^{\text{ind}} = J_k + \partial_t P_k, \tag{18.3-5}$$

and

$$K_j^{\text{ind}} = \mu_0 \partial_t M_j, \tag{18.3-6}$$

where

J_k = volume density of electric (conduction) current (A/m^2),

P_k = electric polarisation (C/m^2),

M_j = magnetisation (A/m).

The latter quantities are particularly useful for describing the electromagnetic properties of matter in case only fields varying slowly in time (quasi-static fields) are present. Substitution of Equations (18.3-4)–(18.3-6) in Equations (18.3-1) and (18.3-2) leads to

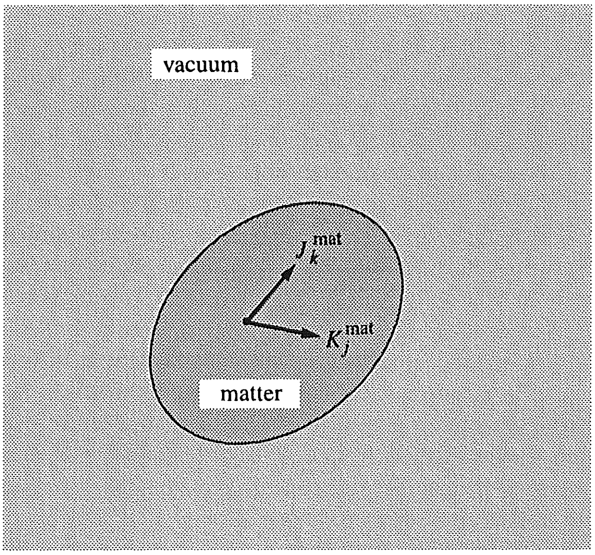

Figure 18.3-1 Piece of matter embedded in vacuum, with volume density of material electric current J_k^{mat} and volume density of material magnetic current K_j^{mat}.

$$-\varepsilon_{k,m,p}\partial_m H_p + \varepsilon_0 \partial_t E_k + J_k + \partial_t P_k = -J_k^{ext}, \qquad (18.3\text{-}7)$$

$$\varepsilon_{j,n,r}\partial_n E_r + \mu_0 \partial_t H_j + \mu_0 \partial_t M_j = -K_j^{ext}. \qquad (18.3\text{-}8)$$

Furthermore, it is customary to introduce the quantities

$$D_k = \varepsilon_0 E_k + P_k\,, \qquad (18.3\text{-}9)$$

and

$$B_j = \mu_0 (H_j + M_j)\,, \qquad (18.3\text{-}10)$$

where

D_k = electric flux density (C/m^2),

B_j = magnetic flux density (T).

With the aid of Equations (18.3-9) and (18.3-10), Equations (18.3-7) and (18.3-8) can be rewritten as

$$-\varepsilon_{k,m,p}\partial_m H_p + J_k + \partial_t D_k = -J_k^{ext}, \qquad (18.3\text{-}11)$$

$$\varepsilon_{j,n,r}\partial_n E_r + \partial_t B_j = -K_j^{ext}, \qquad (18.3\text{-}12)$$

which is the final form of the electromagnetic field equations. Equation (18.3-11) is known as *Maxwell's first equation* (in matter), Equation (18.3-12) is known as *Maxwell's second equation* (in matter). The quantity $J_k + \partial_t D_k$ occurring in Equation (18.3-11) is sometimes denoted as the *Maxwell current density*.

It is clear that through the introduction of the (induced) quantities J_k, P_k and M_j, in addition to the electric field strength E_r and the magnetic field strength H_p, the number of unknown quantities (five vectorial quantities if the source terms are assumed to be known) is larger than the number of equations (two vectorial equations). Consequently, the system of Equations (18.3-11) and (18.3-12) is as yet an underdetermined one. Now, from a physical point of view, the electromagnetic field must be uniquely determined once the sources that generate the field and the distribution of matter are given. Hence, in order to make the number of equations equal to the number of unknown quantities, Maxwell's equations in matter remain to be supplemented by another set of relations that is equivalent to three vectorial relations between the five field quantities occurring in Equations (18.3-11) and (18.3-12). These supplementary relations are known as the *constitutive relations*; they are representative for the electromagnetic response of a passive piece of matter to the presence of an electromagnetic field. As far as the constitutive relations are concerned, we can on one hand adhere to a strictly macroscopic point of view where the constitutive relations follow from a series of appropriate experiments. On the other hand, we can try to arrive at the macroscopic constitutive relations by employing a microscopic model for the kind of matter under consideration. Through a hypothesis about the interaction between the elementary building blocks of matter, the macroscopic constitutive relations then follow after applying an appropriate spatial averaging procedure of the type that is common in statistical physics. One such model is furnished by the Lorentz theory of electrons where, on a microscopic scale, matter is conceived to consist of electrically charged particles that move around in vacuum and interact according to the classical laws of physics (see Lorentz 1909). In some fields of application, the Lorentz model has proved to be successful; examples in this category will be discussed in Sections 19.4–19.7. In other fields, the interaction on a microscopic scale can only be adequately described with the aid of quantum theory; these considerations are outside the scope of the present treatment (see, for example, Kittel 1986).

Compatibility relations

As far as the compatibility relations are concerned, we observe that the application of the operators ∂_k and ∂_j to Equations (18.3-11) and (18.3-12), leads to

$$\partial_k(J_k + \partial_t D_k) = -\partial_k J_k^{\text{ext}}, \tag{18.3-13}$$

and

$$\partial_j \partial_t B_j = -\partial_j K_j^{\text{ext}}, \tag{18.3-14}$$

respectively. Equations (18.3-13) and (18.3-14) replace, in matter, the vacuum domain compatibility relations given in Equations (18.2-12) and (18.2-14), respectively.

Historically again, in the context of Equation (18.3-13) the volume density of electric charge is introduced as

$$\rho = \partial_k D_k\,. \tag{18.3-15}$$

How this quantity is related to the microscopic point of view where electric current is interpreted as a flow of electrically charged particles, will be expounded in Section 19.4.

Exercises

Exercise 18.3-1

Write Maxwell's equations (18.3-11) and (18.3-12) out in terms of their components.

Answer:

$$-\partial_1 H_2 + \partial_2 H_1 + J_3 + \partial_t D_3 = -J_3^{\text{ext}}, \quad (18.3\text{-}16)$$

$$-\partial_2 H_3 + \partial_3 H_2 + J_1 + \partial_t D_1 = -J_1^{\text{ext}}, \quad (18.3\text{-}17)$$

$$-\partial_3 H_1 + \partial_1 H_3 + J_2 + \partial_t D_2 = -J_2^{\text{ext}}, \quad (18.3\text{-}18)$$

and

$$\partial_1 E_2 - \partial_2 E_1 + \partial_t B_3 = -K_3^{\text{ext}}, \quad (18.3\text{-}19)$$

$$\partial_2 E_3 - \partial_3 E_2 + \partial_t B_1 = -K_1^{\text{ext}}, \quad (18.3\text{-}20)$$

$$\partial_3 E_1 - \partial_1 E_3 + \partial_t B_2 = -K_2^{\text{ext}}. \quad (18.3\text{-}21)$$

Exercise 18.3-2

Write the relation given in Equation (18.3-15) out in terms of the components of the electric flux density.

Answer:

$$\rho = \partial_1 D_1 + \partial_2 D_2 + \partial_3 D_3 . \quad (18.3\text{-}22)$$

18.4 The electromagnetic field equations for time-independent fields (quasi-static field equations)

When at a certain instant $t = t_0$ given sources of a constant magnitude in time are switched on, it can on physical grounds be argued that the solutions of the field equations (18.3-11) and (18.3-12) approach a time-independent limit as $t \to \infty$. In this limit, the derivatives with respect to time vanish. As a consequence, Equation (18.3-11) changes into

$$-\varepsilon_{k,m,p}\partial_m H_p + J_k = -J_k^{\text{ext}}, \quad (18.4\text{-}1)$$

and Equation (18.3-12) into

$$\varepsilon_{j,n,r}\partial_n E_r = -K_j^{\text{ext}}. \quad (18.4\text{-}2)$$

From Equations (18.4-1) and (18.4-2) it is clear that through the field equations the electric and the magnetic field are no longer mutually coupled. Also, the compatibility relations (see Equations (18.3-13) and (18.3-14)) are no longer direct consequences of the field equations and, therefore, play now the role of additional field equations. From Equation (18.3-13) we obtain

$$\partial_k J_k = -\partial_k J_k^{\text{ext}}, \qquad (18.4\text{-}3)$$

which is also a direct consequence of Equation (18.4-1), and from Equation (18.3-14) we obtain

$$\partial_j K_j^{\text{ext}} = 0\,, \qquad (18.4\text{-}4)$$

which is also a direct consequence of Equation (18.4-2). Equation (18.3-15), viz.

$$\partial_k D_k = \rho \qquad (18.4\text{-}5)$$

remains as it is. Combining Equation (18.4-4) with Equation (18.3-14) and using the causality argument of Section 18.2, it follows that

$$\partial_j B_j = 0\,. \qquad (18.4\text{-}6)$$

Equations (18.4-5) and (18.4-6) are now independent field equations.

Meanwhile, the relations (18.3-9) and (18.3-10) remain satisfied, in which now, however, both the electric polarisation and the magnetisation in general consist of a field dependent *induced part* (to be indicated by the superscript 'ind') and a field independent *permanent part* (to be indicated by the superscript 'perm'). Hence, in the time-independent (quasi-static) limit

$$D_k = \varepsilon_0 E_k + P_k^{\text{ind}} + P_k^{\text{perm}} \qquad (18.4\text{-}7)$$

and

$$B_j = \mu_0(H_j + M_j^{\text{ind}}) + \mu_0 M_j^{\text{perm}}. \qquad (18.4\text{-}8)$$

The relationship between $\{J_k, P_k^{\text{ind}}, M_j^{\text{ind}}\}$ and $\{E_r, H_p\}$ is now established through the time-independent counterparts of the constitutive relations to be discussed in Chapter 19.

Exercises

Exercise 18.4-1

The electric field associated with a static electric charge Q located in vacuum that is uniformly distributed over the ball $\mathcal{B} = \{x \in \mathcal{R}^3;\ 0 \leqslant |x| < a\}$ of radius a is spherically symmetric around the centre $x = 0$ of the ball. Calculate (a) the volume density of electric charge ρ, (b) the electric field strength E_r of the charge distribution, and (c) the electric force F_r on a point charge q_2 located at the vectorial distance u from the centre of the ball. Prove that the electric field satisfies the equations $\partial_k E_k = \rho/\varepsilon_0$, $\varepsilon_{j,n,r}\partial_n E_r = 0$. (*Hint*: Note that due to the spherical symmetry we can write $E_r = \Psi(|x|)x_r$, where $\Psi = \Psi(|x|)$ is a scalar function of position depending on $|x|$ only.)

Answer: (a) $\rho = 3Q/4\pi a^3$ when $0 \leqslant |x| < a$, $\rho = 0$ when $a \leqslant |x| < \infty$; (b) $E_r = Qx_r/4\pi\varepsilon_0 a^3$ when $0 \leqslant |x| \leqslant a$, $E_r = Qx_r/4\pi\varepsilon_0|x|^3$ when $a \leqslant |x| < \infty$; (c) $F_r = q_2 Q u_r/4\pi\varepsilon_0 a^3$ when $0 < |u| \leqslant a$, $F_r = q_2 Q u_r/4\pi\varepsilon_0|u|^3$ when $a \leqslant |u| < \infty$.

Exercise 18.4-2

Calculate, with the aid of the results of Exercise 18.4-1, the electric field strength E_r associated with a point charge of strength q_1. (*Hint*: Consider the limit $a \downarrow 0$ in the results of Exercise 18.4-1.)

Answer: $E_r = q_1 x_r/4\pi\varepsilon_0|x|^3$ when $0 < |x| < \infty$.

Exercise 18.4-3

Calculate the force F_r that a static point charge q_1 exerts on a different point charge q_2 located at a vectorial distance $\boldsymbol{x}$ from the first one.

Answer: $F_r = q_1 q_2 x_r / 4\pi\varepsilon_0 |\boldsymbol{x}|^3$ (Coulomb's law).

Exercise 18.4-4

Verify Equation (18.4-3) by carrying out on Equation (18.4-1) the operation ∂_k.

Exercise 18.4-5

Verify Equation (18.4-4) by carrying out on Equation (18.4-2) the operation ∂_j.

18.5 SI units of the electromagnetic field quantities

Table 18.5-1 lists the SI units of the electromagnetic field quantities discussed in this chapter.

Table 18.5-1 Electromagnetic quantities and their units in the International System of Units (SI)

Quantity		Unit	
Name	Symbol	Name	Symbol
Electric field strength	E_r	volt/metre	V/m
Magnetic field strength	H_p	ampere/metre	A/m
Volume density of electric current	J_k	ampere/metre^2	A/m^2
Electric flux density	D_k	coulomb/metre^2	C/m^2
Magnetic flux density	B_j	tesla	T
Electric polarisation	P_k	coulomb/metre^2	C/m^2
Magnetization	M_j	ampere/metre	A/m
Volume density of external electric current	J_k^{ext}	ampere/metre^2	A/m^2
Volume density of external magnetic current	K_j^{ext}	volt/metre^2	V/m^2
Electric charge	Q	coulomb	C
Volume density of electric charge	ρ	coulomb/metre^3	C/m^3
Permeability in vacuum	μ_0	henry/metre	H/m
Permittivity in vacuum	ε_0	farad/metre	F/m
Electromagnetic wave speed in vacuum	c_0	metre/second	m/s

Numerical values: $\mu_0 = 4\pi \times 10^{-7}$ H/m;
$c_0 = 299\,792\,458$ m/s;
$\varepsilon_0 = (\mu_0 c_0^2)^{-1} \approx 8.8541878 \times 10^{-12}$ F/m.

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