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Interference and shielding of electromagnetic systems accessible via low-frequency terminations. ElectroMagnetic Compatibility (EMC)

In this chapter we analyse the properties of electromagnetic systems that are accessible via low-frequency terminations (ports). The latter are representative for the presence of electric or electronic systems or devices whose spatial extent is so small that the travel time for electromagnetic waves needed to traverse the systems or devices is negligibly small compared to the characteristic time scale on which the electromagnetic phenomena in them take place. As a consequence, their behaviour can be described in terms of the simpler concepts of voltages and electric currents rather than in terms of the electric and the magnetic field strengths of the pertaining electromagnetic field. In terms of these concepts, the electromagnetic properties of such systems are described by either their impedance matrix or their admittance matrix, both for their local and their remote interaction. Furthermore, fundamental aspects of the electromagnetic interference and shielding of such systems are investigated in relation to their ElectroMagnetic Compatibility (EMC). The reciprocity relations discussed in Chapter 28 play an important role in the analysis.

30.1 The reciprocity surface interaction integral for a low-frequency multiport system

In our analysis of electric and electronic devices and systems that are accessible via low-frequency port terminations, the relation between the electromagnetic field quantities and their low-frequency counterparts (electric) voltage and electric current is needed. This relation holds on a closed surface $\mathcal{S}$ that surrounds the accessible ports and has a maximum diameter that is small compared to the wavelength of the electromagnetic field. The main property of such a field is that, with sufficient accuracy, the electric field strength can be expressed as the gradient of a scalar electric potential. For example, the near-field part of the electric field radiated by a system of distributed electric currents present in a homogeneous, isotropic medium (see Chapter 26) satisfies this requirement. Then, in the complex frequency-domain representation of the field, we have

$$\hat{E}_r \approx -\partial_r \hat{\Phi}, \tag{30.1-1}$$

where $\hat{\Phi} = \hat{\Phi}(x,s)$ is the electric scalar potential. The terminals themselves are assumed to be perfectly conducting. On them, the electric scalar potential has a constant value since the tangential component of the electric field strength on them vanishes. In each low-frequency N-port system further a reference point (located in the interior of the closed surface surrounding the N-port) is chosen where the electric scalar potential is assigned the value zero and the value that $\hat{\Phi}$ then has on the port terminal with label α is denoted as the latter's (electric) voltage $\hat{V}_\alpha = \hat{V}_\alpha(s)$ ($\alpha = 1,\ldots,N$). In this respect it is noted that the assignment of a particular value of the electric scalar potential to a single (reference) point does not influence the value of the electric field strength in the domain surrounding it in view of the fact that in Equation (30.1-1) the electric scalar potential is differentiated, which operation annihilates the influence of an additive constant. With the use of Equation (30.1-1), the surface interaction integral in the complex frequency-domain reciprocity theorem of the time convolution type over the surface $\mathcal{S}$ that bounds the low-frequency N-port termination can then be rewritten as (Figure 30.1-1).

$$\varepsilon_{m,r,p} \int_{x \in \mathcal{S}} \nu_m \hat{E}_r^{\mathrm{A}} \hat{H}_p^{\mathrm{B}} \,\mathrm{d}A = -\varepsilon_{m,r,p} \int_{x \in \mathcal{S}} \nu_m (\partial_r \hat{\Phi}^{\mathrm{A}}) \hat{H}_p^{\mathrm{B}} \,\mathrm{d}A$$

$$= -\varepsilon_{m,r,p} \int_{x \in \mathcal{S}} \nu_m \partial_r (\hat{\Phi}^{\mathrm{A}} H_p^{\mathrm{B}}) \,\mathrm{d}A + \varepsilon_{m,r,p} \int_{x \in \mathcal{S}} \nu_m \hat{\Phi}^{\mathrm{A}} \partial_r \hat{H}_p^{\mathrm{B}} \,\mathrm{d}A . \tag{30.1-2}$$

However, on account of Stokes' theorem (see Exercise 26.10-1) we have

$$\varepsilon_{m,r,p} \int_{x \in \mathcal{S}} \nu_m \partial_r (\hat{\Phi}^{\mathrm{A}} \hat{H}_p^{\mathrm{B}}) \,\mathrm{d}A = 0 , \tag{30.1-3}$$

since $\mathcal{S}$ is a closed surface and $\varepsilon_{m,r,p} \nu_m \hat{\Phi}^{\mathrm{A}} \hat{H}_p^{\mathrm{B}}$ has been assumed to be continuous on $\mathcal{S}$. Furthermore, on account of Maxwell's first equation we have

$$\varepsilon_{m,r,p} \int_{x \in \mathcal{S}} \nu_m \hat{\Phi}^{\mathrm{A}} \partial_r \hat{H}_p^{\mathrm{B}} \,\mathrm{d}A = \int_{x \in \mathcal{S}} \nu_m \hat{\Phi}^{\mathrm{A}} (\hat{J}_m^{\mathrm{B}} + s\hat{D}_m^{\mathrm{B}}) \,\mathrm{d}A . \tag{30.1-4}$$

Now, in the low-frequency approximation, the Maxwell current density $\hat{J}_m + s\hat{D}_m$ on $\mathcal{S}$ is predominantly concentrated in the conduction current density $\hat{J}_m$ carried by the conductors that form the N-port termination. Let $\mathcal{A}_\alpha$ be the cross-section of the terminal with label $\alpha (\alpha = 1,\ldots,N)$, then

$$\int_{x \in \mathcal{S}} \nu_m \hat{\Phi}^{\mathrm{A}} (\hat{J}_m^{\mathrm{B}} + s\hat{D}_m^{\mathrm{B}}) \,\mathrm{d}A \approx \sum_{\alpha=1}^{N} \int_{x \in \mathcal{A}_\alpha} \nu_m \hat{\Phi}^{\mathrm{A}} \hat{J}_m^{\mathrm{B}} \,\mathrm{d}A$$

$$= \sum_{\alpha=1}^{N} \hat{V}_\alpha^{\mathrm{A}} \int_{x \in \mathcal{A}_\alpha} \nu_m \hat{J}_m^{\mathrm{B}} \,\mathrm{d}A = \sum_{\alpha=1}^{N} \hat{V}_\alpha^{\mathrm{A}} \hat{I}_\alpha^{\mathrm{B}} , \tag{30.1-5}$$

where $\hat{V}_\alpha = \hat{V}_\alpha(s)$ is the (constant) potential of the conductor with label α and

$$\hat{I}_\alpha(s) = \int_{x \in \mathcal{A}_\alpha} \nu_m \hat{J}_m(x,s) \,\mathrm{d}A \tag{30.1-6}$$

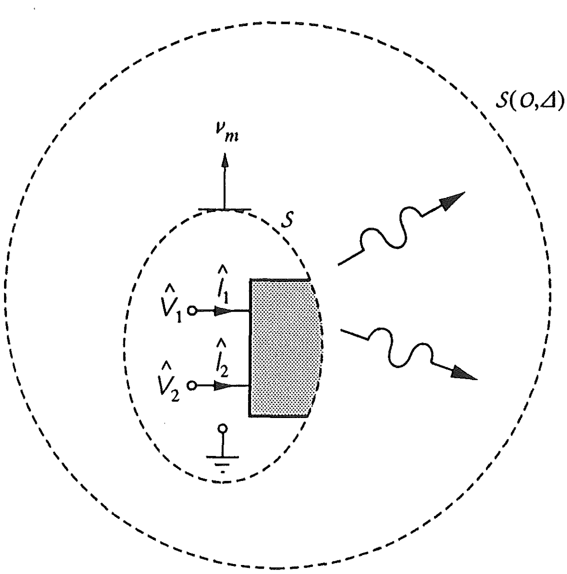


Figure 30.1-1 Electromagnetic system accessible via a low-frequency N-port termination inside a closed surface $\mathcal{S}$.

is the conduction current flowing through the conductor with label α towards (note the orientation of the unit normal ν_m on S) the remainder of the system. In the low-frequency approximation therefore

$$\varepsilon_{m,r,p} \int_{x \in \mathcal{S}} \nu_m \hat{E}_r^{\mathrm{A}}(x,s) \hat{H}_p^{\mathrm{B}}(x,s)\, \mathrm{d}A \approx \sum_{\alpha=1}^{N} \hat{V}_\alpha^{\mathrm{A}}(s) \hat{I}_\alpha^{\mathrm{B}}(s)\,. \tag{30.1-7}$$

Similar results hold for the other combinations of field components occurring in the surface integrals in the reciprocity relations.

Exercises

Exercise 30.1-1

Show that for a low-frequency N-port termination of an electric or electronic device the surface interaction integral occurring in the complex frequency-domain reciprocity theorem of the time convolution type Equation (28.4-7) can be written as

$$\varepsilon_{m,r,p} \int_{x \in \mathcal{S}} \nu_m \left[\hat{E}_r^{\mathrm{A}}(x,s) \hat{H}_p^{\mathrm{B}}(x,s) - \hat{E}_r^{\mathrm{B}}(x,s) \hat{H}_p^{\mathrm{A}}(x,s) \right] \mathrm{d}A$$

$$\approx \sum_{\alpha=1}^{N} \left[\hat{V}_\alpha^{\mathrm{A}}(s) \hat{I}_\alpha^{\mathrm{B}}(s) - \hat{V}_\alpha^{\mathrm{B}}(s) \hat{I}_\alpha^{\mathrm{A}}(s) \right]. \tag{30.1-8}$$

(*Hint*: Approximate the local electric field strength by the gradient of its electric scalar potential, use Maxwell's first equation, and carry out the steps similar to the ones that have led to Equation (30.1-7).)

Exercise 30.1-2

Show that for a low-frequency N-port termination of an electric or electronic device the surface interaction integral occurring in the time-domain reciprocity theorem of the time convolution type Equation (28.2-7) can be written as

$$\varepsilon_{m,r,p}\int_{x\in\mathcal{S}} \nu_m\left[\mathrm{C}_t(E_r^{\mathrm{A}},H_p^{\mathrm{B}},x,t)-\mathrm{C}_t(E_r^{\mathrm{B}},H_p^{\mathrm{A}},x,t)\right]\mathrm{d}A$$

$$\approx\sum_{\alpha=1}^{N}\left[\mathrm{C}_t(V_\alpha^{\mathrm{A}},I_\alpha^{\mathrm{B}},t)-\mathrm{C}_t(V_\alpha^{\mathrm{B}},I_\alpha^{\mathrm{A}},t)\right]. \tag{30.1-9}$$

(*Hint*: Approximate the local electric field strength by the gradient of its electric scalar potential, use Maxwell's first equation, and carry out the steps similar to the ones that have led to Equation (30.1-7).) Note that Equation (30.1-8) is the time Laplace transform of Equation (30.1-9).

Exercise 30.1-3

Show that for a low-frequency N-port termination of an electric or electronic device the surface interaction integral occurring in the complex frequency-domain reciprocity theorem of the time correlation type Equation (28.5-7) can be written as

$$\varepsilon_{m,r,p}\int_{x\in\mathcal{S}} \nu_m\left[\hat{E}_r^{\mathrm{A}}(x,s)\hat{H}_p^{\mathrm{B}}(x,-s)+\hat{E}_r^{\mathrm{B}}(x,-s)\hat{H}_p^{\mathrm{A}}(x,s)\right]\mathrm{d}A$$

$$\approx\sum_{\alpha=1}^{N}\left[\hat{V}_\alpha^{\mathrm{A}}(s)\hat{I}_\alpha^{\mathrm{B}}(-s)+\hat{V}_\alpha^{\mathrm{B}}(-s)\hat{I}_\alpha^{\mathrm{A}}(s)\right]. \tag{30.1-10}$$

(*Hint*: Approximate the local electric field strength by the gradient of its electric scalar potential, use Maxwell's first equation, and carry out the steps similar to the ones that have led to Equation (30.1-7).)

Exercise 30.1-4

Show that for a low-frequency N-port termination of an electric or electronic device the surface interaction integral occurring in the time-domain reciprocity theorem of the time correlation type can be written as

$$\varepsilon_{m,r,p}\int_{x\in\mathcal{S}} \nu_m\left[\mathrm{C}_t(E_r^{\mathrm{A}},\mathrm{J}_t(H_p^{\mathrm{B}}),x,t)+\mathrm{C}_t(\mathrm{J}_t(E_r^{\mathrm{B}}),H_p^{\mathrm{A}},x,t)\right]\mathrm{d}A$$

$$\approx\sum_{\alpha=1}^{N}\left[\mathrm{C}_t(V_\alpha^{\mathrm{A}},\mathrm{J}_t(I_\alpha^{\mathrm{B}}),t)+\mathrm{C}_t(\mathrm{J}_t(V_\alpha^{\mathrm{B}}),I_\alpha^{\mathrm{A}},t)\right]. \tag{30.1-11}$$

(*Hint*: Approximate the local electric field strength by the gradient of its electric scalar potential, use Maxwell's first equation, and carry out the steps similar to the ones that have led to Equation (30.1-7).) Note that Equation (30.1-10) is the time Laplace transform of Equation (30.1-11).

30.2 The electromagnetic N-port system as a transmitting system (electromagnetic emission analysis)

We consider the electromagnetic system or device shown in Figure 30.2-1 that is accessible via a low-frequency N-port termination, the latter being located in the interior of a closed surface $\mathcal{S}$.

The termination is present in an arbitrarily inhomogeneous and anisotropic unbounded embedding. At infinity, the standard provisions of Section 28.1 for unbounded domains are made. The maximum diameter of the domain interior to $\mathcal{S}$ is so small that at the ports the electric voltages $\{\hat{V}_\alpha;\alpha = 1,...,N\}$ and the electric currents $\{\hat{I}_\beta;\beta = 1,...,N\}$ can be defined. The domain containing the termination neednot be connected, i.e. local as well as remote interaction between the ports is included. Let in the domain exterior to $\mathcal{S}$ no sources be present. The electromagnetic field in the domain exterior to $\mathcal{S}$ is considered to be generated by the excitation of the N ports; this situation is denoted as the *transmitting situation* and serves to analyse the *electromagnetic emission* of the system. Let, in this situation, to be denoted by the superscript T, $\{\hat{V}_\alpha^{\mathrm{T}};\ \alpha = 1,...,N\}$ denote the voltages across the ports and $\{\hat{I}_\beta^{\mathrm{T}};\ \beta = 1,...,N\}$ denote the electric currents *fed into* the ports. For the emission analysis under consideration we shall adhere to the convention that the polarity of the voltages and the orientation of the electric currents is such that a positive electromagnetic power flow is oriented towards the embedding.

Now, for any linear, time-invariant, passive, electromagnetic system the property holds that the voltages across the ports of the system are uniquely determined once the electric currents fed into them are known, provided that causality is enforced. In view of the superposition theorem for linear systems (see Section 23.1), the voltages $\{\hat{V}_\alpha^{\mathrm{T}};\ \alpha = 1,...,N\}$ are therefore linearly related to the electric currents $\{\hat{I}_\beta^{\mathrm{T}};\ \beta = 1,...,N\}$. This linear relationship is expressed as

$$\hat{V}_\alpha^{\mathrm{T}}(s) = \sum_{\beta=1}^{N} \hat{Z}_{\alpha,\beta}^{\mathrm{T}}(s)\hat{I}_\beta^{\mathrm{T}}(s) \qquad \text{for} \quad \alpha = 1,...,N, \tag{30.2-1}$$

where $\{\hat{Z}_{\alpha,\beta}^{\mathrm{T}};\ \alpha = 1,...,N;\beta = 1,...,N\}$ are the elements of the *input impedance matrix* $[\hat{Z}^{\mathrm{T}}]$ of the N-port system.

Under the same conditions, the electric currents $\{\hat{I}_\beta^{\mathrm{T}};\ \beta = 1,...,N\}$ are linearly related to the voltages $\{\hat{V}_\alpha^{\mathrm{T}};\ \alpha = 1,...,N\}$. This relationship is expressed as

$$\hat{I}_\beta^{\mathrm{T}}(s) = \sum_{\alpha=1}^{N} \hat{Y}_{\beta,\alpha}^{\mathrm{T}}(s)\hat{V}_\alpha^{\mathrm{T}}(s) \qquad \text{for} \quad \beta = 1,...,N, \tag{30.2-2}$$

where $\{\hat{Y}_{\beta,\alpha}^{\mathrm{T}};\ \beta = 1,...,N;\alpha = 1,...,N\}$ are the elements of the *input admittance matrix* $[\hat{Y}^{\mathrm{T}}]$ of the N-port system.

Both the input impedance matrix and the input admittance matrix can be used to characterise the N-port system. Substitution of Equation (30.2-2) in Equation (30.2-1) and requiring identity in $\{\hat{V}_\alpha^{\mathrm{T}}\}$ leads to the relation

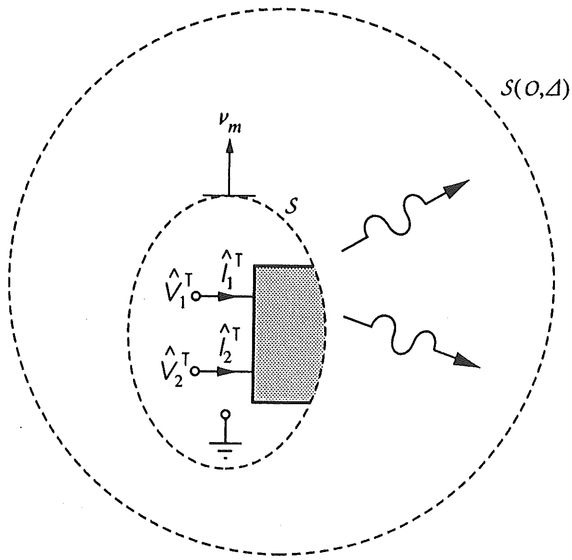


Figure 30.2-1 Emission analysis of an electromagnetic system or device accessible via a low-frequency N-port termination inside a closed surface $\mathcal{S}$. The system is activated by applying either voltages or electric currents to its ports (transmitting situation).

$$\sum_{\beta=1}^{N} \hat{Z}^{\mathrm{T}}_{\alpha,\beta}(s)\hat{Y}^{\mathrm{T}}_{\beta,\gamma}(s) = \delta_{\alpha,\gamma} \qquad \text{for} \quad \alpha = 1,\ldots,N;\ \gamma = 1,\ldots,N\,, \tag{30.2-3}$$

where $\delta_{\alpha,\gamma}$ are the elements of the unit matrix $[I]$: $\delta_{\alpha,\gamma} = 1$ for $\alpha = \gamma$, $\delta_{\alpha,\gamma} = 0$ for $\alpha \neq \gamma$. Substitution of Equation (30.2-1) in Equation (30.2-2) and requiring identity in $\{\hat{I}^{\mathrm{T}}_{\beta};\beta = 1,\ldots,N\}$ leads to the relation

$$\sum_{\alpha=1}^{N} \hat{Y}^{\mathrm{T}}_{\beta,\alpha}(s)\hat{Z}^{\mathrm{T}}_{\alpha,\gamma}(s) = \delta_{\beta,\gamma} \qquad \text{for} \quad \beta = 1,\ldots,N;\ \gamma = 1,\ldots,N\,. \tag{30.2-4}$$

Equations (30.2-3) and (30.2-4) express that the input impedance matrix and the input admittance matrix are each other's inverses.

The input impedance matrix and the input admittance matrix of an N-port system are configurational quantities; they are independent of the values of the voltages across or the electric currents fed into the ports. To investigate their reciprocity properties, consider two electromagnetic transmitting states, A and B, such that the media in the two states are each other's adjoint and apply the complex frequency-domain reciprocity theorem of the time convolution type Equation (28.4-7) to the domain exterior to $\mathcal{S}$. Then, the domain integral in Equation (28.4-7) vanishes. Since the two states are causal, the contribution from the sphere "at infinity" also vanishes. Furthermore, on $\mathcal{S}$, the relation of Equation (30.1-8) between the electromagnetic field quantities, and the voltages and the electric currents holds. As a consequence, we have

$$\sum_{\alpha=1}^{N} \hat{V}^{\mathrm{T;A}}_{\alpha}(s)\hat{I}^{\mathrm{T;B}}_{\alpha}(s) = \sum_{\beta=1}^{N} \hat{V}^{\mathrm{T;B}}_{\beta}(s)\hat{I}^{\mathrm{T;A}}_{\beta}(s)\,. \tag{30.2-5}$$

Let, now, in states A and B the input impedance matrices be $[\hat{Z}^{\mathrm{T;A}}]$ and $[\hat{Z}^{\mathrm{T;B}}]$, respectively. Then, substituting the relations corresponding to Equation (30.2-1) in Equation (30.2-5), we obtain

$$\sum_{\alpha=1}^{N}\sum_{\beta=1}^{N} \hat{Z}_{\alpha,\beta}^{\mathrm{T;A}}(s)\hat{I}_{\beta}^{\mathrm{T;A}}(s)\hat{I}_{\alpha}^{\mathrm{T;B}}(s) = \sum_{\alpha=1}^{N}\sum_{\beta=1}^{N} \hat{Z}_{\beta,\alpha}^{\mathrm{T;B}}(s)\hat{I}_{\alpha}^{\mathrm{T;B}}(s)\hat{I}_{\beta}^{\mathrm{T;A}}(s) . \tag{30.2-6}$$

Since Equation (30.2-6) has to hold for arbitrary values of the electric currents in the states A and B, it follows that

$$\hat{Z}_{\alpha,\beta}^{\mathrm{T;A}}(s) = \hat{Z}_{\beta,\alpha}^{\mathrm{T;B}}(s) . \tag{30.2-7}$$

Hence, the input impedance matrices corresponding to adjoint media surrounding one and the same electromagnetic N-port system in the transmitting situation are each other's transpose; if the *medium* in the configuration is self-adjoint or *reciprocal*, the input impedance matrix is a *symmetrical matrix*.

Let, similarly, $[\hat{Y}^{\mathrm{T;A}}]$ and $[\hat{Y}^{\mathrm{T;B}}]$ denote the input admittance matrices in the states A and B, respectively. Then, Equation (30.2-5) leads, by using the relations similar to Equation (30.2-2), to

$$\sum_{\alpha=1}^{N}\sum_{\beta=1}^{N} \hat{Y}_{\alpha,\beta}^{\mathrm{T;B}}(s)\hat{V}_{\alpha}^{\mathrm{T;A}}(s)\hat{V}_{\beta}^{\mathrm{T;B}}(s) = \sum_{\alpha=1}^{N}\sum_{\beta=1}^{N} \hat{Y}_{\beta,\alpha}^{\mathrm{T;A}}(s)\hat{V}_{\beta}^{\mathrm{T;B}}(s)\hat{V}_{\alpha}^{\mathrm{T;A}}(s) . \tag{30.2-8}$$

Since Equation (30.2-8) has to hold for arbitrary values of the voltages in the states A and B, it follows that

$$\hat{Y}_{\alpha,\beta}^{\mathrm{T;B}}(s) = \hat{Y}_{\beta,\alpha}^{\mathrm{T;A}}(s) . \tag{30.2-9}$$

Hence, the input admittance matrices corresponding to adjoint media surrounding one and the same electromagnetic N-port system in the transmitting situation are each other's transpose; if the *medium* in the configuration is self-adjoint or *reciprocal*, the input admittance matrix is a *symmetrical matrix*.

Once the electromagnetic N-port system is excited by either voltages across or electric currents into its ports, its excitation causes an electromagnetic field to be present everywhere in space. This field will, in general, interfere with any other electromagnetic system present somewhere else in space. The case where the latter system is again an electromagnetic system that is accessible via low-frequency terminations will be investigated in Section 30.3; this system is then in a receiving situation.

Exercises

Exercise 30.2-1

An electromagnetic one-port system is radiating into an unbounded medium that is otherwise free from sources. What are the relationships between the voltage $\hat{V}^{\mathrm{T}} = \hat{V}^{\mathrm{T}}(s)$ across the port and the electric current $\hat{I}^{\mathrm{T}} = \hat{I}^{\mathrm{T}}(s)$ fed into it? (In this case, it is not necessary to use Greek subscripts to distinguish between the different ports.)

Answer: $\hat{V}^{\mathrm{T}} = \hat{Z}^{\mathrm{T}}\hat{I}^{\mathrm{T}}$, where $\hat{Z}^{\mathrm{T}} = \hat{Z}^{\mathrm{T}}(s)$ is the input impedance; $\hat{I}^{\mathrm{T}} = \hat{Y}^{\mathrm{T}}\hat{V}^{\mathrm{T}}$, where $\hat{Y}^{\mathrm{T}} = \hat{Y}^{\mathrm{T}}(s)$ is the input admittance. (Note that $\hat{Z}^{\mathrm{T}}\hat{Y}^{\mathrm{T}} = 1$.)

Exercise 30.2-2

Give the time-domain relations between electric voltages and electric currents of a low-frequency N-port electromagnetic system in the transmitting situation.

Answer:

$$V_{\alpha}^{\mathrm{T}}(t) = \sum_{\beta=1}^{N} \mathrm{C}_t(Z_{\alpha,\beta}^{\mathrm{T}}, I_{\beta}^{\mathrm{T}}, t) \quad \text{for} \quad \alpha = 1,\ldots,N, \tag{30.2-10}$$

or

$$I_{\beta}^{\mathrm{T}}(t) = \sum_{\alpha=1}^{N} \mathrm{C}_t(Y_{\beta,\alpha}^{\mathrm{T}}, V_{\alpha}^{\mathrm{T}}, t) \quad \text{for} \quad \beta = 1,\ldots,N. \tag{30.2-11}$$

Exercise 30.2-3

Give the time-domain relation between the input impedance matrix $Z_{\alpha,\beta}^{\mathrm{T}} = Z_{\alpha,\beta}^{\mathrm{T}}(t)$ and the input admittance matrix $Y_{\beta,\alpha}^{\mathrm{T}} = Y_{\beta,\alpha}^{\mathrm{T}}(t)$ of a low-frequency N-port electromagnetic system in the transmitting situation, expressing that these matrices are each other's inverses.

Answer:

$$\sum_{\beta=1}^{N} \mathrm{C}_t(Z_{\alpha,\beta}^{\mathrm{T}}, Y_{\beta,\gamma}^{\mathrm{T}}, t) = \delta_{\alpha,\gamma}\delta(t) \quad \text{for} \quad \alpha = 1,\ldots,N; \gamma = 1,\ldots,N, \tag{30.2-12}$$

or

$$\sum_{\alpha=1}^{N} \mathrm{C}_t(Y_{\beta,\alpha}^{\mathrm{T}}, Z_{\alpha,\gamma}^{\mathrm{T}}, t) = \delta_{\beta,\gamma}\delta(t) \quad \text{for} \quad \beta = 1,\ldots,N; \gamma = 1,\ldots,N. \tag{30.2-13}$$

30.3 The electromagnetic N-port system as a receiving system (electromagnetic susceptibility analysis)

Again we consider an electromagnetic system that is accessible via a low-frequency N-port termination, the latter being located in the interior of a closed surface $\mathcal{S}$. The domain interior to $\mathcal{S}$ need not be connected, i.e. local as well as remote interaction between the ports is included. The termination is present in an arbitrarily inhomogeneous and anisotropic unbounded embedding. At infinity, the standard provisions of Section 28.1 for unbounded domains are made. The maximum diameter of the domain interior to $\mathcal{S}$ is so small that at the ports the electric voltages $\{\hat{V}_{\alpha}; \alpha = 1,\ldots,N\}$ and the electric currents $\{\hat{I}_{\beta}; \beta = 1,\ldots,N\}$ can be defined. In the domain exterior to $\mathcal{S}$, sources of electromagnetic radiation are present. Under these conditions, the N-port system is in the *receiving situation*, which situation serves to analyse the *electromagnetic susceptibility* of the system. Let, in the receiving situation, to be denoted by the superscript R, the electric voltages at the ports be $\{\hat{V}_{\alpha}^{\mathrm{R}}; \alpha = 1,\ldots,N\}$ and the electric currents flowing *out of* the ports be $\{\hat{I}_{\beta}^{\mathrm{R}}; \beta = 1,\ldots,N\}$. In general, the accessible terminals are then connected to an

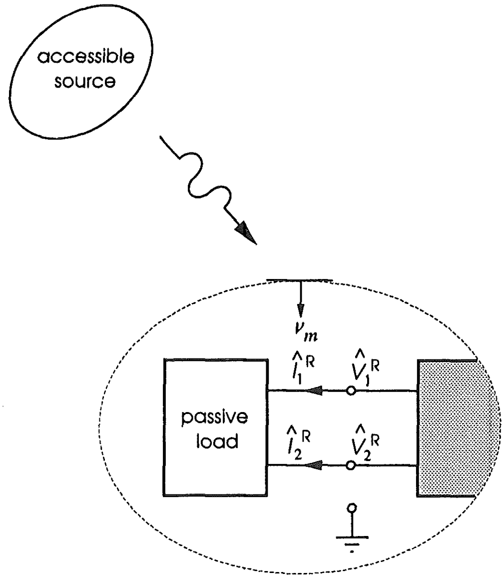


Figure 30.3-1 Susceptibility analysis of an electromagnetic system or device accessible via a low-frequency N-port termination inside a closed surface $\mathcal{S}$. The ports are connected to a passive load (receiving situation).

N-port load. In the susceptibility analysis under consideration, we shall adhere to the customary convention that the polarity of the voltages and the orientation of the electric currents is such that a positive power flow is into the load (Figure 30.3-1).

The load is characterised either by its impedance matrix $[\hat{Z}^{\mathrm{L}}]$ with elements $\hat{Z}^{\mathrm{L}}_{\alpha,\beta} = \hat{Z}^{\mathrm{L}}_{\alpha,\beta}(s)$ or by its admittance matrix $[\hat{Y}^{\mathrm{L}}]$ with elements $\hat{Y}^{\mathrm{L}}_{\beta,\alpha} = \hat{Y}^{\mathrm{L}}_{\beta,\alpha}(s)$. The linear relationship between the voltages across the terminals and the electric currents fed through them into the load are then either expressed by

$$\hat{V}^{\mathrm{R}}_{\alpha}(s) = \sum_{\beta=1}^{N} \hat{Z}^{\mathrm{L}}_{\alpha,\beta}(s)\hat{I}^{\mathrm{R}}_{\beta}(s) \qquad \text{for} \quad \alpha = 1,\ldots,N, \tag{30.3-1}$$

or by

$$\hat{I}^{\mathrm{R}}_{\beta}(s) = \sum_{\beta=1}^{N} \hat{Y}^{\mathrm{L}}_{\beta,\alpha}(s)\hat{V}^{\mathrm{R}}_{\alpha}(s) \qquad \text{for} \quad \beta = 1,\ldots,N. \tag{30.3-2}$$

Substitution of Equation (30.3-2) in Equation (30.3-1) and requiring identity in $\{\hat{V}^{\mathrm{R}}_{\alpha}\}$ leads to the relation

$$\sum_{\beta=1}^{N} \hat{Z}^{\mathrm{L}}_{\alpha,\beta}(s)\hat{Y}^{\mathrm{L}}_{\beta,\gamma}(s) = \delta_{\alpha,\gamma} \qquad \text{for} \quad \alpha = 1,\ldots,N;\ \gamma = 1,\ldots,N. \tag{30.3-3}$$

Substitution of Equation (30.3-1) in Equation (30.3-2) and requiring identity in $\{\hat{I}^{\mathrm{R}}_{\beta}; \beta = 1,\ldots,N\}$ leads to the relation

$$\sum_{\alpha=1}^{N} \hat{Y}^{\rm L}_{\beta,\alpha}(s)\hat{Z}^{\rm L}_{\alpha,\gamma}(s) = \delta_{\beta,\gamma} \quad \text{for} \quad \beta = 1,\ldots,N;\ \gamma = 1,\ldots,N\,. \tag{30.3-4}$$

Equations (30.3-3) and (30.3-4) express that the impedance matrix of the load $[\hat{Z}^{\rm L}]$ and its admittance matrix $[\hat{Y}^{\rm L}]$ are each other's inverses.

It will now be shown that the receiving properties of the N-port system under consideration are intimately related to its transmitting properties on one hand and the exciting sources in the receiving situation on the other hand. As far as the latter are concerned, two cases can be distinguished: the case where the sources that generate the electromagnetic field are known in location and properties (*accessible sources*), and the case where either the location or the properties of the sources that generate the electromagnetic field are unknown (*inaccessible sources*). The two cases will be discussed separately below.

Excitation by accessible sources

Let the exciting field be generated by sources located in the bounded domain $\mathcal{D}^{\rm R}$; $\mathcal{D}^{\rm R}$ is situated in the domain exterior to the closed surface $\mathcal{S}$ that bounds our N-port system (Figure 30.3-2). The action of the sources is accounted for by the volume source densities of electric current $\hat{J}_k^{\rm R}$ and magnetic current $\hat{K}_j^{\rm R}$ with the common support $\mathcal{D}^{\rm R}$. We now apply the complex frequency-domain reciprocity theorem of the time convolution type Equation (28.4-7) to the domain exterior to the closed surface $\mathcal{S}$ and identify state A in with the receiving state, for which the superscript R is used and state B with the transmitting state as discussed in Section 30.2, for which the superscript T is used. Both states are causally related to sources that activate them and hence the contribution from the "sphere at infinity" vanishes. Furthermore, at $\mathcal{S}$, the relation of Equation (30.1-8) between the fields on $\mathcal{S}$ and the electric voltages and currents at the ports holds. Let the medium in the transmitting state be the adjoint of the one in the actual receiving state, then the first integral on the right-hand side of Equation (28.4-7) vanishes. (Note that in this analysis the occurrence of non-reciprocal transmitting/receiving-state switches in, for example, antenna systems is included.) Furthermore, of the second integral on the right-hand side of Equation (28.4-7) only the terms containing $\hat{J}_k^{\rm R}$ and $\hat{K}_j^{\rm R}$ remain. As a result, we have

$$-\sum_{\alpha=1}^{N} \hat{V}^{\rm R}_{\alpha}\hat{I}^{\rm T}_{\alpha} - \sum_{\beta=1}^{N} \hat{V}^{\rm T}_{\beta}\hat{I}^{\rm R}_{\beta} = \int_{x\in\mathcal{D}^{\rm R}} (\hat{J}_r^{\rm R}\hat{E}_r^{\rm T} - \hat{K}_p^{\rm R}\hat{H}_p^{\rm T})\,{\rm d}V\,, \tag{30.3-5}$$

where it has been taken into account that the unit vector along the normal to $\mathcal{S}$ is oriented towards the interior of $\mathcal{S}$ and that the polarities of the voltages across the ports in the transmitting and the receiving states have been chosen the same, while in the transmitting state the orientation of the electric currents is into the system's ports, whereas in the receiving state this orientation is out of the system's ports.

To arrive at a first result, we substitute Equation (30.2-1) (but for the adjoint medium) in Equation (30.3-5) and further use the fact that the generated electromagnetic field in the transmitting state $\{\hat{E}_r^{\rm T},\hat{H}_p^{\rm T}\}$ is linearly related to the values of the electric currents $\{\hat{I}_\alpha^{\rm T};\ \alpha = 1,\ldots,N\}$ fed into the ports that excite it. The latter property we express by writing (note again the orientation of the unit vector along the normal to $\mathcal{S}$)

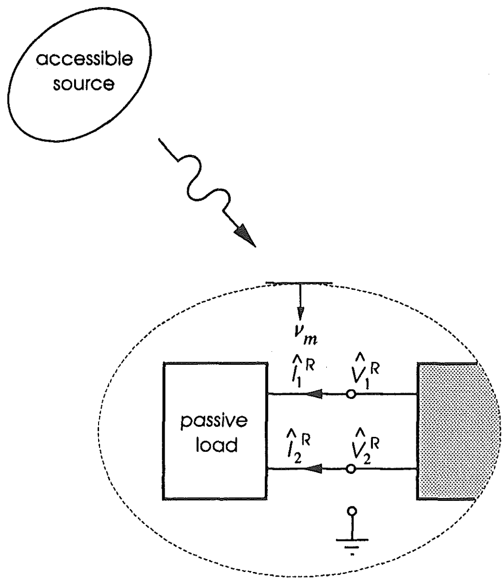


Figure 30.3-2 Susceptibility analysis of an electromagnetic system or device accessible via a low-frequency N-port termination inside a closed surface $\mathcal{S}$. The ports are connected to a passive load (receiving situation). Exciting sources are known in location and properties (accessible sources).

$$\int_{x\in\mathcal{D}^{\mathrm{R}}} \left[\hat{J}_r^{\mathrm{R}}(x,s)\hat{E}_r^{\mathrm{T}}(x,s) - \hat{K}_p^{\mathrm{R}}(x,s)\hat{H}_p^{\mathrm{T}}(x,s)\right] \mathrm{d}V = -\sum_{\alpha=1}^{N} \hat{V}_\alpha^{\mathrm{e}}(s)\hat{I}_\alpha^{\mathrm{T}}(s)\,. \tag{30.3-6}$$

With this result the reciprocity relation leads to

$$\sum_{\alpha=1}^{N}\left[\hat{V}_\alpha^{\mathrm{R}}(s) + \sum_{\beta=1}^{N} \hat{Z}_{\beta,\alpha}^{\mathrm{T}}(s)\hat{I}_\beta^{\mathrm{R}}(s)\right]\hat{I}_\alpha^{\mathrm{T}}(s) = \sum_{\alpha=1}^{N} \hat{V}_\alpha^{\mathrm{e}}(s)\hat{I}_\alpha^{\mathrm{T}}(s)\,. \tag{30.3-7}$$

Keeping in mind that the resulting equation has to hold for arbitrary values of $\{\hat{I}_{\alpha=1}^{\mathrm{T}};\ \alpha = 1,\ldots,N\}$, we arrive at

$$\hat{V}_\alpha^{\mathrm{R}}(s) + \sum_{\beta=1}^{N} \hat{Z}_{\beta,\alpha}^{\mathrm{T}}(s)\hat{I}_\beta^{\mathrm{R}}(s) = \hat{V}_\alpha^{\mathrm{e}}(s) \qquad \text{for} \quad \alpha = 1,\ldots,N\,. \tag{30.3-8}$$

Equation (30.3-8) is representative for an N-port network with internal voltage sources $\{\hat{V}_\alpha^{\mathrm{e}};\alpha = 1,\ldots,N\}$ and an internal impedance matrix which is the transpose of the input impedance matrix applying to the same N-port in the transmitting situation, but with the adjoint medium, i.e. the internal impedance matrix is the input impedance matrix that the N-port would actually experience if it would be excited at its ports. The corresponding representation is known as the *Thevenin representation*. By combining Equation (30.3-8) with the loading conditions of Equation (30.3-2), the values of the voltages and the electric currents in the receiving situation can be calculated.

A second result is arrived at upon substituting Equation (30.2-2) (but for the adjoint medium) in Equation (30.3-3) and use the fact that the electromagnetic field $\{\hat{E}_r^{\mathrm{T}},\hat{H}_p^{\mathrm{T}}\}$ in the transmitting state is linearly related to the values of the voltages $\{\hat{V}_\alpha^{\rm T};\alpha=1,\dots,N\}$ that excite it. The latter property we express by writing (note again the orientation of the unit vector along the normal to $\mathcal{S}$)

$$\int_{x\in\mathcal{D}^{\rm R}}\left[\hat{J}_r^{\rm R}(x,s)\hat{E}_r^{\rm T}(x,s)-\hat{K}_p^{\rm R}(x,s)\hat{H}_p^{\rm T}(x,s)\right]\mathrm{d}V=-\sum_{\beta=1}^{N}\hat{V}_\beta^{\rm T}(s)\hat{I}_\beta^{\rm e}(s)\,. \tag{30.3-9}$$

With this result the reciprocity relation leads to

$$\sum_{\beta=1}^{N}\hat{V}_\beta^{\rm T}(s)\left[\sum_{\alpha=1}^{N}\hat{Y}_{\alpha,\beta}^{\rm T}(s)\hat{V}_\alpha^{\rm R}(s)+\hat{I}_\beta^{\rm R}(s)\right]=\sum_{\beta=1}^{N}\hat{V}_\beta^{\rm T}(s)\hat{I}_\beta^{\rm e}(s)\,. \tag{30.3-10}$$

Keeping in mind that the resulting equation has to hold for arbitrary values of $\{\hat{V}_\beta^{\rm T};\beta=1,\dots,N\}$, we arrive at

$$\sum_{\alpha=1}^{N}\hat{Y}_{\alpha,\beta}^{\rm T}(s)\hat{V}_\alpha^{\rm R}(s)+\hat{I}_\beta^{\rm R}(s)=\hat{I}_\beta^{\rm e}(s)\qquad\text{for}\quad\beta=1,\dots,N\,. \tag{30.3-11}$$

Equation (30.3-11) is representative for an N-port network with internal electric current sources $\{\hat{I}_\alpha^{\rm e};\alpha=1,\dots,N\}$ and an internal admittance matrix which is the transpose of the input admittance matrix applying to the same N-port in the transmitting situation, but with the adjoint medium, i.e. the internal admittance matrix is the input admittance matrix that the N-port actually would experience if it would be excited at its ports. The corresponding representation is known as the *Norton representation.* By combining Equation (30.3-11) with the loading conditions of Equation (30.3-1), the values of the electric currents and voltages in the receiving situation can be calculated.

For a one-port system the Thevenin representation is shown in Figure 30.3-3 and the Norton representation in Figure 30.3-4.

Excitation by inaccessible sources

In case the sources that generate the field are inaccessible, or if for some reason the procedure outlined for accessible sources is undesirable, one can proceed differently. Let the electromagnetic N-port system, which terminates at the surface $\mathcal{S}$ at which the electric voltages and currents can be defined, be surrounded by another closed surface $\mathcal{S}^{\rm R}$ that is amenable for field measurements to be carried out (for example, the walls of the laboratory space in which some sensitive measuring apparatus is to be installed). On $\mathcal{S}^{\rm R}$ the field description need not be reducible to the equivalent circuit quantities of electric voltages and currents. The unit vectors to the normals to $\mathcal{S}$ and $\mathcal{S}^{\rm R}$ are oriented as shown in Figure 30.3-5.

The domain in between $\mathcal{S}$ and $\mathcal{S}^{\rm R}$ is assumed to be free from sources. To this domain we apply the complex frequency-domain reciprocity theorem of the time convolution type Equation (28.4-7). Again, state A is identified with the actual state in the receiving situation, but now the total electromagnetic field is written as the sum of the *incident field* $\{\hat{E}_r^{\rm i},\hat{H}_p^{\rm i}\}$ that is emitted by unspecified sources and that would be the total field if the load were "absent",

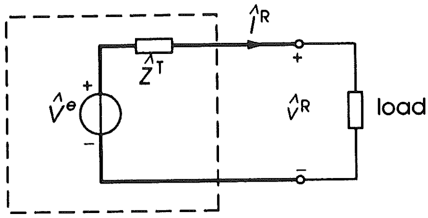


Figure 30.3-3 Thevenin (voltage source) representation of a one-port electromagnetic system in the receiving state.

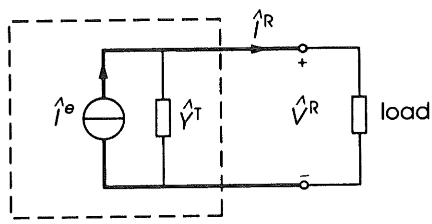


Figure 30.3-4 Norton (electric current source) representation of a one-port electromagnetic system in the receiving state.

and the *scattered field* $\{\hat{E}_r^{s},\hat{H}_p^{s}\}$ that is the field that must be superimposed on the incident field to yield the total field in the presence of the load. Correspondingly,

$$\{\hat{E}_r^{R},\hat{H}_p^{R}\} = \{\hat{E}_r^{i} + \hat{E}_r^{s},\hat{H}_p^{i} + \hat{H}_p^{s}\} \,. \tag{30.3-12}$$

What actually "absence" of the load implies for the description of the receiving system will be specified further on. State B is chosen as an arbitrary transmitting state of the system (see Section 30.2). The media in the two states are taken to be each other's adjoint. Under these conditions, the domain integrals in Equation (28.4-7) vanish. Furthermore, on $\mathcal{S}$ the relation of Equation (30.1-8) between the field quantities and the electric voltages and currents applies. From Equation (28.4-7) we then obtain

$$\varepsilon_{m,r,p}\int_{x\in\mathcal{S}^{R}} \nu_m \left\{\left[\hat{E}_r^{i}(x,s) + \hat{E}_r^{s}(x,s)\right]\hat{H}_p^{T}(x,s) - \hat{E}_r^{T}(x,s)\left[\hat{H}_p^{i}(x,s) + \hat{H}_p^{s}(x,s)\right]\right\} \mathrm{d}A$$
$$= -\sum_{\alpha=1}^{N} \hat{V}_\alpha^{R}(s)\hat{I}_\alpha^{T}(s) - \sum_{\beta=1}^{N} \hat{V}_\beta^{T}(s)\hat{I}_\beta^{R}(s) \,. \tag{30.3-13}$$

Now, both $\{\hat{E}_r^{s},\hat{H}_p^{s}\}$ and $\{\hat{E}_r^{T},\hat{H}_p^{T}\}$ are source-free in the domain exterior to $\mathcal{S}^{R}$, and both are causally related to the action of sources located in the bounded domain interior to $\mathcal{S}$. Consequently,

$$\varepsilon_{m,r,p}\int_{x\in\mathcal{S}^{R}} \nu_m \left[\hat{E}_r^{s}(x,s)\hat{H}_p^{T}(x,s) - \hat{E}_r^{T}(x,s)\hat{H}_p^{s}(x,s)\right] \mathrm{d}A$$
$$= \varepsilon_{m,r,p}\int_{x\in\mathcal{S}(O,\Delta)} \nu_m \left[\hat{E}_r^{s}(x,s)\hat{H}_p^{T}(x,s) - \hat{E}_r^{T}(x,s)\hat{H}_p^{s}(x,s)\right] \mathrm{d}A \,, \tag{30.3-14}$$

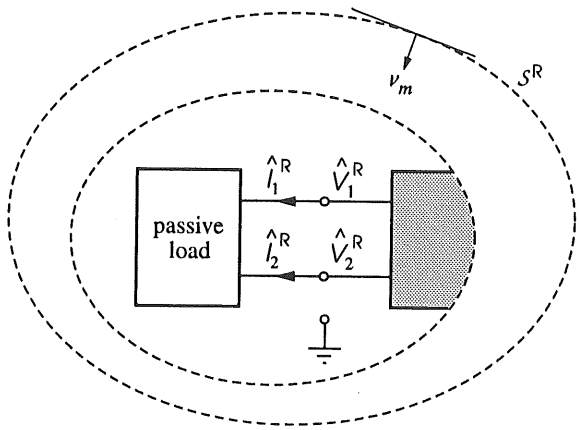


Figure 30.3-5 Susceptibility analysis of an electromagnetic system or device accessible via a low-frequency N-port termination inside a closed surface $\mathcal{S}$. The ports are connected to a passive load (receiving situation). Exciting sources are unknown in location and/or properties (inaccessible sources). The closed surface $\mathcal{S}^{\mathrm{R}}$ is accessible for carrying out field measurements.

where $\mathcal{S}(O,\Delta)$ is the sphere with radius Δ and centre at the origin O of the chosen reference frame and is assumed to completely surround $\mathcal{S}^{\mathrm{R}}$. In the limit $\Delta\to 0$, however, the right-hand side of Equation (30.3-14) vanishes in view of the causality condition at infinity. Now, the left-hand side of Equation (30.3-14) is independent of Δ, and consequently,

$$\varepsilon_{m,r,p}\int_{x\in\mathcal{S}^{\mathrm{R}}}\nu_m\left[\hat{E}_r^{\mathrm{s}}(x,s)\hat{H}_p^{\mathrm{T}}(x,s)-\hat{E}_r^{\mathrm{T}}(x,s)\hat{H}_p^{\mathrm{s}}(x,s)\right]\mathrm{d}A=0\,. \tag{30.3-15}$$

Using Equation (30.3-15) in Equation (30.3-13) we obtain the desired relation

$$\varepsilon_{m,r,p}\int_{x\in\mathcal{S}^{\mathrm{R}}}\nu_m\left[\hat{E}_r^{\mathrm{i}}(x,s)\hat{H}_p^{\mathrm{T}}(x,s)-\hat{E}_r^{\mathrm{T}}(x,s)\hat{H}_p^{\mathrm{i}}(x,s)\right]\mathrm{d}A$$

$$=-\sum_{\alpha=1}^{N}\hat{V}_\alpha^{\mathrm{R}}(s)\hat{I}_\alpha^{\mathrm{T}}(s)-\sum_{\beta=1}^{N}\hat{V}_\beta^{\mathrm{T}}(s)\hat{I}_\beta^{\mathrm{R}}(s)\,. \tag{30.3-16}$$

From this result we arrive at the Thevenin representation by using Equation (30.2-1) (but for the adjoint medium) and expressing the linear relationship that exists between the electromagnetic field in the transmitting state and the electric currents that excite it through

$$\varepsilon_{m,r,p}\int_{x\in\mathcal{S}^{\mathrm{R}}}\nu_m\left[\hat{E}_r^{\mathrm{i}}(x,s)\hat{H}_p^{\mathrm{T}}(x,s)-\hat{E}_r^{\mathrm{T}}(x,s)\hat{H}_p^{\mathrm{i}}(x,s)\right]\mathrm{d}A$$

$$=-\sum_{\alpha=1}^{N}\hat{V}_\alpha^{\mathrm{e}}(s)\hat{I}_\alpha^{\mathrm{T}}(s)\,. \tag{30.3-17}$$

From the condition that the resulting identity has to hold for arbitrary values of $\{\hat{I}_{\alpha}^{\mathrm{T}}; \alpha = 1,\ldots,N\}$, again Equation (30.3-8) results. As Equation (30.3-8) shows, "absence" of the load means in the Thevenin representation that the loading terminals of the N-port are left open. (Note that $\hat{V}_{\alpha}^{\mathrm{R}}(s) = \hat{V}_{\alpha}^{\mathrm{e}}(s)$ if $\hat{I}_{\beta}^{\mathrm{R}}(s) = 0$ for all $\beta = 1,\ldots,N$.)

From Equation (30.3-16) we arrive at the Norton representation by using Equation (30.2-2) (but for the adjoint medium) and expressing the linear relationship that exists between the electromagnetic field in the transmitting state and the electric voltages that excite it through

$$\varepsilon_{m,r,p} \int_{x \in \mathcal{S}^{\mathrm{R}}} \nu_m \left[\hat{E}_r^{\mathrm{i}}(x,s)\hat{H}_p^{\mathrm{T}}(x,s) - \hat{E}_r^{\mathrm{T}}(x,s)\hat{H}_p^{\mathrm{i}}(x,s)\right] \mathrm{d}A$$

$$= -\sum_{\beta=1}^{N} \hat{V}_{\beta}^{\mathrm{T}}(s)\hat{I}_{\beta}^{\mathrm{e}}(s) . \tag{30.3-18}$$

From the condition that the resulting identity has to hold for arbitrary values of $\{\hat{V}_{\beta}^{\mathrm{T}}; \beta = 1,\ldots,N\}$, again Equation (30.3-11) results. As Equation (30.3-11) shows, "absence" of the load means in the Norton representation that the loading terminals of the N-port are short-circuited. (Note that $\hat{I}_{\beta}^{\mathrm{R}}(s) = \hat{I}_{\beta}^{\mathrm{e}}(s)$ if $\hat{V}_{\alpha}^{\mathrm{R}}(s) = 0$ for all $\alpha = 1,\ldots,N$.)

Exercises

Exercise 30.3-1

What is, in the Thevenin representation of an electromagnetic one-port system in the receiving state, the value of (a) the internal source voltage and (b) the internal impedance in case the system is excited by accessible sources located in the domain $\mathcal{D}^{\mathrm{R}}$?

Answer:

(a) $\hat{V}^{\mathrm{e}}(s) = -\int_{x \in \mathcal{D}^{\mathrm{R}}} \left\{\hat{J}_r^{\mathrm{R}}(x,s)\left[\hat{E}_r^{\mathrm{T}}(x,s)/\hat{I}^{\mathrm{T}}(s)\right] - \hat{K}_p^{\mathrm{R}}(x,s)\left[\hat{H}_p^{\mathrm{T}}(x,s)/\hat{I}^{\mathrm{T}}(s)\right]\right\} \mathrm{d}V$;

(b) $\hat{Z}^{\mathrm{T}} = \hat{Z}^{\mathrm{T}}(s)$.

Exercise 30.3-2

What is, in the Thevenin representation of an electromagnetic one-port system in the receiving state, the value of (a) the internal source voltage and (b) the internal impedance in case the system is excited by inaccessible sources and the closed surface $\mathcal{S}^{\mathrm{R}}$ that completely surrounds the closed surface that bounds the one-port, is available for carrying out field measurements?

Answer:

(a) $\hat{V}^{\mathrm{e}}(s) = -\varepsilon_{m,r,p} \int_{x \in \mathcal{S}^{\mathrm{R}}} \nu_m \left\{\hat{E}_r^{\mathrm{i}}(x,s)\left[\hat{H}_p^{\mathrm{T}}(x,s)/\hat{I}^{\mathrm{T}}(s)\right] - \left[\hat{E}_r^{\mathrm{T}}(x,s)/\hat{I}^{\mathrm{T}}(s)\right]\hat{H}_p^{\mathrm{i}}(x,s)\right\} \mathrm{d}A$;

(b) $\hat{Z}^{\mathrm{T}} = \hat{Z}^{\mathrm{T}}(s)$.

Exercise 30.3-3

What is, in the Norton representation of an electromagnetic one-port system in the receiving state, the value of (a) the internal source current and (b) the internal admittance in case the system is excited by accessible sources located in the domain $\mathcal{D}^{\mathrm{R}}$?

Answer:

(a) $$\hat{I}^{\mathrm{e}}(s) = -\int_{x\in\mathcal{D}^{\mathrm{R}}} \left\{\hat{J}_r^{\mathrm{R}}(x,s)\left[\hat{E}_r^{\mathrm{T}}(x,s)/\hat{V}^{\mathrm{T}}(s)\right] - \hat{K}_p^{\mathrm{R}}(x,s)\left[\hat{H}_p^{\mathrm{T}}(x,s)/\hat{V}^{\mathrm{T}}(s)\right]\right\} \mathrm{d}V \, ;$$

(b) $\hat{Y}^{\mathrm{T}} = \hat{Y}^{\mathrm{T}}(s)$.

Exercise 30.3-4

What is, in the Norton representation of an electromagnetic one-port system in the receiving state, the value of (a) the internal source current and (b) the internal admittance in case the system is excited by inaccessible sources and the closed surface $\mathcal{S}^{\mathrm{R}}$ that completely surrounds the closed surface that bounds the one-port, is available for carrying out field measurements?

Answer:

(a) $$\hat{I}^{\mathrm{e}}(s) = -\varepsilon_{m,r,p}\int_{x\in\mathcal{S}^{\mathrm{R}}} \nu_m \left\{\hat{E}_r^{\mathrm{i}}(x,s)\left[\hat{H}_p^{\mathrm{T}}(x,s)/\hat{V}^{\mathrm{T}}(s)\right] - \left[\hat{E}_r^{\mathrm{T}}(x,s)/\hat{V}^{\mathrm{T}}(s)\right]\hat{H}_p^{\mathrm{i}}(x,s)\right\} \mathrm{d}A \, ;$$

(b) $\hat{Y}^{\mathrm{T}} = \hat{Y}^{\mathrm{T}}(s)$.

Exercise 30.3-5

Give the equations governing the Thevenin description of an electromagnetic N-port system in the receiving state in the time domain.

Answer:

$$V_\alpha^{\mathrm{R}}(t) + \sum_{\beta=1}^{N} \mathrm{C}_t(Z_{\beta,\alpha}^{\mathrm{T}}, I_\beta^{\mathrm{R}}, t) = V_\alpha^{\mathrm{e}}(t) \qquad \text{for} \quad \alpha = 1,\ldots,N \, . \tag{30.3-19}$$

Exercise 30.3-6

Give the equations governing the Norton description of an electromagnetic N-port system in the receiving state in the time domain.

Answer:

$$I_\beta^{\mathrm{R}}(t) + \sum_{\alpha=1}^{N} \mathrm{C}_t(Y_{\alpha,\beta}^{\mathrm{T}}, V_\alpha^{\mathrm{R}}, t) = I_\beta^{\mathrm{e}}(t) \qquad \text{for} \quad \beta = 1,\ldots,N \, . \tag{30.3-20}$$

30.4 Remote interaction between an *M*-port system and an *N*-port system

The domain interior to the closed surface $\mathcal{S}$ introduced in Section 30.1 where the low-frequency approximation applies, need not be connected, i.e. the low-frequency approximation may in several disjoint domains locally apply, and local as well as remote interaction between the different ports is included in the description. As an example, we consider the remote interaction between an M-port system that is accessible via the closed surface $\mathcal{S}_1$ and an N-port system that is accessible via the closed surface $\mathcal{S}_2$. The intersection of the domain interior to $\mathcal{S}_1$ and the domain interior to $\mathcal{S}_2$ is empty (Figure 30.4-1).

The maximum diameter of the domain interior to $\mathcal{S}_1$ and the maximum diameter of the domain interior to $\mathcal{S}_2$ are both so small that on $\mathcal{S}_1$ as well as on $\mathcal{S}_2$ the field description in terms of equivalent voltages and electric currents locally applies, be it that for the electric scalar potential on $\mathcal{S}_1$ a reference point in the interior of $\mathcal{S}_1$ (needed to define the voltages at the ports in $\mathcal{S}_1$) has to be chosen and for the electric scalar potential on $\mathcal{S}_2$ a reference point in the interior of $\mathcal{S}_2$ (needed to define the voltages at the ports in $\mathcal{S}_2$). In view of the linearity and the time invariance of the system, still the $(M + N)$-port impedance matrix $[Z^{\mathrm{T}}]$ and the $(M + N)$-port admittance matrix $[Y^{\mathrm{T}}]$ can be introduced. To distinguish the local interactions from the remote ones, these matrices will be *partitioned.* We shall analyse the electromagnetic interference between the M-port system and the N-port system by first considering the case where the M-port system is in its transmitting state (i.e. its ports are activated by either given values of the voltages or given values of the electric currents), while the N-port system is in its receiving state (i.e. its ports are terminated into a passive load), and secondly the case where the N-port system is in its transmitting state (i.e. its ports are activated by either given values of the voltages or given values of the electric currents), while the M-port system is in its receiving state (i.e. its ports are terminated into a passive load). To avoid the confusion that could arise from one part of the

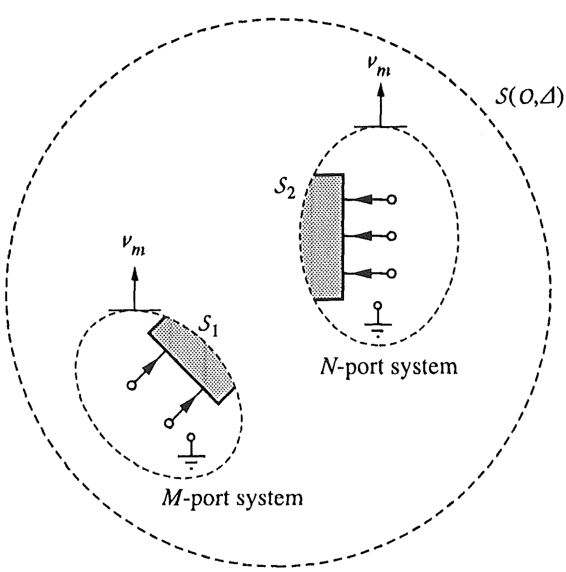


Figure 30.4-1 Remote interaction between an M-port system accessible via the closed surface $\mathcal{S}_1$ and an N-port system accessible via the closed surface $\mathcal{S}_2$.

total system to be in a transmitting state and the other part to be in a receiving state, the superscripts on the voltages and the electric currents as well as the superscripts on the impedance and admittance matrices will be omitted and all impedance and admittance matrices will be taken to refer to the transmitting situation of the total system. Since, further, the partitioning will be based on the impedance matrix or the admittance matrix of the total system, the unit vectors along the normals to $\mathcal{S}_1$ and $\mathcal{S}_2$ are both oriented into the ports (see Figure 30.4-1) and the results of Section 30.2 apply.

Starting from the impedance matrix as given in Equation (30.2-1), the partitioning is effected as follows:

$$\hat{V}_\alpha(s) = \sum_{\beta=1}^{M} \hat{Z}_{\alpha,\beta}(s)\hat{I}_\beta(s) + \sum_{\beta=M+1}^{M+N} \hat{Z}_{\alpha,\beta}(s)\hat{I}_\beta(s) \quad \text{for} \quad \alpha = 1,\dots,M\,, \tag{30.4-1}$$

and

$$\hat{V}_\alpha(s) = \sum_{\beta=1}^{M} \hat{Z}_{\alpha,\beta}(s)\hat{I}_\beta(s) + \sum_{\beta=M+1}^{M+N} \hat{Z}_{\alpha,\beta}(s)\hat{I}_\beta(s) \quad \text{for} \quad \alpha = M+1,\dots,M+N\,. \tag{30.4-2}$$

The partial impedance matrix with elements $\{\hat{Z}_{\alpha,\beta};\alpha = 1,\dots,M;\beta = 1,\dots,M\}$ describes the local interaction of the M-port system. Similarly, the partial impedance matrix with elements $\{\hat{Z}_{\alpha,\beta};\alpha = M+1,\dots,M+N;\beta = M+1,\dots,M+N\}$ describes the local interaction of the N-port system. The partial impedance matrices with elements $\{\hat{Z}_{\alpha,\beta};\alpha = 1,\dots,M;\beta = M+1,\dots,M+N\}$ and $\{\hat{Z}_{\alpha,\beta};\alpha = M+1,\dots,M+N;\beta = 1,\dots,M\}$ describe the remote interactions from the N-port to the M-port and from the M-port to the N-port, respectively.

To analyse the electromagnetic interference between the two systems we first consider the M-port system as the transmitting one and the N-port system as the receiving one. Elimination of the electric currents $\{\hat{I}_\beta;\beta = 1,\dots,M\}$ pertaining to the transmitting M-port from Equations (30.4-1) and (30.4-2) by expressing them, by way of solving Equation (30.4-1), in terms of $\{\hat{V}_\alpha;\alpha = 1,\dots,M\}$ and $\{\hat{I}_\beta;\beta = M+1,\dots,M+N\}$, leads to the Thevenin or voltage source representation of the receiving N-port system.

The same procedure applies to the case where the N-port system is the transmitting one and the M-port system the receiving one. In that case, elimination of the electric currents $\{\hat{I}_\beta;\beta = M+1,\dots,M+N\}$ pertaining to the transmitting N-port system from Equations (30.4-1) and (30.4-2) by expressing them, by way of solving Equation (30.4-2), in terms of $\{\hat{V}_\alpha;\alpha = M+1,\dots,M+N\}$ and $\{\hat{I}_\beta;\beta = 1,\dots,M\}$ leads to the Thevenin or voltage source representation of the receiving M-port system.

The same kind of analysis can be carried out by partitioning the admittance matrix as given in Equation (30.2-2) according to

$$\hat{I}_\beta(s) = \sum_{\alpha=1}^{M} \hat{Y}_{\beta,\alpha}(s)\hat{V}_\alpha(s) + \sum_{\alpha=M+1}^{M+N} \hat{Y}_{\beta,\alpha}(s)\hat{V}_\alpha(s) \quad \text{for} \quad \beta = 1,\dots,M\,, \tag{30.4-3}$$

and

$$\hat{I}_\beta(s) = \sum_{\alpha=1}^{M} \hat{Y}_{\beta,\alpha}(s)\hat{V}_\alpha(s) + \sum_{\alpha=M+1}^{M+N} \hat{Y}_{\beta,\alpha}(s)\hat{V}_\alpha(s) \quad \text{for} \quad \beta = M+1,\dots,M+N\,. \tag{30.4-4}$$

The partial admittance matrix with elements $\{\hat{Y}_{\beta,\alpha};\beta = 1,\dots,M;\alpha = 1,\dots,M\}$ describes the local interaction of the M-port system. Similarly, the partial admittance matrix with elements

$\{\hat{Y}_{\beta,\alpha};\beta = M+1,\ldots,M+N;\alpha = M+1,\ldots,M+N\}$ describes the local interaction of the N-port system. The partial admittance matrices with elements $\{\hat{Y}_{\beta,\alpha};\beta = 1,\ldots,M;\alpha = M+1,\ldots,M+N\}$ and $\{\hat{Y}_{\beta,\alpha};\beta = M+1,\ldots,M+N;\alpha = 1,\ldots,M\}$ describe the remote interactions from the N-port to the M-port and from the M-port to the N-port, respectively.

To analyse the electromagnetic interference between the two systems we first consider the M-port system as the transmitting one and the N-port system as the receiving one. Elimination of the voltages $\{\hat{V}_{\alpha};\alpha = 1,\ldots,M\}$ pertaining to the transmitting M-port from Equations (30.4-3) and (30.4-4) by expressing them, by way of solving Equation (30.4-3), in terms of $\{\hat{I}_{\beta};\beta = 1,\ldots,M\}$ and $\{\hat{V}_{\alpha};\alpha = M+1,\ldots,M+N\}$, leads to the Norton or electric current source representation of the receiving N-port system.

The same procedure applies to the case where the N-port system is the transmitting one and the M-port system the receiving one. In that case, elimination of the voltages $\{\hat{V}_{\alpha};\alpha = M+1,\ldots,M+N\}$ pertaining to the transmitting N-port system from Equations (30.4-3) and (30.4-4) by expressing them, by way of solving Equation (30.4-4), in terms of $\{\hat{I}_{\beta};\beta = M+1,\ldots,M+N\}$ and $\{\hat{V}_{\alpha};\alpha = 1,\ldots,M\}$ leads to the Norton or electric current source representation of the receiving M-port system.

Note that in the analysis of the present section it is, for reasons of symmetry, advantageous to keep the orientation of the unit vectors along the normals to $\mathcal{S}_1$ and $\mathcal{S}_2$ towards the exterior of both surfaces. In the Thevenin and Norton representations this leads to some changes in sign as compared with Equations (30.3-8) and (30.3-11).

The case $M = 1$, $N = 1$ (Thevenin representation)

By way of illustration, the case of the (remote) interaction between two one-ports is further discussed. We assume that the two-port system is embedded in a self-adjoint or reciprocal medium. In the Thevenin representation we then have

$$\hat{V}_1(s) = \hat{Z}_{1,1}(s)\hat{I}_1(s) + \hat{Z}_{1,2}(s)\hat{I}_2(s)\,, \tag{30.4-5}$$

$$\hat{V}_2(s) = \hat{Z}_{2,1}(s)\hat{I}_1(s) + \hat{Z}_{2,2}(s)\hat{I}_2(s)\,, \tag{30.4-6}$$

in which, in view of the self-adjointness of the medium,

$$\hat{Z}_{1,2}(s) = \hat{Z}_{2,1}(s)\,. \tag{30.4-7}$$

Under this latter condition, Equations (30.4-5) and (30.4-6) can be envisaged as to apply to the equivalent T-circuit shown in Figure 30.4-2. Both, from Equations (30.4-5) and (30.4-6) and from Figure 30.4-2 it is clear that the two interacting one-ports decouple if $\hat{Z}_{1,2} = \hat{Z}_{2,1} = 0$, for under this condition Equation (30.4-5) reduces to $\hat{V}_1 = \hat{Z}_{1,1}\hat{I}_1$ for the local one-port 1, which one-port then yields zero contribution to the voltage at one-port 2 (see also Exercise 30.4-1, case (b)), while Equation (30.4-6) reduces to $\hat{V}_2 = \hat{Z}_{2,2}\hat{I}_2$ for the local one-port 2, which one-port then yields zero contribution to the voltage at one-port 1 (see also Exercise 30.4-1, case (a)).

For non-zero coupling between the two one-ports ($\hat{Z}_{1,2} = \hat{Z}_{2,1} \neq 0$) it can happen that these coupling impedances are predominantly of the *resistive/inductive* type. Then, $\hat{Z}_{1,2} = \hat{R}_{1,2} + s\hat{L}_{1,2}$ and $\hat{Z}_{2,1} = \hat{R}_{2,1} + s\hat{L}_{2,1}$, where $R_{1,2} = R_{2,1}$ is the *coupling resistance* and $L_{1,2} = L_{2,1}$ is the *coupling inductance*. In this case, the Thevenin description is the more appropriate one to describe the remote interaction.

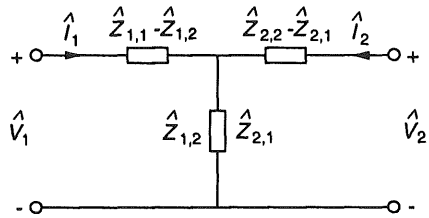


Figure 30.4-2 Equivalent T-circuit for two (remotely) interacting one-ports embedded in a self-adjoint (reciprocal) medium.

The case $M = 1$, $N = 1$ (Norton representation)

In the Norton representation of two interacting one-ports embedded in a self-adjoint or reciprocal medium we have

$$\hat{I}_1(s) = \hat{Y}_{1,1}(s)\hat{V}_1(s) + \hat{Y}_{1,2}(s)\hat{V}_2(s)\,, \tag{30.4-8}$$

$$\hat{I}_2(s) = \hat{Y}_{2,1}(s)\hat{V}_1(s) + \hat{Y}_{2,2}(s)\hat{V}_2(s)\,, \tag{30.4-9}$$

in which, in view of the self-adjointness of the medium,

$$\hat{Y}_{1,2}(s) = \hat{Y}_{2,1}(s)\,. \tag{30.4-10}$$

Under this latter condition, Equations (30.4-8) and (30.4-9) can be envisaged as to apply to the equivalent Π-circuit shown in Figure 30.4-3. Both, from Equations (30.4-8) and (30.4-9) and from Figure 30.4-3, it is clear that the two interacting one-ports decouple if $\hat{Y}_{1,2} = \hat{Y}_{2,1} = 0$, for under this condition Equation (30.4-8) reduces to $\hat{I}_1 = \hat{Y}_{1,1}\hat{V}_1$ for the local one-port 1, which one-port then yields zero contribution to the electric current at one-port 2 (see also Exercise 30.4-2, case (b)), while Equation (30.4-9) reduces to $\hat{I}_2 = \hat{Y}_{2,2}\hat{V}_2$ for the local one-port 2, which one-port then yields zero contribution to the electric current at one-port 1 (see also Exercise 30.4-2, case (a)).

For non-zero coupling between the two one-ports ($\hat{Y}_{1,2} = \hat{Y}_{2,1} \neq 0$) it can happen that these coupling admittances are predominantly of the *conductive/capacitive* type. Then, $\hat{Y}_{1,2} = \hat{G}_{1,2} + s\hat{C}_{1,2}$ and $\hat{Y}_{2,1} = \hat{G}_{2,1} + s\hat{C}_{2,1}$, where $G_{1,2} = G_{2,1}$ is the *coupling conductance* and $C_{1,2} = C_{2,1}$ is the *coupling capacitance.* In this case, the Norton description is the more appropriate one to describe the remote interaction.

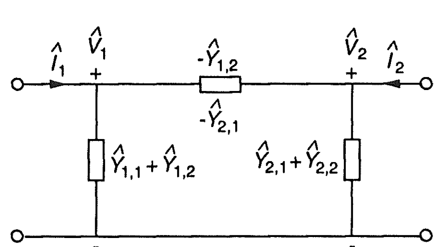


Figure 30.4-3 Equivalent Π-circuit for two (remotely) interacting one-ports embedded in a self-adjoint (reciprocal) medium.

Exercises

Exercise 30.4-1

Consider the remote interaction between two one-port systems, to be denoted as System 1 and System 2, respectively. Give the equations pertaining to the Thevenin representation of the receiving port (a) if System 1 is transmitting and System 2 is receiving; (b) if System 2 is transmitting and System 1 is receiving.

Answer:

(a) $\hat{V}_2 - (\hat{Z}_{2,2} - \hat{Z}_{2,1}\hat{Z}_{1,2}/\hat{Z}_{1,1})\hat{I}_2 = (\hat{Z}_{2,1}/\hat{Z}_{1,1})\hat{V}_1$,

(b) $\hat{V}_1 - (\hat{Z}_{1,1} - \hat{Z}_{1,2}\hat{Z}_{2,1}/\hat{Z}_{2,2})\hat{I}_1 = (\hat{Z}_{1,2}/\hat{Z}_{2,2})\hat{V}_2$.

(Note that in case (a) the interference is small if $\hat{Z}_{2,1}/\hat{Z}_{1,1} \ll 1$, while in case (b) the interference is small if $\hat{Z}_{1,2}/\hat{Z}_{2,2} \ll 1$.)

Exercise 30.4-2

Consider the remote interaction between two one-port systems, to be denoted as System 1 and System 2, respectively. Give the equations pertaining to the Norton representation of the receiving port (a) if System 1 is transmitting and System 2 is receiving; (b) if System 2 is transmitting and System 1 is receiving.

Answer:

(a) $\hat{I}_2 - (\hat{Y}_{2,2} - \hat{Y}_{2,1}\hat{Y}_{1,2}/\hat{Y}_{1,1})\hat{V}_2 = (\hat{Y}_{2,1}/\hat{Y}_{1,1})\hat{I}_1$,

(b) $\hat{I}_1 - (\hat{Y}_{1,1} - \hat{Y}_{1,2}\hat{Y}_{2,1}/\hat{Y}_{2,2})\hat{V}_1 = (\hat{Y}_{1,2}/\hat{Y}_{2,2})\hat{I}_2$.

(Note that in case (a) the interference is small if $\hat{Y}_{2,1}/\hat{Y}_{1,1} \ll 1$, while in case (b) the interference is small if $\hat{Y}_{1,2}/\hat{Y}_{2,2} \ll 1$.)

Exercise 30.4-3

The passive, single-channel transmission line section of finite length is an example of a system of two non-locally reacting one-ports. Let the section extend along the interval $0 < z < L$ of the z axis of an orthogonal Cartesian reference frame. Let $\hat{Z}_L = \hat{Z}_L(s)$ be the longitudinal impedance per length of the section and $\hat{Y}_T = \hat{Y}_T(s)$ its transverse admittance per length and denote by $\hat{\gamma}_0 = (\hat{Z}_L\hat{Y}_T)^{1/2}$ its propagation coefficient, by $\hat{Z}_0 = (\hat{Z}_L/\hat{Y}_T)^{1/2}$ its characterisic impedance and by $\hat{Y}_0 = (\hat{Y}_T/\hat{Z}_L)^{1/2}$ its characteristic admittance. The voltage $\hat{V} = \hat{V}(z,s)$ across the line and the electric current $\hat{I} = \hat{I}(z,s)$ along the line, where the polarity of $\hat{V}$ and the orientation of $\hat{I}$ are chosen such that the transfer of electromagnetic power is in the direction of increasing z, satisfy the system of differential equations (see Exercise 24.4-1)

$$\partial_z\hat{V} + \hat{Z}_L\hat{I} = 0 \quad \text{for} \quad 0 < z < L, \tag{30.4-11}$$

$$\partial_z\hat{I} + \hat{Y}_T\hat{V} = 0 \quad \text{for} \quad 0 < z < L. \tag{30.4-12}$$

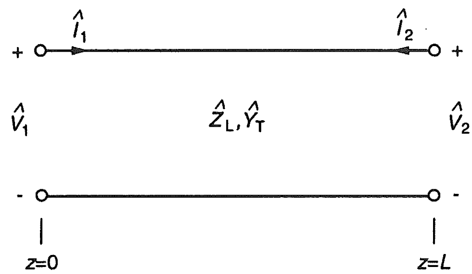


Figure 30.4-4 Passive section of a single-channel transmission line as an example of two remotely interacting one-ports.

Carry out the following steps that lead to the impedance matrix description of the remote interaction of the one-port at $z = 0$ and the one-port at $z = L$, where the polarities of the voltages $\hat{V}_1(s) = \hat{V}(0,s)$ and $\hat{V}_2(s) = \hat{V}(L,s)$, and the orientation of the electric currents $\hat{I}_1(s) = \hat{I}(0,s)$ and $\hat{I}_2(s) = -\hat{I}(L,s)$ are chosen as shown in Figure 30.4-4.

(a) Write $\hat{I}(z,s) = A^+(s)\exp(-\hat{\gamma}_0 z) + A^-(s)\exp[-\hat{\gamma}_0(L-z)]$ for $0<z<L$ and give the corresponding expression for $\hat{V}(z,s)$. (b) Express A^+ and A^- in terms of $\hat{I}_1$ and $\hat{I}_2$. (c) Express $\hat{V}_1$ and $\hat{V}_2$ in terms of $\hat{I}_1$ and $\hat{I}_2$ through

$$\hat{V}_1 = \hat{Z}_{1,1}\hat{I}_1 + \hat{Z}_{1,2}\hat{I}_2 , \tag{30.4-13}$$

$$\hat{V}_2 = \hat{Z}_{2,1}\hat{I}_1 + \hat{Z}_{2,2}\hat{I}_2 , \tag{30.4-14}$$

and determine the elements of the impedance matrix $[\hat{Z}]$. The equivalent T-circuit of $[\hat{Z}]$ is shown in Figure 30.4-5. (d) Determine the leading term in the expansions for $\hat{Z}_{1,1} - \hat{Z}_{1,2}$, $\hat{Z}_{2,2} - \hat{Z}_{2,1}$, $\hat{Z}_{1,2}$ and $\hat{Z}_{2,1}$ as $\hat{\gamma}_0 L \to 0$ (i.e. for a short section of the transmission line) and show that Figure 30.4-6 is the corresponding equivalent T-circuit. (Observe that the equivalent T-circuit of Figure 30.4-6 agrees with the physical picture on which the construction of the differential equations (30.4-13) and (30.4-14) (see Exercise 24.4-1) was based.)

Answers:

(a) $\hat{V}(z,s) = A^+(s)\hat{Z}_0 \exp(-\hat{\gamma}_0 z) - A^-(s)\hat{Z}_0 \exp[-\hat{\gamma}_0(L-z)]$ for $0 < z < L$;

(b) $A^+ = [\hat{I}_1 + \hat{I}_2 \exp(-\hat{\gamma}_0 L)]/[1 - \exp(-2\hat{\gamma}_0 L)]$,

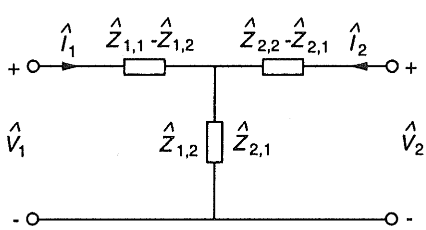


Figure 30.4-5 Equivalent T-circuit for the impedance matrix describing the remote interaction of the one-port at $z = 0$ and the one-port at $z = L$ of the transmission line section shown in Figure 30.4-4.

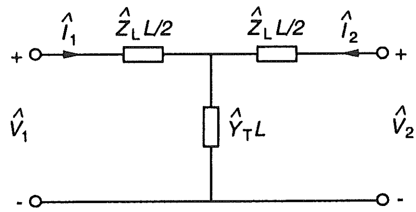


Figure 30.4-6 Equivalent T-circuit for the impedance matrix describing the remote interaction of the one-port at $z = 0$ and the one-port at $z = L$ for a short section of the transmission line shown in Figure 30.4-4 ($\hat{\gamma}_0 L \to 0$).

$$A^- = -[\hat{I}_1 \exp(-\hat{\gamma}_0 L) + \hat{I}_2]/[1 - \exp(-2\hat{\gamma}_0 L)] ;$$

(c) $$\hat{Z}_{1,1} = \hat{Z}_{2,2} = \hat{Z}_0 [1 + \exp(-2\hat{\gamma}_0 L)]/[1 - \exp(-2\hat{\gamma}_0 L)] ,$$

$$\hat{Z}_{1,2} = \hat{Z}_{2,1} = 2\hat{Z}_0 \exp(-\hat{\gamma}_0 L) / [1 - \exp(-2\hat{\gamma}_0 L)] ;$$

(d) $$\hat{Z}_{1,1} = \hat{Z}_{1,2} = \hat{Z}_{2,2} - \hat{Z}_{2,1} = \hat{Z}_L L/2 + O\left[(\hat{\gamma}_0 L)^2\right] \quad \text{as} \quad \hat{\gamma}_0 L \to 0 ,$$

$$\hat{Z}_{1,2}^{-1} = \hat{Z}_{2,1}^{-1} = Y_T L + O\left[(\hat{\gamma}_0 L)^2\right] \quad \text{as} \quad \hat{\gamma}_0 L \to 0 .$$

Exercise 30.4-4

The passive, single-channel transmission line section of finite length is an example of a system of two non-locally reacting one-ports. Let the section extend along the interval $0 < z < L$ of the z axis of an orthogonal Cartesian reference frame. Let $\hat{Z}_L = \hat{Z}_L(s)$ be the longitudinal impedance per length of the section and $\hat{Y}_T = \hat{Y}_T(s)$ its transverse admittance per length and denote by $\hat{\gamma}_0 = (\hat{Z}_L \hat{Y}_T)^{1/2}$ its propagation coefficient, by $\hat{Z}_0 = (\hat{Z}_L / \hat{Y}_T)^{1/2}$ its characterisic impedance and by $\hat{Y}_0 = (\hat{Y}_T / \hat{Z}_L)^{1/2}$ its characteristic admittance. The voltage $\hat{V} = \hat{V}(z,s)$ across the line and the electric current $\hat{I} = \hat{I}(z,s)$ along the line, where the polarity of $\hat{V}$ and the orientation of $\hat{I}$ are chosen such that the transfer of electromagnetic power is in the direction of increasing z, satisfy the system of differential equations (see Exercise 24.4-1)

$$\partial_z \hat{V} + \hat{Z}_L \hat{I} = 0 \quad \text{for} \quad 0 < z < L , \tag{30.4-15}$$

$$\partial_z \hat{I} + \hat{Y}_T \hat{V} = 0 \quad \text{for} \quad 0 < z < L . \tag{30.4-16}$$

Carry out the following steps that lead to the admittance matrix description of the remote interaction of the one-port at $z = 0$ and the one-port at $z = L$, where the polarities of the voltages $\hat{V}_1(s) = \hat{V}(0,s)$ and $\hat{V}_2(s) = \hat{V}(L,s)$, and the orientation of the electric currents $\hat{I}_1(s) = \hat{I}(0,s)$ and $\hat{I}_2(s) = -\hat{I}(L,s)$ are chosen as shown in Figure 30.4-7.

(a) Write $\hat{V}(z,s) = B^+(s) \exp(-\hat{\gamma}_0 z) + B^-(s) \exp[-\hat{\gamma}_0 (L - z)]$ for $0 < z < L$ and give the corresponding expression for $\hat{I}(z,s)$. (b) Express B^+ and B^- in terms of $\hat{V}_1$ and $\hat{V}_2$. (c) Express $\hat{I}_1$ and $\hat{I}_2$ in terms of $\hat{V}_1$ and $\hat{V}_2$ through

$$\hat{I}_1 = \hat{Y}_{1,1} \hat{V}_1 + \hat{Y}_{1,2} \hat{V}_2 , \tag{30.4-17}$$

$$\hat{I}_2 = \hat{Y}_{2,1} \hat{V}_1 + \hat{Y}_{2,2} \hat{V}_2 , \tag{30.4-18}$$

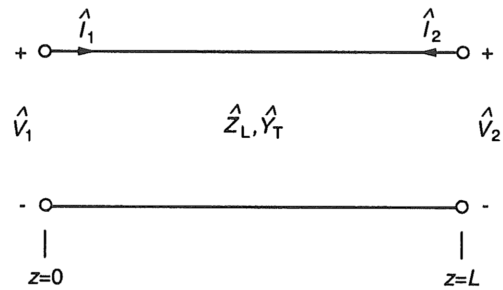


Figure 30.4-7 Passive section of a single-channel transmission line as an example of two remotely interacting one-ports.

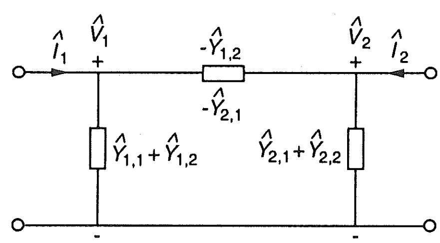


Figure 30.4-8 Equivalent Π-circuit for the admittance matrix describing the remote interaction of the one-port at $z = 0$ and the one-port at $z = L$ of the transmission line section shown in Figure 30.4-7.

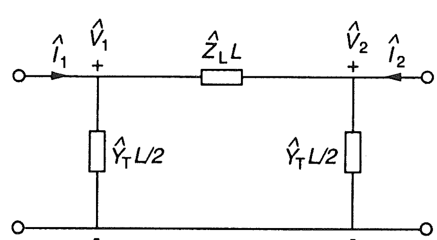


Figure 30.4-9 Equivalent Π-circuit for the admittance matrix describing the remote interaction of the one-port at $z = 0$ and the one-port at $z = L$ for a short section of the transmission line shown in Figure 30.4-7 ($\hat{\gamma}_0 L \to 0$).

and determine the elements of the admittance matrix $[\hat{Y}]$. The equivalent Π-circuit of $[\hat{Y}]$ is shown in Figure 30.4-8. (d) Determine the leading term in the expansions for $\hat{Y}_{1,1} + \hat{Y}_{1,2}$, $\hat{Y}_{2,2} + \hat{Y}_{2,1}$, $-\hat{Y}_{1,2}$ and $-\hat{Y}_{2,1}$ as $\hat{\gamma}_0 L \to 0$ (i.e. for a short section of the transmission line) and show that Figure 30.4-9 is the corresponding equivalent Π-circuit. (Observe that the equivalent Π-circuit of Figure 30.4-9 agrees with the physical picture on which the construction of the differential equations (30.4-17) and (30.4-18) (see Exercise 24.4-1) was based.)

Answers:

(a) $\hat{I}(z,s) = B^+(s)\hat{Y}_0 \exp(-\hat{\gamma}_0 z) - B^-(s)\hat{Y}_0 \exp[-\hat{\gamma}_0(L-z)]$ for $0 < z < L$;

(b) $B^{+}=[\hat{V}_1-\hat{V}_2\exp(-\hat{\gamma}_0L)]/[1-\exp(-2\hat{\gamma}_0L)]$,

$B^{-}=-[\hat{V}_1\exp(-\hat{\gamma}_0L)-\hat{V}_2]/[1-\exp(-2\hat{\gamma}_0L)]$;

(c) $\hat{Y}_{1,1}=\hat{Y}_{2,2}=\hat{Y}_0[1+\exp(-2\hat{\gamma}_0L)]/[1-\exp(-2\hat{\gamma}_0L)]$,

$-\hat{Y}_{1,2}=-\hat{Y}_{2,1}=2\hat{Y}_0\exp(-\hat{\gamma}_0L)/[1-\exp(-2\hat{\gamma}_0L)]$;

(d) $\hat{Y}_{1,1}+\hat{Y}_{1,2}=\hat{Y}_{2,2}+\hat{Y}_{2,1}=\hat{Y}_{\mathrm{T}}L/2+O\left[(\hat{\gamma}_0L)^2\right]$ as $\hat{\gamma}_0L\to 0$,

$-\hat{Y}_{1,2}^{-1}=-\hat{Y}_{2,1}^{-1}=Z_{\mathrm{L}}L+O\left[(\hat{\gamma}_0L)^2\right]$ as $\hat{\gamma}_0L\to 0$.

30.5 Electromagnetic interference

Owing to the omniperviousness of the electromagnetic field, each electromagnetic device, equipment or system can, in principle, influence any other electromagnetic device, equipment or system and thereby possibly degrade the latter's performance through *electromagnetic interference*. The basic ingredients of an ElectroMagnetic Interference problem are therefore: a source emitting electromagnetic energy (emitter), a susceptible device (susceptor), and a coupling path (either conductive or radiative or both) in between them (Figure 30.5-1).

The ever-increasing number of applications of electric and electronic systems (that often have been designed to function properly on their own, but often find themselves in each other's immediate neighbourhood) has led to the necessity of legislature through which the *electromagnetic compatiblility* (EMC) of such systems can be legally enforced. In view of the legal aspects of this domain of electromagnetics, a strict terminology has internationally been set up; the main items of this terminology are given below.

EMC—terminology

ElectroMagnetic Compatiblity (EMC)

The ability of an equipment or system to function satisfactorily in its electromagnetic environment without introducing intolerable electromagnetic disturbances to anything in that environment. (*Note*:"Anything" includes both living and inert matter.)

Electromagnetic environment

The totality of electromagnetic phenomena existing at a given location. (*Note*: In general, this totality is time dependent and its description may need a statistical approach.)

ElectroMagnetic Interference (EMI)

The degradation of the performance of a device, equipment or system caused by an electromagnetic disturbance.

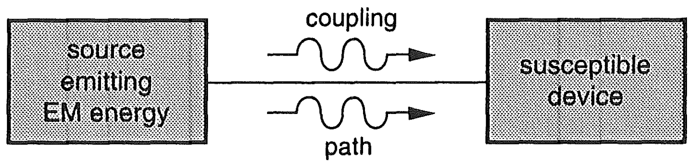


Figure 30.5-1 Basic ingredients in an ElectroMagnetic Interference problem.

Electromagnetic disturbance

Any electromagnetic phenomenon which may degrade the performance of a device, equipment or system, or adversely affect living or inert matter. (*Note*: Disturbance and Interference are, respectively, cause and effect.)

Degradation

An undesired departure in the operational performance of any device, equipment or system from its intended performance.

Electromagnetic emission

The phenomenon by which electromagnetic energy emanates from a source.

Emitters

Devices, equipment or systems which emit potentially disturbing voltages, currents or fields.

Susceptors

Devices, equipment or systems whose operation may be degraded by electromagnetic emissions.

Susceptibility

The inability of a device, equipment or system to perform without degradation in the presence of an electromagnetic disturbance.

Immunity

The ability of a device, equipment or system to perform without degradation in the presence of an electromagnetic disturbance.

Level (of a quantity)

The magnitude of a quantity in a specified manner.

Electromagnetic emission level

Emission level at the emitter's site.

Electromagnetic emission limit

The specified maximum emission level of a source of electromagnetic disturbance.

Electromagnetic disturbance level

Disturbance level at the susceptor's site.

Electromagnetic immunity level

The maximum level of a given electromagnetic disturbance incident on a particular device, equipment or system for which it remains capable of operation at a required degree of performance.

Electromagnetic immunity limit

The specified minimum immunity level.

Level of a quantity

The *level* of a quantity is often expressed in the ratio, on a logarithmic scale (for example, in decibel (dB)) with respect to a reference value. For its specification, the notational rule applies that the symbol for the unit of the level of that quantity consists of the unit of the pertaining reference value, concatenated with the symbol dB. Furthermore, the logarithmic scale is standardly taken at the base of the number 10 and, in order to arrive at the decibel, multiplied by a factor of 10 in case electromagnetic power flow or energy quantities are involved, whereas the multiplying factor is taken to be 20 in case amplitude values of voltages, electric currents, electric field strengths or magnetic field strengths are involved. So, for the level of the electromagnetic power flow P with respect to the reference value P_{ref} we have

$$P\ \text{dB}P\text{ref} = 10\ \log_{10}(P/P_{\text{ref}})\ , \qquad (30.5\text{-}1)$$

where Pref is the symbol for the reference value P_{ref} of the level of P. For example, for $P_{\text{ref}} =$ 1 μW, the level of P is expressed as

$$P\ \text{dB}\mu\text{W} = 10\ \log_{10}(P/1\mu\text{W})\ . \qquad (30.5\text{-}2)$$

Inversely, the actual value of the electromagnetic power flow P can be reconstructed from its level P dBPref and its reference value P_{ref} by the expression

$$P = P_{\text{ref}} 10^{P\,\text{dB}P\text{ref}/10} . \tag{30.5-3}$$

Note: By convention, due to its frequent occurrence, the symbol for the level unit dBmW is abbreviated to dBm.

For the level of an electric field strength E with respect to the reference value E_{ref} we have

$$E\,\text{dB}E\text{ref} = 20 \log_{10}(E/E_{\text{ref}}) , \tag{30.5-4}$$

where Eref is the symbol for the reference value E_{ref} of the level of E. For example, for E_{ref} = 1 mV/m the level of E is expressed as

$$E\,\text{dBmV/m} = 20 \log_{10}(E/1\ \text{mV/m}) . \tag{30.5-5}$$

Inversely, the actual value of the electric field strength E can be reconstructed from its level E dBEref and its reference value E_{ref} by the expression

$$E = E_{\text{ref}} 10^{E\,\text{dB}E\text{ref}/20} . \tag{30.5-6}$$

Similarly, for the level of a voltage V with respect to the reference value V_{ref} we have

$$V\,\text{dB}V\text{ref} = 20 \log_{10}(V/V_{\text{ref}}) , \tag{30.5-7}$$

where Vref is the symbol for the reference value V_{ref} of the level of V. For example, for V_{ref} = 1 mV the level of V is expressed as

$$V\,\text{dBmV} = 20 \log_{10}(V/1\ \text{mV}) . \tag{30.5-8}$$

Inversely, the actual value of the voltage V can be reconstructed from its level V dBVref and its reference value V_{ref} by the expression

$$V = V_{\text{ref}} 10^{V\,\text{dB}V\text{ref}/20} . \tag{30.5-9}$$

Standard test pulse shapes

For the characterisation of electromagnetic emission and susceptibility levels of systems and devices several standard test pulse shapes are in use. So, for the testing of the immunity against lightning surges, the standard test pulse shape of the electric current is *the double exponential pulse* (Figure 30.5-2).

$$I(t) = A[\exp(-at) - \exp(-bt)] \qquad \text{with} \quad A > 0, \quad 0 < a < b . \tag{30.5-10}$$

By applying some elementary rules of the time Laplace transformation its complex frequency-domain representation follows as

$$\hat{I}(s) = A\left(\frac{1}{s+a} - \frac{1}{s+b}\right) = A\left(\frac{b-a}{(s+a)(s+b)}\right) \qquad \text{for} \quad \text{Re}(s) > -a . \tag{30.5-11}$$

For real frequencies (i.e. for $s = \text{j}\omega$ with $\omega \in \mathcal{R}$) Equation (30.5-11) yields

$$\hat{I}(\text{j}\omega) = A\left(\frac{1}{\text{j}\omega + a} - \frac{1}{\text{j}\omega + b}\right) = A\left(\frac{b-a}{(\text{j}\omega + a)(\text{j}\omega + b)}\right) . \tag{30.5-12}$$

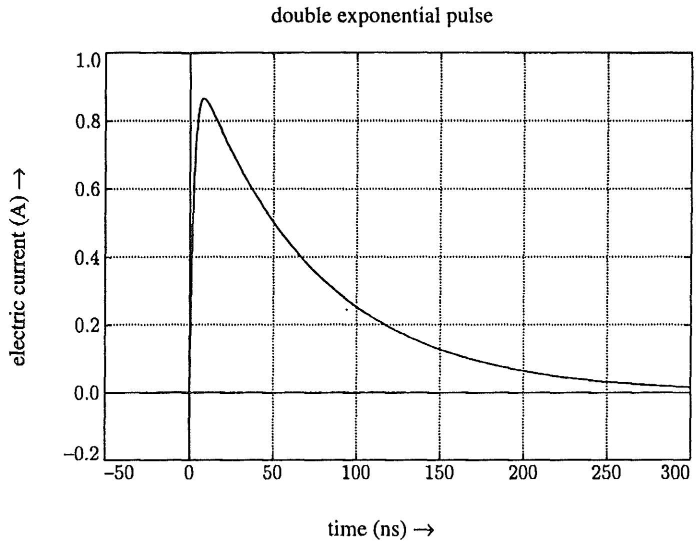


Figure 30.5-2 The double exponential electric current pulse as the standard pulse shape for the testing of the immunity of a system against a lightning surge.

The corresponding amplitude spectrum $|\hat{I}(\mathrm{j}\omega)|$ follows as

$$|\hat{I}(\mathrm{j}\omega)| = A\left(\frac{b-a}{[(\omega^2+a^2)(\omega^2+b^2)]^{1/2}}\right). \tag{30.5-13}$$

Furthermore,

$$\hat{I}(0) = A\left(\frac{1}{a}-\frac{1}{b}\right). \tag{30.5-14}$$

The graphical representation where $\log_{10}|\hat{I}(\mathrm{j}\omega)|$ is plotted against $\log_{10}(f)$, where $\omega = 2\pi f$, is denoted as the *spectral plot*. Figure 30.5-3 shows the spectral plot of the standard pulse given by Equation (30.5-10).

The straight lines that bound the *spectral plot* are denoted as *spectral bounds*. They play an important role because of the circumstance that electromagnetic emission limits and electromagnetic compatibility limits are standardly specified in terms of such straight lines. From Equation (30.5-13) it follows that spectral bounds for the doubly exponential pulse are given by

$$|\hat{I}(\mathrm{j}\omega)| < A\left(\frac{1}{a}-\frac{1}{b}\right), \tag{30.5-15}$$

which bound is in the spectral plot a horizontal straight line through the value of $|\hat{I}(0)|$ and which bound is useful at low frequencies, and by

$$|\hat{I}(\mathrm{j}\omega)| < A\left(\frac{b-a}{\omega^2}\right), \tag{30.5-16}$$

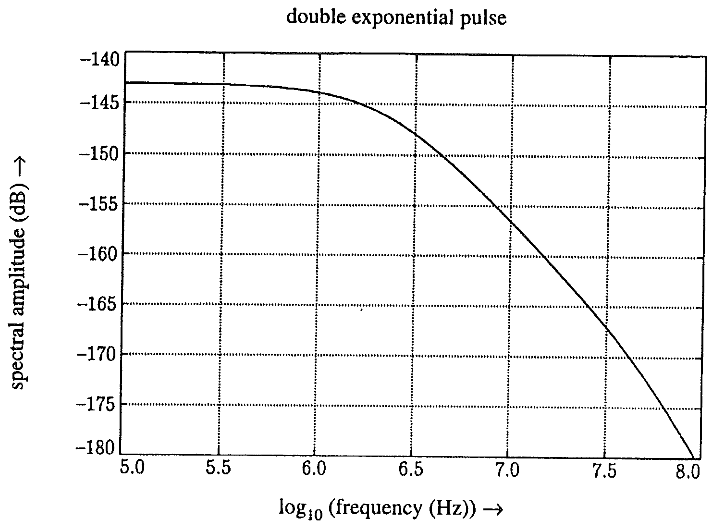


Figure 30.5-3 Spectral plot of the double exponential electric current pulse that is the standard pulse of testing the immunity of a system against a lightning surge.

which bound is in the spectral plot a line at a slope of −2 that intersects the horizontal spectral bound at $\omega = (ab)^{1/2}$ and which bound is useful at high frequencies. Note that the spectral bound given in Equation (30.5-15) also follows from the application of the result of Exercise 30.5-7 and Equation (30.5-14).

A standard pulse shape for the characterisation of the emission levels of digital electronic equipment (in particular, electronic computers) is the *trapezoidal electric current pulse* (Figure 30.5-4):

$$I(t) = \begin{cases} 0 \quad \text{for} \quad t < t_0\,, \\ I_{\max}\left(\dfrac{t-t_0}{t_1-t_0}\right) \quad \text{for} \quad t_0 < t < t_1\,, \\ I_{\max} \quad \text{for} \quad t_1 < t < t_2\,, \\ I_{\max}\left(\dfrac{t_3-t}{t_3-t_2}\right) \quad \text{for} \quad t_2 < t < t_3\,, \\ 0 \quad \text{for} \quad t_3 < t\,, \end{cases} \tag{30.5-17}$$

where $I_{\max}$ = pulse height, t_0 = starting time of the pulse, $t_1 - t_0 = t_r$ = pulse rise time, $t_3 - t_2 = t_f$ = pulse fall time and $(t_2 + t_3 - t_0 - t_1)/2 = t_w$ = pulse width (taken at half of the pulse height).

With the aid of the Heaviside unit step function this electric current pulse can also be written as

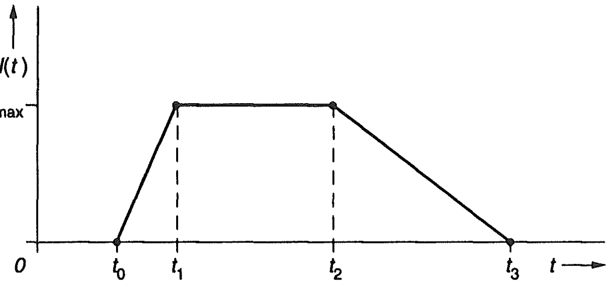


Figure 30.5-4 Trapezoidal electric current pulse for the determination of the emission level of digital electronic equipment. I_{max} = pulse height, t_0 = pulse starting time, $t_r = t_1 - t_0$ = pulse rise time, $t_w = (t_2 + t_3 - t_0 - t_1)/2$ = pulse width (at half the pulse height), $t_f = t_3 - t_2$ = pulse fall time.

$$
\begin{aligned}
I(t) = I_{max}\Bigg[& \left(\frac{t-t_0}{t_r} H(t-t_0)\right) - \left(\frac{t-t_0-t_r}{t_r}\right) H(t-t_0-t_r) \\
& - \frac{t-t_0-t_w-(t_r-t_f)/2}{t_f} H[t-t_0-t_w-(t_r-t_f)/2] \\
& + \frac{t-t_0-t_w-(t_r+t_f)/2}{t_f} H[t-t_0-t_w-(t_r+t_f)/2] \,. \qquad (30.5\text{-}18)
\end{aligned}
$$

Both expressions lead to the complex frequency-domain representation

$$
\hat{I}(s) = \frac{I_{max}}{s^2} \exp(-st_0) \left[\frac{1-\exp(-st_r)}{t_r} - \frac{1-\exp(-st_f)}{t_f} \exp\{-s[t_w - (t_r - t_f)/2]\} \right]
$$
$$
\text{for} \quad \mathrm{Re}(s) > 0 \,. \qquad (30.5\text{-}19)
$$

The spectral plot of this pulse is shown in Figure 30.5-5.

A Taylor expansion about $s = 0$ of the right-hand side of Equation (30.5-19) shows that

$$
\hat{I}(0) = I_{max} t_w \,. \qquad (30.5\text{-}20)
$$

which, in view of the result of Exercise 30.5-7 leads to the spectral bound useful at low frequencies

$$
|\hat{I}(j\omega)| \leqslant |I_{max}| t_w \,, \qquad (30.5\text{-}21)
$$

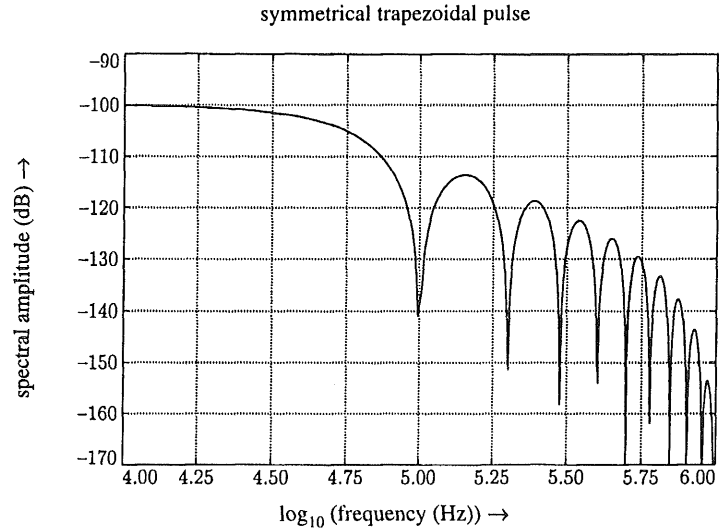
while for all frequencies

$$
|\hat{I}(j\omega)| \leqslant I_{max} \frac{1}{\omega^2} \left(\frac{2}{t_r} + \frac{2}{t_f} \right), \qquad (30.5\text{-}22)
$$

which yields a spectral bound at high frequencies. Note that the results for the trapezoidal pulse are in accordance with the results of Exercise 30.5-8 for the more general piecewise linear pulse.

For an electromagnetically compatible design of electric or electronic devices, equipment or systems it is almost a necessity to incorporate in the design procedures an analysis of the pertaining emission and susceptiblity levels. Now, the complete calculation of the electromagnetic performance of a configurtation where all field interactions governed by Maxwell's

30.5-5(a)



30.5-5(b)

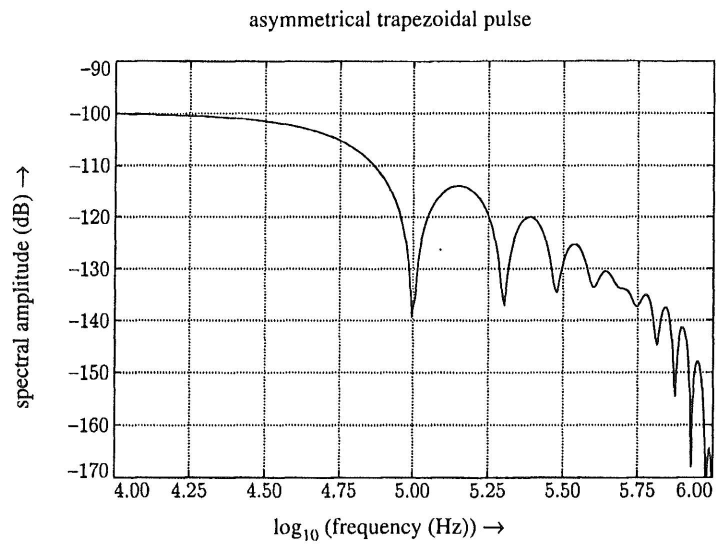


Figure 30.5-5 Normalised spectral plot of the trapezoidal electric current pulse of Figure 30.5-4. (a) $t_r = t_f = 10\,\mu s$, $t_w = 100\,\mu s$; (b) $t_r = 10\,\mu s$, $t_f = 20\,\mu s$, $t_w = 100\,\mu s$.

equations are exactly taken into account requires high-level software and an enormous amount of computational facilities. Therefore, a first impression as to the EMC of the configuration is usually arrived at by the application of an iterative procedure, whereby the configuration is initially designed on the basis of Kirchhoff's voltage and electric current laws that govern the behaviour of low-frequency electric or electronic circuits, i.e. without taking into account the interaction of the different branches, meshes and components of the circuit other than those subject to these laws (i.e. neglecting the interaction due to electromagnetic radiation). This procedure results into a configuration of electric current carrying wire segments and loops. Next, the electromagnetic field emission of these wire segments and loops is evaluated with the aid of the expressions derived in Sections 26.9 and 26.10. These emitted fields couple, in their turn, back into the wire segments and the loops of the configuration, and the susceptibility of these constituents can quantitatively be determined with the aid of the expressions derived in Section 30.3. In case the procedure results into an electromagnetic compatibility level that meets the specifications, the circuit configuration is acceptable for manufacturing. If this is not the case, either the circuit configuration has to be redesigned or *shielding measures* have to be taken. Two simple examples of the latter are analysed in Sections 30.6 and 30.7. To characterise the performance of a shielding configuration the *shielding effectiveness* is introduced as:

Shielding effectiveness ($S_{E,H}$)

For a given external source, the ratio of the electric or the magnetic field strength at a point before and after placement of the shield.

Customarily, the shielding effectiveness is expressed on a logarithmic scale in decibel (dB), i.e. the value of $S_{E,H}\,\mathrm{dB} = 20\log_{10}|S_{E,H}|$ is taken (and usually expressed as a function of frequency).

A useful introduction to the further aspects of electromagnetic compatibility is Paul (1992).

Exercises

Exercise 30.5-1

(a) Give the formula for the level of the electromagnetic power flow density (Poynting vector) S with respect to the reference value S_{ref} of S. (b) Give the formula for the level of the electromagnetic power flow density (Poynting vector) S with respect to the reference value $S_{\mathrm{ref}} = 1\ \mathrm{mW/m^2}$ of S.

Answer: (a) $S\ \mathrm{dB}S\mathrm{ref} = 10\log_{10}(S/S_{\mathrm{ref}})$; (b) $S\ \mathrm{dBmW/m^2} = 10\log_{10}(S/1\ \mathrm{mW/m^2})$.

Exercise 30.5-2

(a) Give the formula for the level of the magnetic field strength H with respect to the reference value H_{ref} of H. (b) Give the formula for the level of the magnetic field strength H with respect to the reference value $H_{\mathrm{ref}} = 1$ A/m of H.

Answer: (a) $H\ \mathrm{dB}H\mathrm{ref} = 20\log_{10}(H/H_{\mathrm{ref}})$; (b) $H\ \mathrm{dBA/m} = 20\log_{10}(H/1\ \mathrm{A/m})$.

Exercise 30.5-3

(a) Give the formula for the level of the electric current I with respect to the reference value I_{ref} of I. (b) Give the formula for the level of the electric current I with respect to the reference value $I_{\mathrm{ref}} = 1\ \mu\mathrm{A}$ of I.

Answer: (a) $I\ \mathrm{dB}I\mathrm{ref} = 20\log_{10}(I/I_{\mathrm{ref}})$; (b) $I\ \mathrm{dB}\mu\mathrm{A} = 20\log_{10}(I/1\ \mu\mathrm{A})$.

Exercise 30.5-4

Give the formula that expresses the actual value of the electromagnetic power flow density (Poynting vector) S in terms of its level S dBSref with respect to the reference value S_{ref}.

Answer: $S = S_{\text{ref}} 10^{S\,\text{dB}S\text{ref}/10}$.

Exercise 30.5-5

Give the formula that expresses the actual value of the magnetic field strength H in terms of its level H dBHref with respect to the reference value H_{ref}.

Answer: $H = H_{\text{ref}} 10^{H\,\text{dB}H\text{ref}/20}$.

Exercise 30.5-6

Give the formula that expresses the actual value of the electric current I in terms of its level I dBIref with respect to the reference value I_{ref}.

Answer: $I = I_{\text{ref}} 10^{I\,\text{dB}I\text{ref}/20}$.

Exercise 30.5-7

Let $I = I(t)$ be an electric current pulse with the property $I(t) > 0$ for all $t \in \mathcal{R}$. Assume that $\int_{\tau\in\mathcal{R}} I(t)\,\mathrm{d}t$ exists. Show that $|\hat{I}(\mathrm{j}\omega)| \leqslant \hat{I}(0)$, i.e. in the doubly logarithmic spectral plot the horizontal line through $\mathrm{j}\omega = 0$ is a spectral bound for any pulse of the indicated type. (*Hint*: Employ the inequality

$$\left| \int_{t\in\mathcal{R}} \exp(\mathrm{j}\omega t) I(t)\,\mathrm{d}t \right| \leqslant \int_{t\in\mathcal{R}} |\exp(\mathrm{j}\omega t)|\, I(t)\,\mathrm{d}t = \int_{t\in\mathcal{R}} I(t)\,\mathrm{d}t$$

and observe that

$$\int_{t\in\mathcal{R}} I(t)\,\mathrm{d}t = \hat{I}(0)\ .)$$

Exercise 30.5-8

Consider the piecewise linear, continuous electric current pulse (Figure 30.5-6)

$$I(t) = \begin{cases} 0 & \text{for} \quad t \leqslant t_0\,, \\ I_n \left(\dfrac{t - t_{n-1}}{t_n - t_{n-1}} \right) + I_{n-1} \left(\dfrac{t_n - t}{t_n - t_{n-1}} \right) & \text{for} \quad t_{n-1} \leqslant t \leqslant t_n \text{ and } n = 1,\ldots,N, \\ 0 & \text{for} \quad t_N \leqslant t\,. \end{cases} \tag{30.5-23}$$

with $I_0 = 0$ and $I_N = 0$.

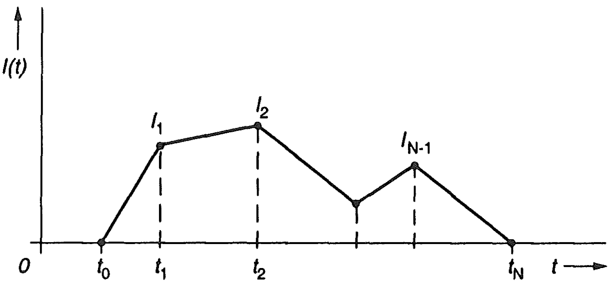


Figure 30.5-6 Piecewise linear, continuous electric current pulse of Equation (30.5-23).

(a) Show, by using the definition integral of the Laplace transform and integrating by parts two times, i.e.

$$\hat{I}(s) = s^{-2} \int_{t \in \mathcal{R}} \exp(-st) \partial_t^{\,2} I(t) \, \mathrm{d}t \,, \tag{30.5-24}$$

and observing that

$$\partial_t^{\,2} I(t) = \sum_{n=1}^{N} \left[\frac{I_n - I_{n-1}}{t_n - t_{n-1}} \delta(t - t_{n-1}) - \frac{I_n - I_{n-1}}{t_n - t_{n-1}} \delta(t - t_n) \right], \tag{30.5-25}$$

that

$$\hat{I}(s) = s^{-2} \sum_{n=1}^{N} \left[\frac{I_n - I_{n-1}}{t_n - t_{n-1}} \exp(-st_{n-1}) - \frac{I_n - I_{n-1}}{t_n - t_{n-1}} \exp(-st_n) \right]. \tag{30.5-26}$$

(b) Show, by observing that $|\exp(-\mathrm{j}\omega t_n)| = 1$, that from this result the following spectral bound, useful at high frequencies in the doubly logarithmic spectral plot, is obtained:

$$|\hat{I}(\mathrm{j}\omega)| \leqslant \omega^{-2} \sum_{n=1}^{N} 2 \frac{|I_n - I_{n-1}|}{t_n - t_{n-1}} \quad \text{with} \quad I_0 = 0 \quad \text{and} \quad I_N = 0 \,. \tag{30.5-27}$$

(c) Show, by using the Taylor expansion about $s = 0$, that from Equation (30.5-26) it follows that

$$\hat{I}(0) = -\frac{1}{2} \sum_{n=1}^{N} (I_n - I_{n-1})(t_{n-1} + t_n) \,. \tag{30.5-28}$$

and that in view of the relation

$$\sum_{n=1}^{N} (I_n t_n - I_{n-1} t_{n-1}) = 0 \tag{30.5-29}$$

Equation (30.5-28) can be rewritten as

$$\hat{I}(0) = \frac{1}{2} \sum_{n=1}^{N} (I_n + I_{n-1})(t_n - t_{n-1}) = \int_{t \in \mathcal{R}} I(t) \, \mathrm{d}t \,, \tag{30.5-30}$$

where the latter equality results from the application of the repeated trapezoidal integration rule which is exact for piecewise linear functions. (Note that if $I_n \geqslant 0$ for all $n = 0,\ldots,N$, the result of Exercise 30.5-7 provides a spectral bound in the doubly logarithmic spectral plot which is useful at low frequencies.)

Exercise 30.5-9

Consider the triangular electric current pulse (Figure 30.5-7)

$$I(t) = \begin{cases} 0 \quad \text{for} \quad t \leqslant t_0 , \\ I_1\left(\dfrac{t-t_0}{t_1-t_0}\right) \quad \text{for} \quad t_0 \leqslant t \leqslant t_1 , \\ I_1\left(\dfrac{t_2-t}{t_2-t_1}\right) \quad \text{for} \quad t_1 \leqslant t \leqslant t_2 , \\ 0 \quad \text{for} \quad t_2 \leqslant t . \end{cases} \tag{30.5-31}$$

(a) Show, by using the definition integral of the Laplace transform and integrating by parts two times, i.e.

$$\hat{I}(s) = s^{-2}\int_{t\in\mathcal{R}} \exp(-st)\partial_t^2 I(t)\,\mathrm{d}t , \tag{30.5-32}$$

and observing that

$$\partial_t^2 I(t) = \frac{I_1}{t_1-t_0}\delta(t-t_0) - \frac{I_1}{t_1-t_0}\delta(t-t_1) - \frac{I_1}{t_2-t_1}\delta(t-t_1) + \frac{I_1}{t_2-t_1}\delta(t-t_2) , \tag{30.5-33}$$

that

$$\hat{I}(s) = s^{-2}\left[\frac{I_1}{t_1-t_0}\exp(-st_0) - \frac{I_1}{t_1-t_0}\exp(-st_1) \right.$$
$$\left. - \frac{I_1}{t_2-t_1}\exp(-st_1) + \frac{I_1}{t_2-t_1}\exp(-st_2)\right] . \tag{30.5-34}$$

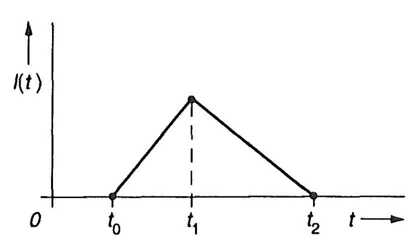


Figure 30.5-7 Triangular electric current pulse of Equation (30.5-31).

(b) Show, by observing that $|\exp(-j\omega t_n)| = 1$, that from this result the following spectral bound, useful at high frequencies in the doubly logarithmic spectral plot, is obtained:

$$|\hat{I}(j\omega)| \leqslant \omega^{-2} 2\left[\frac{|I_1|}{t_1 - t_0} + \frac{|I_1|}{t_2 - t_1}\right]. \tag{30.5-35}$$

(c) Show, by using the Taylor expansion about $s = 0$, that from Equation (30.5-34) it follows that

$$\hat{I}(0) = -\tfrac{1}{2} I_1(t_0 - t_2) \tag{30.5-36}$$

and that this result can be rewritten as

$$\hat{I}(0) = \tfrac{1}{2} I_1(t_2 - t_0) = \int_{t \in \mathcal{R}} I(t)\,\mathrm{d}t\,, \tag{30.5-37}$$

where the latter equality results from the application of the repeated trapezoidal integration rule which is exact for piecewise linear functions. (Note that if $I_1 \geqslant 0$, the result of Exercise 30.5-7 provides a spectral bound in the doubly logarithmic spectral plot which is useful at low frequencies.)

Exercise 30.5-10

Verify Equations (30.5-11), (30.5-13), (30.5-14), (30.5-15) and (30.5-16).

30.6 The shielding effectiveness of a spherical shield for a radiating electric dipole placed at its centre (complex frequency-domain analysis)

In case the interference level between two electromagnetic systems is too high, shielding measures have to be taken. Now, one of the shielding problems that can be solved by rather elementary means is the determination of the shielding effectiveness of a spherical shield for either an electric dipole (short segment of electric current carrying wire) or a magnetic dipole (small electric current carrying loop) placed at its centre. In the present section we analyse the case of an electric dipole (Figure 30.6-1); the case of a magnetic dipole is investigated in Section 30.7.

A Cartesian reference frame is chosen, the origin of which coincides with the centre of the spherical shield. As a consequence of the fact that the emitting electric dipole is placed at the centre of the spherical shield, its electromagnetic field in the absence of the shield is fully specified by an electric source current vector potential (see Section 26.9) that is oriented along the vectorial length L_r of the wire segment, depends on the spatial coordinates only via $|\boldsymbol{x}|$, i.e. the distance from the origin to the point of observation (see Equation (26.9-4)), while the scalar wave function multiplying $\hat{I}L_r$ satisfies the modified Helmholtz equation, Equation (26.2-4), and is bounded as $|\boldsymbol{x}| \to \infty$. It will be shown that under these circumstances the total electromagnetic field in the presence of the shield can be constructed by superimposing on the already existing electric current source vector potential appropriate terms that are also oriented

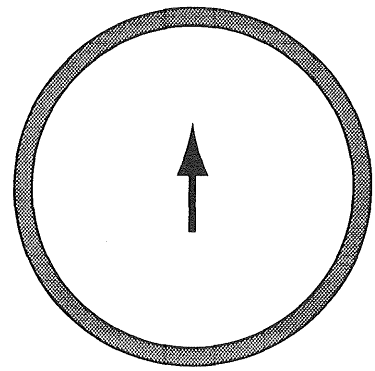


Figure 30.6-1 Electric dipole place at the centre of a spherical shield.

along L_r, also depend on the spatial coordinates only via $|x|$, while the scalar wave functions that multiply $\hat{I}L_r$ satisfy the source-free modified Helmholtz equation in the pertaining domain.

To prove this, we first observe that for any electric-current source vector potential of the type

$$\hat{\Phi}_r^J = \hat{I}L_r\hat{U}\,, \tag{30.6-1}$$

where $\hat{U} = \hat{U}(|x|)$, we have

$$\partial_m\hat{\Phi}_r^J = \xi_m\partial_{|x|}\hat{\Phi}_r^J \tag{30.6-2}$$

and

$$\partial_k\partial_r\hat{\Phi}_r^J = \partial_{k,r}|x|^{-1}\partial_{|x|}\hat{\Phi}_r^J - \xi_k\xi_r|x|^{-1}\partial_{|x|}\hat{\Phi}_r^J + \xi_k\xi_r\partial_{|x|}\partial_{|x|}\hat{\Phi}_r^J\,, \tag{30.6-3}$$

where

$$\xi_m = x_m/|x| \tag{30.6-4}$$

is the unit vector in the radial direction. Using Equations (30.6-1)–(30.6-4) in the expressions for the electric and the magnetic field strengths (see Equations (26.9-2) and (26.9-3)), we obtain

$$\hat{E}_k = (-\hat{\zeta}\hat{U} + \hat{\eta}^{-1}|x|^{-1}\partial_{|x|}\hat{U})\hat{I}L_k + \hat{\eta}^{-1}(-|x|^{-1}\partial_{|x|}\hat{U} + \partial_{|x|}\partial_{|x|}\hat{U})\xi_k\xi_r\hat{I}L_r \tag{30.6-5}$$

and

$$\hat{H}_j = \varepsilon_{j,n,r}\xi_n\hat{I}L_r\partial_{|x|}\hat{U}\,, \tag{30.6-6}$$

respectively. Next, we observe that the second term on the right-hand side of Equation (30.6-5) points along ξ_k; i.e. it has no component tangential to a sphere $|x| = r$. Furthermore, the right-hand side of Equation (30.6-6) has only components tangential to a sphere $|x| = r$. At a spherical interface between two adjacent domains with different homogeneous, isotropic media (note that our analysis only applies to media of this kind) the tangential components of the electric and the magnetic field strengths are therefore continuous across the interface if we make $-\hat{\zeta}\hat{U} + \hat{\eta}^{-1}|x|^{-1}\partial_{|x|}\hat{U}$ and $\partial_{|x|}\hat{U}$ continuous across this interface. These considerations will further be used in the analysis of the shielding problem.

To put the analysis in a rather general setting, we consider a configuration consisting of an arbitrary number of different domains $\{\mathcal{D}_M\}$, bounded by concentric, spherical shells. In each subdomain $\mathcal{D}_M = \{x \in \mathcal{R}^3; r_{M-1} < |x| < r_M\}$ bounded internally by a sphere of radius r_{M-1} and externally by a sphere of radius r_M, a homogeneous, isotropic medium is present with scalar transverse admittance per length $\hat{\eta}_M = \hat{\sigma}_M + s\hat{\varepsilon}_M$ and scalar longitudinal impedance per length $\hat{\zeta}_M = s\hat{\mu}_M$ ($\hat{\sigma}_M$ is the conductivity, $\hat{\varepsilon}_M$ the permittivity and $\hat{\mu}_M$ the permeability of the medium in $\mathcal{D}_M$, s is the time Laplace-transform parameter or complex frequency). The representation of the scalar wave function $\hat{U} = \hat{U}(|x|)$ in the domain $\mathcal{D}_M$ is written as

$$\hat{U}_M = \hat{U}_M^{+} \frac{\exp[-\hat{\gamma}_M(|x| - r_{M-1})]}{4\pi|x|} + \hat{U}_M^{-} \frac{\exp[-\hat{\gamma}_M(r_M - |x|)]}{4\pi|x|}$$

$$\text{for} \quad r_{M-1} < |x| < r_M , \tag{30.6-7}$$

in which

$$\hat{\gamma}_M = (\hat{\eta}_M \hat{\zeta}_M)^{1/2} \tag{30.6-8}$$

with $\mathrm{Re}(\hat{\gamma}_M) > 0$ for $\mathrm{Re}(s) > 0$. In Equation (30.6-7), $\hat{U}_M^{+}$ and $\hat{U}_M^{-}$ are arbitrary constant coefficients. Each term on the right-hand side of Equation (30.6-7) satisfies the source-free modified Helmholtz equation as long as $|x| \neq 0$. The reference values of $|x|$ (viz. r_{M-1} and r_M) in the arguments of the exponential functions have been included to ensure that all exponential functions have arguments with non-positive real parts which avoids the loss of significant figures in the numerical evaluation of the expressions. Similarly, the representation of the scalar wave function $\hat{U} = \hat{U}(|x|)$ in the domain $\mathcal{D}_{M+1}$ is written as

$$\hat{U}_{M+1} = \hat{U}_{M+1}^{+} \frac{\exp[-\hat{\gamma}_{M+1}(|x| - r_M)]}{4\pi|x|} + \hat{U}_{M+1}^{-} \frac{\exp[-\hat{\gamma}_{M+1}(r_{M+1} - |x|)]}{4\pi|x|}$$

$$\text{for} \quad r_M < |x| < r_{M+1} . \tag{30.6-9}$$

In view of Equations (30.6-5) and (30.6-6) the continuity of the tangential components of the electric and the magnetic field strengths across the common interface $|x| = r_M$ of $\mathcal{D}_M$ and $\mathcal{D}_{M+1}$ is now guaranteed if

$$\lim_{|x| \downarrow r_M}(-\hat{\zeta}\hat{U} + \hat{\eta}^{-1}|x|^{-1}\partial_{|x|}\hat{U}) = \lim_{|x| \uparrow r_M}(-\hat{\zeta}\hat{U} + \hat{\eta}^{-1}|x|^{-1}\partial_{|x|}\hat{U}) \tag{30.6-10}$$

and

$$\lim_{|x| \downarrow r_M}\partial_{|x|}\hat{U} = \lim_{|x| \uparrow r_M}\partial_{|x|}\hat{U} . \tag{30.6-11}$$

Using Equations (30.6-7) and (30.6-9) in Equations (30.6-10) and (30.6-11) and noting that

$$\partial_{|x|}\hat{U}_M = (-\hat{\gamma}_M - |x|^{-1})\hat{U}_M^{+} \frac{\exp[-\hat{\gamma}_M(|x| - r_{M-1})]}{4\pi|x|} + (\hat{\gamma}_M - |x|^{-1})\hat{U}_M^{-} \frac{\exp[-\hat{\gamma}_M(r_M - |x|)]}{4\pi|x|}$$

$$\text{for} \quad r_{M-1} < |x| < r_M , \tag{30.6-12}$$

with a similar expression for $\partial_{|x|}\hat{U}_{M+1}$, we arrive at the conditions

$$\begin{aligned}
&(-\hat{\zeta}_{M+1} - \hat{\eta}_{M+1}^{-1}\hat{\gamma}_{M+1}/r_M - \hat{\eta}_{M+1}^{-1}/r_M^2)\hat{U}_{M+1}^{+} \\
&+ (-\hat{\zeta}_{M+1} + \hat{\eta}_{M+1}^{-1}\hat{\gamma}_{M+1}/r_M - \hat{\eta}_{M+1}^{-1}/r_M^2)\hat{U}_{M+1}^{-} \exp(-\hat{\gamma}_{M+1}d_{M+1}) \\
&= (-\hat{\zeta}_M - \hat{\eta}_M^{-1}\hat{\gamma}_M/r_M - \hat{\eta}_M^{-1}/r_M^2)\hat{U}_M^{+} \exp(-\hat{\gamma}_M d_M) \\
&\quad + (-\hat{\zeta}_M + \hat{\eta}_M^{-1}\hat{\gamma}_M/r_M - \hat{\eta}_M^{-1}/r_M^2)\hat{U}_M^{-}
\end{aligned} \tag{30.6-13}$$

and

$$(-\hat{\gamma}_{M+1} - r_M^{-1})\hat{U}_{M+1}^{+} + (\hat{\gamma}_{M+1} - r_M^{-1})\hat{U}_{M+1}^{-} \exp(-\hat{\gamma}_{M+1} d_{M+1})$$
$$= (-\hat{\gamma}_M - r_M^{-1})\hat{U}_M^{+} \exp(-\hat{\gamma}_M d_M) + (\hat{\gamma}_M - r_M^{-1})\hat{U}_M^{-}, \tag{30.6-14}$$

where

$$d_M = r_M - r_{M-1} \tag{30.6-15}$$

is the thickness of the shell that occupies the domain $\mathcal{D}_M$.

We now return to our shielding problem. Here, we distinguish the domain $\mathcal{D}_1 = \{x \in \mathcal{R}^3; 0 \leqslant |x| < r_1\}$ interior to the shield (where the emitting dipole is placed), the shielding domain $\mathcal{D}_2 = \{x \in \mathcal{R}^3; r_1 < |x| < r_2\}$, and the domain $\mathcal{D}_3 = \{x \in \mathcal{R}^3; r_2 < |x| < \infty\}$ exterior to the shield (Figure 30.6-1). In $\mathcal{D}_1$, the wave function that must be superimposed on the one of the transmitting dipole to account for the presence of the shield must be bounded at $|x| = 0$. This is accomplished by taking

$$\hat{U}_1 = \frac{\exp(-\hat{\gamma}_1(|x|)}{4\pi|x|} + 2R\exp(-\hat{\gamma}_1 r_1)\frac{\sinh(-\hat{\gamma}_1|x|)}{4\pi|x|}$$
$$= [1 - R\exp(-\hat{\gamma}_1 r_1)]\frac{\exp(-\hat{\gamma}_1(|x|)}{4\pi|x|} + R\frac{\exp[-\hat{\gamma}_1(r_1 - |x|)]}{4\pi|x|} \quad \text{for } 0 \leqslant |x| < r_1. \tag{30.6-16}$$

In $\mathcal{D}_2$, we take (see Equation (30.6-7))

$$\hat{U}_2 = \hat{U}_2^{+}\frac{\exp[-\hat{\gamma}_2(|x| - r_1)]}{4\pi|x|} + \hat{U}_2^{-}\frac{\exp[-\hat{\gamma}_2(r_2 - |x|)]}{4\pi|x|} \quad \text{for } r_1 < |x| < r_2. \tag{30.6-17}$$

In $\mathcal{D}_3$, the wave function must, because of causality, remain bounded as $|x| \to \infty$, and hence the term containing $\exp(\hat{\gamma}_3|x|)/4\pi|x|$ must be absent. Accordingly, we take

$$\hat{U}_3 = T\frac{\exp[-\hat{\gamma}_3(|x| - r_2)]}{4\pi|x|} \quad \text{for } r_2 < |x| < \infty. \tag{30.6-18}$$

The boundary conditions at the interface $|x| = r_1$ lead to the conditions (see Equations (30.6-13) and (30.6-14))

$$(-\hat{\xi}_2 - \hat{\eta}_2^{-1}\hat{\gamma}_2/r_1 - \hat{\eta}_2^{-1}/r_1^2)\hat{U}_2^{+} + (-\hat{\xi}_2 + \hat{\eta}_2^{-1}\hat{\gamma}_2/r_1 - \hat{\eta}_2^{-1}/r_1^2)\hat{U}_2^{-}\exp(-\hat{\gamma}_2 d_2)$$
$$= (-\hat{\xi}_1 - \hat{\eta}_1^{-1}\hat{\gamma}_1/r_1 - \hat{\eta}_1^{-1}/r_1^2)\,[1 - R\exp(-\hat{\gamma}_1 d_1)]\exp(-\hat{\gamma}_1 r_1)$$
$$+ (-\hat{\xi}_1 + \hat{\eta}_1^{-1}\hat{\gamma}_1/r_1 - \hat{\eta}_1^{-1}/r_1^2)R \tag{30.6-19}$$

and

$$(-\hat{\gamma}_2 - r_1^{-1})\hat{U}_2^{+} + (\hat{\gamma}_2 - r_1^{-1})\hat{U}_2^{-}\exp(-\hat{\gamma}_2 d_2)$$
$$= (-\hat{\gamma}_1 - r_1^{-1})\,[1 - R\exp(-\hat{\gamma}_1 r_1)]\exp(-\hat{\gamma}_1 r_1) + (\hat{\gamma}_1 - r_1^{-1})R. \tag{30.6-20}$$

The boundary conditions at the interface $|x| = r_2$ lead to the conditions

$$(-\hat{\xi}_3 - \hat{\eta}_3^{-1}\hat{\gamma}_3/r_2 - \hat{\eta}_3^{-1}/r_2^2)T$$
$$= (-\hat{\xi}_2 - \hat{\eta}_2^{-1}\hat{\gamma}_2/r_2 - \hat{\eta}_2^{-1}/r_2^2)\,\hat{U}_2^{+}\exp(-\hat{\gamma}_2 d_2) + (-\hat{\xi}_2 + \hat{\eta}_2^{-1}\hat{\gamma}_2/r_2 - \hat{\eta}_2^{-1}/r_2^2)\hat{U}_2^{-} \tag{30.6-21}$$

and

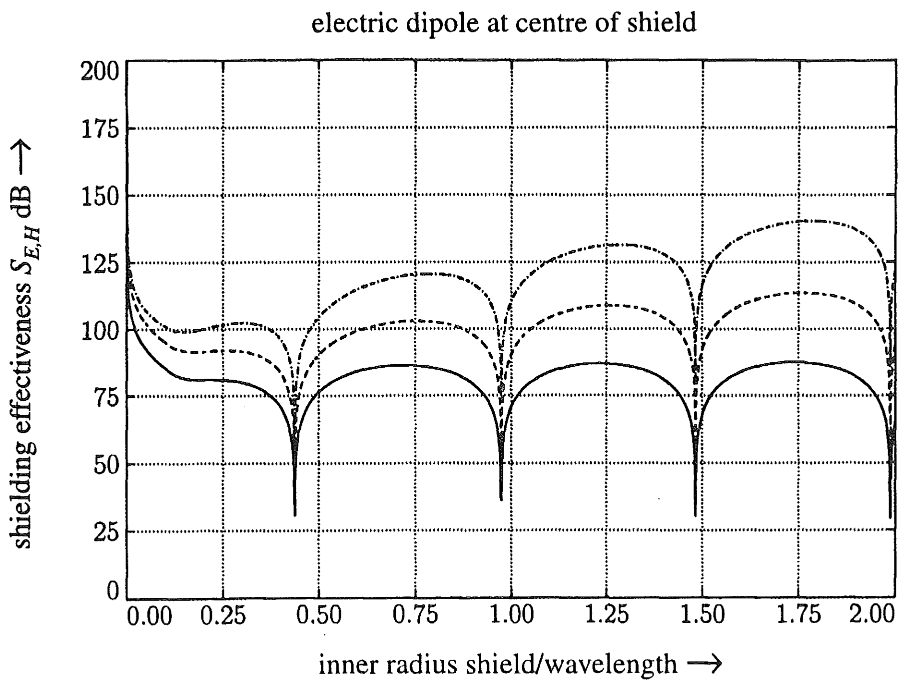


Figure 30.6-2 Shielding effectiveness of a copper spherical shield present in vacuum for an electric dipole placed at its centre. Shield parameters values: conductivity 5.65×10^7 S/m, permittivity ε_0, permeability μ_0, inner radius 5.0×10^{-2} m, thickness 1.0×10^{-6} m (lower curve)/ 3.0×10^{-6} m (middle curve)/ 5.0×10^{-6} m (upper curve).

$$(-\hat{\gamma}_3 - r_2^{-1})T = (-\hat{\gamma}_2 - r_2^{-1})\hat{U}_2^+ \exp(-\hat{\gamma}_2 d_2) + (\hat{\gamma}_2 - r_2^{-1})\hat{U}_2^- . \tag{30.6-22}$$

Equations (30.6-19)–(30.6-22) constitute an inhomogeneous system of four linear, algebraic equations from which the four unknown coefficients R, $\hat{U}_2^+$, $\hat{U}_2^-$ and T are to be solved. For each particular case this is done numerically.

To characterise the performance of the shield, its shielding effectiveness $S_{E,H}$ is calculated (this quantity is defined in Section 30.5). To apply the definition, we take $\hat{\eta}_3 = \hat{\eta}_1$, $\hat{\zeta}_3 = \hat{\zeta}_1$, and hence $\hat{\gamma}_3 = \hat{\gamma}_1$. Then, for any point $x \in \mathcal{D}_3$ in the exterior of the shield we have

$$S_{E,H} = T^{-1} \tag{30.6-23}$$

and hence

$$S_{E,H}\,\mathrm{dB} = -20\log_{10}|T| . \tag{30.6-24}$$

Figure 30.6-2 shows $S_{E,H}$ dB as a function of normalised frequency for a sinusoidally in time, with angular frequency ω, oscillating dipole (hence, $s = \mathrm{j}\omega$) for a copper shield placed in air (vacuum). The dips in the shielding effectiveness occur at frequencies that correspond to a resonant frequency of the innermost spherical cavity.

30.7 The shielding effectiveness of a spherical shield for a radiating magnetic dipole placed at its centre (complex frequency-domain analysis)

In this section we investigate the shielding properties of a spherical shield for a magnetic dipole (small electric current-carrying loop) placed at its centre (Figure 30.7-1). The analysis runs parallel to the one for the electric dipole considered in Section 30.6. Therefore, we confine ourselves to presenting the major steps.

With the origin of the Cartesian reference frame at the centre of the shield, the electromagnetic field of the magnetic dipole in the absence of the shield is fully specified by a magnetic-current source vector potential (see Section 26.10) that is oriented along the vectorial area A_p of the magnetic dipole, depends on the spatial coordinates only via $|x|$, i.e. the distance from the origin to the point of observation (see Equation (26.10-13)), while the scalar wave function multiplying $\hat{\zeta}\hat{I}A_p$ satisfies the modified Helmholtz equation, Equation (26.2-4). As in Section 30.6, the total electromagnetic field in the presence of the shield can be constructed by superimposing on the already existing magnetic current source vector potential appropriate terms that are also oriented along A_p, also depend on the spatial coordinates only via $|x|$, while the scalar wave functions multiplying $\hat{\zeta}\hat{I}A_p$ satisfy the source-free modified Helmholtz equation in the pertaining domain.

Let

$$\hat{\Phi}_p^K = \hat{\zeta}\hat{I}A_p\hat{V}, \tag{30.7-1}$$

where $\hat{V} = \hat{V}(|x|)$, then we have

$$\partial_m\hat{\Phi}_p^K = \xi_m\partial_{|x|}\hat{\Phi}_p^K \tag{30.7-2}$$

and

$$\partial_j\partial_p\hat{\Phi}_p^K = \partial_{j,p}|x|^{-1}\partial_{|x|}\hat{\Phi}_p^K - \xi_j\xi_p|x|^{-1}\partial_{|x|}\hat{\Phi}_p^K + \xi_j\xi_p\partial_{|x|}\partial_{|x|}\hat{\Phi}_p^K, \tag{30.7-3}$$

where

$$\xi_m = x_m/|x| \tag{30.7-4}$$

is the unit vector in the radial direction. Using Equations (30.7-1)–(30.7-4) in the expressions for the electric and the magnetic field strengths (see Equations (26.10-14) and (26.10-15)), we obtain

$$\hat{E}_k = -\varepsilon_{k,m,p}\xi_m\hat{\zeta}\hat{I}A_p\partial_{|x|}\hat{V} \tag{30.7-5}$$

and

$$\hat{H}_j = (-\hat{\eta}\hat{V} + \hat{\zeta}^{-1}|x|^{-1}\partial_{|x|}\hat{V})\hat{\zeta}\hat{I}A_j + \hat{\zeta}^{-1}(-|x|^{-1}\partial_{|x|}\hat{V} + \partial_{|x|}\partial_{|x|}\hat{V})\xi_j\xi_p\hat{\zeta}\hat{I}A_p, \tag{30.7-6}$$

respectively. The right-hand side of Equation (30.7-5) has only components tangential to a sphere $|x| = r$. The second term on the right-hand side of Equation (30.7-6) points along ξ_j, i.e. it has no component tangential to a sphere $|x| = r$. At a spherical interface between two adjacent domains with different homogeneous, isotropic media the tangential components of the electric and the magnetic field strengths are therefore continuous if we make $\hat{\zeta}\partial_{|x|}\hat{V}$ and $\hat{\zeta}(-\hat{\eta}\hat{V} + \hat{\zeta}^{-1}|x|^{-1}\partial_{|x|}\hat{V}$ continuous across this interface. These considerations will be further used in the analysis of our shielding problem.

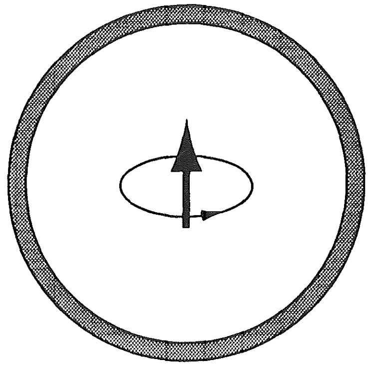


Figure 30.7-1 Magnetic dipole place at the centre of a spherical shield.

Again, we put the analysis in the setting of a configuration consisting of an arbitrary number of different domains $\{\mathcal{D}_M\}$, bounded by concentric, spherical shells. In each subdomain $\mathcal{D}_M = \{x \in \mathcal{R}^3; r_{M-1} < |x| < r_M\}$ bounded internally by a sphere of radius r_{M-1} and externally by a sphere of radius r_M, a homogeneous, isotropic medium is present with scalar transverse admittance per length $\hat{\eta}_M = \hat{\sigma}_M + s\hat{\varepsilon}_M$ and scalar longitudinal impedance per length $\hat{\zeta}_M = s\hat{\mu}_M$ ($\hat{\sigma}_M$ is the conductivity, $\hat{\varepsilon}_M$ the permittivity and $\hat{\mu}_M$ the permeability of the medium in $\mathcal{D}_M$, s is the time Laplace-transform parameter or complex frequency). The representation of the scalar wave function $\hat{V} = \hat{V}(|x|)$ in the domain $\mathcal{D}_M$ is written as

$$\hat{V}_M = \hat{V}_M^+ \frac{\exp[-\hat{\gamma}_M(|x| - r_{M-1})]}{4\pi|x|} + \hat{V}_M^- \frac{\exp[-\hat{\gamma}_M(r_M - |x|)]}{4\pi|x|}$$

$$\text{for} \quad r_{M-1} < |x| < r_M , \tag{30.7-7}$$

in which

$$\hat{\gamma}_M = (\hat{\eta}_M \hat{\zeta}_M)^{1/2} \tag{30.7-8}$$

with $\mathrm{Re}(\hat{\gamma}_M) > 0$ if $\mathrm{Re}(s) > 0$. In Equation (30.7-7), $\hat{V}_M^+$ and $\hat{V}_M^-$ are arbitrary constant coefficients. Each term on the right-hand side of Equation (30.7-7) satisfies the source-free modified Helmholtz equation as long as $|x| \neq 0$. The reference values of $|x|$ (viz. r_{M-1} and r_M) in the arguments of the exponential functions have been included to ensure that all exponential functions have arguments with non-positive real parts which avoids the loss of significant figures in the numerical evaluation of the expressions. Similarly, the representation of the scalar wave function $\hat{V} = \hat{V}(|x|)$ in the domain $\mathcal{D}_{M+1}$ is written as

$$\hat{V}_{M+1} = \hat{V}_{M+1}^+ \frac{\exp[-\hat{\gamma}_{M+1}(|x| - r_M)]}{4\pi|x|} + \hat{V}_{M+1}^- \frac{\exp[-\hat{\gamma}_{M+1}(r_{M+1} - |x|)]}{4\pi|x|}$$

$$\text{for} \quad r_M < |x| < r_{M+1} . \tag{30.7-9}$$

In view of Equations (30.7-5) and (30.7-6) the continuity of the tangential components of the electric and the magnetic field strengths across the common interface $|x| = r_M$ is now guaranteed if

$$\lim_{|x| \downarrow r_M} \hat{\zeta} \partial_{|x|} \hat{V} = \lim_{|x| \uparrow r_M} \hat{\zeta} \partial_{|x|} \hat{V} \tag{30.7-10}$$

and

$$\lim_{|x|\downarrow r_M} \hat{\xi}(-\hat{\eta}\hat{V} + \hat{\xi}^{-1}|x|^{-1}\partial_{|x|}\hat{V}) = \lim_{|x|\uparrow r_M} \hat{\xi}(-\hat{\eta}\hat{V} + \hat{\xi}^{-1}|x|^{-1}\partial_{|x|}\hat{V}) . \qquad (30.7\text{-}11)$$

Using Equations (30.7-7) and (30.7-9) in Equations (30.7-10) and (30.7-11) and noting that

$$\partial_{|x|}\hat{V}_M = (-\hat{\gamma}_M - |x|^{-1})\hat{V}_M^+ \frac{\exp[-\hat{\gamma}_M(|x| - r_{M-1})]}{4\pi|x|} + (\hat{\gamma}_M - |x|^{-1})\hat{V}_M^- \frac{\exp[-\hat{\gamma}_M(r_M - |x|)]}{4\pi|x|}$$

$$\text{for} \quad r_{M-1} < |x| < r_M , \qquad (30.7\text{-}12)$$

with a similar expression for $\partial_{|x|}\hat{V}_{M+1}$, we arrive at the conditions

$$\hat{\xi}_{M+1}\left[(-\hat{\gamma}_{M+1} - r_M^{-1})\hat{V}_{M+1}^+ + (\hat{\gamma}_{M+1} - r_M^{-1})\hat{V}_{M+1}^- \exp(-\hat{\gamma}_{M+1}d_{M+1})\right]$$
$$= \hat{\xi}_M\left[(-\hat{\gamma}_M - r_M^{-1})\hat{V}_M^+ \exp(-\hat{\gamma}_M d_M) + (\hat{\gamma}_M - r_M^{-1})\hat{V}_M^-\right], \qquad (30.7\text{-}13)$$

and

$$\hat{\xi}_{M+1}\left[(-\hat{\eta}_{M+1} - \hat{\xi}_{M+1}^{-1}\hat{\gamma}_{M+1}/r_M - \hat{\xi}_{M+1}^{-1}/r_M^2)\hat{V}_{M+1}^+\right.$$
$$\left. + (-\hat{\eta}_{M+1} + \hat{\xi}_{M+1}^{-1}\hat{\gamma}_{M+1}/r_M - \hat{\xi}_{M+1}^{-1}/r_M^2)\hat{V}_{M+1}^- \exp(-\hat{\gamma}_{M+1}d_{M+1})\right]$$
$$= \hat{\xi}_M\left[(-\hat{\eta}_M - \hat{\xi}_M^{-1}\hat{\gamma}_M/r_M - \hat{\xi}_M^{-1}/r_M^2)\hat{V}_M^+ \exp(-\hat{\gamma}_M d_M)\right.$$
$$\left. + (-\hat{\eta}_M + \hat{\xi}_M^{-1}\hat{\gamma}_M/r_M - \hat{\xi}_M^{-1}/r_M^2)\hat{V}_M^-\right], \qquad (30.7\text{-}14)$$

where

$$d_M = r_M - r_{M-1} \qquad (30.6\text{-}15)$$

is the thickness of the shell occupying the domain $\mathcal{D}_M$.

We now return to our shielding problem. Here, we distinguish the domain $\mathcal{D}_1 = \{x \in \mathcal{R}^3; 0 \leqslant |x| < r_1\}$ interior to the shield (where the emitting dipole is placed), the shielding domain $\mathcal{D}_2 = \{x \in \mathcal{R}^3; r_1 < |x| < r_2\}$, and the domain $\mathcal{D}_3 = \{x \in \mathcal{R}^3; r_2 < |x| < \infty\}$ exterior to the shield (Figure 30.7-1). In $\mathcal{D}_1$, the wave function that must be superimposed on the one of the transmitting dipole to account for the presence of the shield must be bounded at $|x| = 0$. This is accomplished by taking

$$\hat{V}_1 = \frac{\exp(-\hat{\gamma}_1(|x|)}{4\pi|x|} + 2R\exp(-\hat{\gamma}_1 r_1)\frac{\sinh(-\hat{\gamma}_1|x|)}{4\pi|x|}$$
$$= [1 - R\exp(-\hat{\gamma}_1 r_1)]\frac{\exp(-\hat{\gamma}_1(|x|)}{4\pi|x|} + R\frac{\exp[-\hat{\gamma}_1(r_1 - |x|)]}{4\pi|x|} \quad \text{for} \quad 0 \leqslant |x| < r_1 . \qquad (30.7\text{-}16)$$

In $\mathcal{D}_2$, we take (see Equation (30.7-7))

$$\hat{V}_2 = \hat{V}_2^+ \frac{\exp[-\hat{\gamma}_2(|x| - r_1)]}{4\pi|x|} + \hat{V}_2^- \frac{\exp[-\hat{\gamma}_2(r_2 - |x|)]}{4\pi|x|} \quad \text{for} \quad r_1 < |x| < r_2 . \qquad (30.7\text{-}17)$$

In $\mathcal{D}_3$, the wave function must, because of causality, remain bounded as $|x| \to \infty$, and hence the term containing $\exp(\hat{\gamma}_3|x|)/4\pi|x|$ must be absent. Accordingly, we take

$$\hat{V}_3 = T\frac{\exp[-\hat{\gamma}_3(|\boldsymbol{x}| - r_2)]}{4\pi|\boldsymbol{x}|} \quad \text{for} \quad r_2 < |\boldsymbol{x}| < \infty . \tag{30.7-18}$$

The boundary conditions at the interface $|\boldsymbol{x}| = r_1$ lead to the conditions (see Equations (30.7-13) and (30.7-14))

$$\hat{\zeta}_2\left[(-\hat{\gamma}_2 - r_1^{-1})\hat{V}_2^+ + (\hat{\gamma}_2 - r_1^{-1})\hat{V}_2^- \exp(-\hat{\gamma}_2 d_2)\right]$$
$$= \hat{\zeta}_1\left[(-\hat{\gamma}_1 - r_1^{-1})\,[1 - R\exp(-\hat{\gamma}_1 r_1)]\exp(-\hat{\gamma}_1 r_1) + (\hat{\gamma}_1 - r_1^{-1})R\right], \tag{30.7-19}$$

and

$$\hat{\zeta}_2\left[(-\hat{\eta}_2 - \hat{\zeta}_2^{-1}\hat{\gamma}_2/r_1 - \hat{\zeta}_2^{-1}/r_1^2)\hat{V}_2^+ + (-\hat{\eta}_2 + \hat{\zeta}_2^{-1}\hat{\gamma}_2/r_1 - \hat{\zeta}_2^{-1}/r_1^2)\hat{V}_2^- \exp(-\hat{\gamma}_2 d_2)\right]$$
$$= \hat{\zeta}_1\left[(-\hat{\eta}_1 - \hat{\zeta}_1^{-1}\hat{\gamma}_1/r_1 - \hat{\zeta}_1^{-1}/r_1^2)\,[1 - R\exp(-\hat{\gamma}_2 r_1)]\exp(-\hat{\gamma}_1 r_1)\right.$$
$$\left. + (-\hat{\eta}_1 + \hat{\zeta}_1^{-1}\hat{\gamma}_1/r_1 - \hat{\zeta}_1^{-1}/r_1^2)R\right]. \tag{30.7-20}$$

The boundary conditions at the interface $|\boldsymbol{x}| = r_2$ lead to the conditions

$$\hat{\zeta}_3(-\hat{\gamma}_3 - r_2^{-1})T = \hat{\zeta}_2\left[(-\hat{\gamma}_2 - r_2^{-1})\hat{V}_2^+ \exp(-\hat{\gamma}_2 d_2) + (\hat{\gamma}_2 - r_2^{-1})\hat{V}_2^-\right]. \tag{30.7-21}$$

and

$$\hat{\zeta}_3(-\hat{\eta}_3 - \hat{\zeta}_3^{-1}\hat{\gamma}_3/r_2 - \hat{\zeta}_3^{-1}/r_2^2)T$$
$$= \hat{\zeta}_2\left[(-\hat{\eta}_2 - \hat{\zeta}_2^{-1}\hat{\gamma}_2/r_2 - \hat{\zeta}_2^{-1}/r_2^2)\,\hat{V}_2^+ \exp(-\hat{\gamma}_2 d_2)\right.$$
$$\left. + (-\hat{\eta}_2 + \hat{\zeta}_2\hat{\gamma}_2^{-1}/r_2 - \hat{\zeta}_2^{-1}/r_2^2)\hat{V}_2^-\right]. \tag{30.7-22}$$

Equations (30.7-19)–(30.7-22) constitute an inhomogeneous system of four linear, algebraic equations from which the four unknown coefficients R, $\hat{V}_2^+$, $\hat{V}_2^-$ and T are to be solved. For each particular case this is done numerically.

To characterise the performance of the shield, its shielding effectiveness $S_{E,H}$ is calculated (this quantity is defined in Section 30.5). To apply this definition, we take $\hat{\eta}_3 = \hat{\eta}_1$, $\hat{\zeta}_3 = \hat{\zeta}_1$, and hence $\hat{\gamma}_3 = \hat{\gamma}_1$. Then, for any point $\boldsymbol{x} \in \mathcal{D}_3$ in the exterior of the shield we have

$$S_{E,H} = T^{-1} \tag{30.7-23}$$

and hence

$$S_{E,H}\,\text{dB} = -20\log_{10}|T| . \tag{30.7-24}$$

Figure 30.7-2 shows $S_{E,H}$ dB as a function of normalised frequency for a sinusoidally in time, with angular frequency ω, oscillating dipole (hence, $s = j\omega$) for a copper shield placed in air (vacuum). The dips in the shielding effectiveness occur at frequencies that correspond to a resonant frequency of the innermost spherical cavity.

Note that the resonant frequencies for magnetic-dipole excitation differ from the ones for electric-dipole excitation. Also, the low-frequency behaviour is different for the two cases. For further discussion, see Quak and De Hoop (1989).

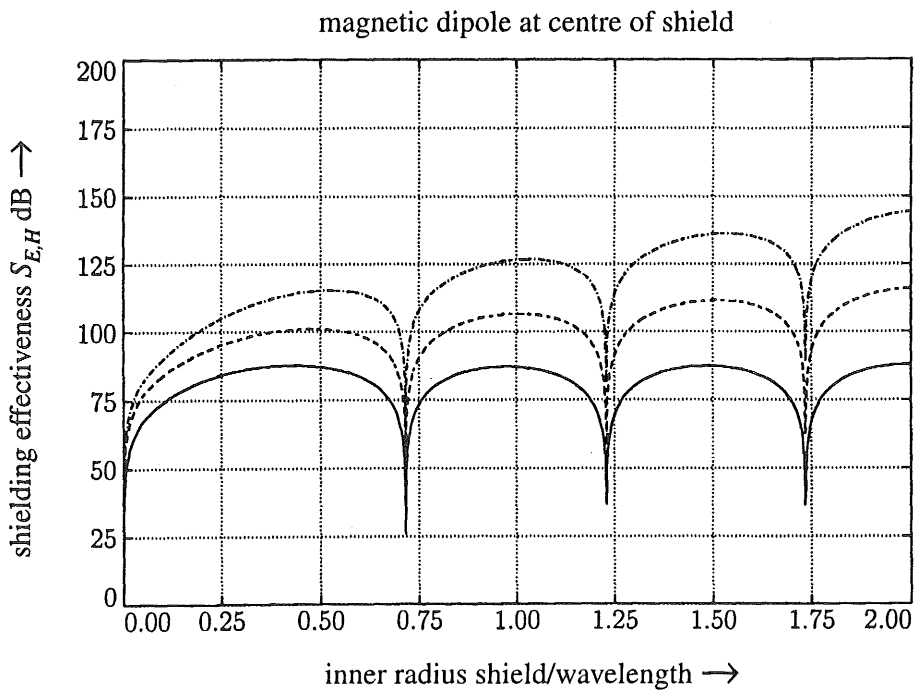


Figure 30.7-2 Shielding effectiveness of a copper spherical shield present in vacuum for a magnetic dipole placed at its centre. Shield parameters values: conductivity 5.65×10^7 S/m, permittivity ε_0, permeability μ_0, inner radius 5.0×10^{-2} m, thickness 1.0×10^{-6} m (lower curve)/ 3.0×10^{-6} m (middle curve)/ 5.0×10^{-6} m (upper curve).

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