General introduction

This *Handbook* covers the fundamental aspects and a number of applications of the theory of three wave phenomena, related in their mathematical structure, viz. *Radiation and Scattering of Acoustic Waves in Fluids* (Part 1), *Radiation and Scattering of Elastic Waves in Solids* (Part 2) and *Radiation and Scattering of Electromagnetic Waves* (Part 3). Each of the three parts can be read and studied independently. Some general remarks applying to all three kinds of wave motion are given in this General Introduction. Some mathematical preliminaries are collected in Appendices A and B. Appendix A deals with the elementary properties of Cartesian vectors and tensors, for which the subscript notation, supplemented with the summation convention, is used as an interdisciplinary notational tool. Appendix B deals with the description of signals in linear, time-invariant systems. For such systems, the time-domain characterisation and the complex frequency-domain characterisation yield complementary descriptions, that are related through the time Laplace transformation. In addition, the spatial Fourier transformation is discussed, which finds its application in spatially shift-invariant configurations (in one, two or three dimensions). This transformation interrelates the spatial-domain and spectral-domain descriptions of wave phenomena.

The physical laws that underlie the properties of acoustic, elastic or electromagnetic waves are deduced from a series of basic standard experiments. To carry out these experiments, an observer must be able to register both the position in space and the instant at which an observation is made. To register the position in space, the existence of a three-dimensional, isotropic, Euclidean, background space $\mathcal{R}^3$ is presumed. In this space, distance can be measured along three mutually perpendicular directions with the same position- and orientation-independent standard measuring rod. To register instants, the existence of a position- and orientation-independent standard clock is presumed.

The standard measuring rod is used to define, at a certain position that is denoted as the origin O, an orthogonal Cartesian reference frame consisting of three *base vectors* $\{\boldsymbol{i}(1),\boldsymbol{i}(2),\boldsymbol{i}(3)\}$ that are mutually perpendicularly oriented, are each of unit length, and form in the indicated order a right-handed system (Figure 1). (The property that each base vector specifies geometrically a length and an orientation makes it a vectorial quantity, or a *vector*; notationally, vectors will, whenever appropriate, be represented by bold-face symbols.) Let $\{x_1,x_2,x_3\}$ denote the ordered sequence of the three real numbers that are needed to specify the position of an observer, then the vectorial position $\boldsymbol{x}$ of the observer is the linear combination

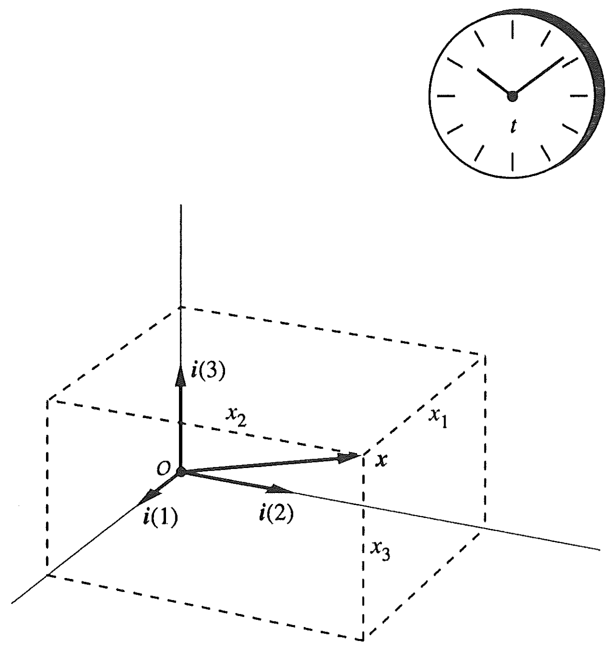

Figure 1 Orthogonal Cartesian reference frame with origin O and three mutually perpendicular base vectors $\{\boldsymbol{i}(1),\boldsymbol{i}(2),\boldsymbol{i}(3)\}$ of unit length each, position vector $\boldsymbol{x} = x_1\boldsymbol{i}(1) + x_2\boldsymbol{i}(2) + x_3\boldsymbol{i}(3)$, and time coordinate t.

$$\boldsymbol{x} = x_1\boldsymbol{i}(1) + x_2\boldsymbol{i}(2) + x_3\boldsymbol{i}(3)$$
$$= \sum_{m=1}^{3} x_m\boldsymbol{i}(m) . \qquad (1)$$

The numbers $\{x_1,x_2,x_3\}$ are denoted as the right-handed, orthogonal Cartesian coordinates of the point of observation. The time coordinate is real and is denoted by t. In the notation of the theory of sets we write $\boldsymbol{x}\in\mathcal{R}^3$ and $t\in\mathcal{R}$.

One of the purposes of the basic standard experiments is to define the units in terms of which the measured physical quantities are expressed. In accordance with international convention, the International System of Units (Système International d'Unités) – abbreviated to SI – is employed. This system is based on seven basic quantities. The basic quantities, the basic units and the basic dimensions of SI are shown in Table 1 (see also, *Symbols, Units, Nomenclature and Fundamental Constants in Physics*, 1987 Revision, Document IUPAP-25 (SUNAMCO 87-1) prepared by Richard Cohen and Pierre Giacomo, International Union of Pure and Applied Physics, SUNAMCO Commission, Sevres).

In accordance with international standardisation, a simple unit is denoted in writing either by its full name written in lower-case roman (upright) type letters, and always in singular, or by its normalised symbol (which also remains unaltered in the plural). The latter is a lower-case letter, or a succession of lower-case letters, if the relevant unit is not derived from a proper name

Table 1 Basic quantities, units and dimensions of the International System of Units (SI)

Basic quantity		Basic unit		Basic dimension
Name	Symbol	Name	Symbol	Symbol
Length	l	metre	m	L
Mass	m	kilogramme	kg	M
Time	t	second	s	T
Electric current	I	ampere	A	I
Thermodynamic temperature	T	kelvin	K	θ
Amount of substance	n	mole	mol	N
Luminous intensity	I	candela	cd	J

(such as second (s), kilogramme (kg)), and an upper-case letter or an upper-case letter followed by one or more lower-case letters if the relevant unit is derived from a proper name (such as ampere (A), pascal (Pa)). Either simple or compound quantities that are derived from the basic quantities will occur in the sequel. The compound units are formed from the pertinent simple units by using the dot (•) as the multiplication sign and the solidus (/) as the division sign. Examples are: newton•second (N•s) for the unit of momentum and metre/second (m/s) for the unit of velocity. The only exception occurs if a simple unit would need a division sign; in this case, the notation 'to the power −1' is used. An example of the latter is second^{-1} (s^{-1}) for the unit 'per time'.

In SI the basic unit of length (m) is derived from the basic unit of time (s) via the speed of light (or electromagnetic wave speed) in vacuum: c_0 = 299 792 458 m/s (exactly).

The prefixes that should be used to indicate decimal multiples or submultiples of a unit are listed in Table 2.

When a prefix symbol is used with a unit symbol the combination should be considered as a single new symbol that can be raised to a positive or negative power without using parentheses; for example, 1 mm^3 = (10^{-3} m)^3 = 10^{-9} m^3, 1 km^3 = (10^3 m)^3 = 10^9 m^3. Compound prefixes formed by the juxtaposition of two or more prefixes should not be used.

The mathematical framework by which the results from the standard basic experiments are cast into the macroscopic physical laws that govern the wave motion is furnished by *tensor calculus*. In fact, a postulate to this effect has been used by Einstein to arrive at the theory of relativity (Einstein, 1956). In addition to this, the corresponding subscript notation has the important advantage that expressions and equations can be copied almost effortlessly to produce the corresponding statements in any of the high-level programming languages (for example, Fortran 77 or Fortran 90), while they are also directly amenable to symbolic manipulation in programs like Mathematica™. For this reason, Appendix A gives an introduction to the notation and the properties of the Cartesian tensors that are needed in our further analysis.

In a number of canonical configurations the acoustic, elastic or electromagnetic wave motion can adequately be analysed with the aid of integral-transformation methods of the time Laplace or spatial Fourier type. For this reason, the main features of these transforms are surveyed in Appendix B.

Table 2 Prefixes for use with SI units

	Prefix	
Power of 10	Name	Symbol
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	µ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{1}	deca	da
10^{2}	hecto	h
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E

Exercises

Exercise 1

Do symbols for SI units include a full stop (period)?

Answer: No.

Exercise 2

Are symbols for SI units altered in the plural?

Answer: No.

References

Einstein, A., 1956, *The Meaning of Relativity*, Princeton NJ: Princeton University Press, 5th edn.