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IMPULSIVE RADIATION FROM A VERTICAL ELECTRIC DIPOLE SITUATED ABOVE A PLANE, NON-CONDUCTING, EARTH

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SUMMARY

A vertical electric dipole, situated at a height h above a plane, non-conducting, earth, emits an impulsive electromagnetic wave. An expression in closed form is derived for the Hertzian vector of the resulting electromagnetic field in the air.

1. Introduction

The electromagnetic radiation from an oscillatory electric dipole in the presence of a plane earth has been investigated by a large number of authors. The current in the dipole is usually assumed to vary harmonically in time. The basic solutions of this (pure) boundary value problem are due to SOMMERFELD (1) and to WEYL (2). Recently, however, the case has become of interest where the time dependence of the current in the dipole is impulsive rather than harmonic. This leads to a mixed initial-boundary value problem. Electromagnetic problems of this type have been studied by PORITSKY (3), VAN DER POL (4), PEKERIS and ALTERMAN (5), BREMMER (6) and LEVELT (7). The method employed in the present paper is a simplified version (8) of techniques developed by CAGNIARD (9) and by PEKERIS (10) in connection with seismic wave propagation problems. A combination of a Laplace transform with respect to time and a two-dimensional Fourier transform with respect to the horizontal space coordinates leads to an expression in closed form for the Hertzian vector in the air.

The analogous problem for the horizontal dipole has been solved by FRANKENA (11).

2. Formulation of the problem

A Cartesian coordinate system x, y, z is introduced such that the air occupies the half-space $0 < z < \infty$, whilst the earth occupies the half-space $-\infty < z < 0$. The electromagnetic properties of the two media are characterized by their permittivity ε and their permeability μ; their conductivity is assumed to be zero. In the air we have $\varepsilon = \varepsilon_1$, $\mu = \mu_1$; in the earth we have $\varepsilon = \varepsilon_2$, $\mu = \mu_2$. Further we introduce the velocities of propagation

$$v_1 = (\varepsilon_1\mu_1)^{-1/2}\,, \tag{2.1}$$

$$v_2 = (\varepsilon_2\mu_2)^{-1/2}\,. \tag{2.2}$$

At $x = 0$, $y = 0$, $z = h$ $(h > 0)$ a vertical electric dipole starts to radiate at the instant $t = 0$; prior to this instant all field quantities are assumed to vanish identically. The electromagnetic field generated by this vertical dipole can be derived from a Hertzian vector $\mathbf{\Pi}$ of which only the z-component is different from zero. The electric field vector $\boldsymbol{E}$ and the magnetic field vector $\boldsymbol{H}$ are expressed in terms of $\mathbf{\Pi}$ through the relations

$$\boldsymbol{E} = \text{grad div}\,\mathbf{\Pi} - \varepsilon\mu\,\frac{\partial^2\mathbf{\Pi}}{\partial t^2}\,, \tag{2.3}$$

$$\boldsymbol{H} = \varepsilon\,\text{curl}\,\frac{\partial\mathbf{\Pi}}{\partial t}\,. \tag{2.4}$$

In the region $z > 0$ we write

$$\mathbf{\Pi} = (u_0 + u_1)\,\boldsymbol{i}_z\,, \tag{2.5}$$

in the region $z < 0$ we write

$$\mathbf{\Pi} = u_2\,\boldsymbol{i}_z\,. \tag{2.6}$$

The function u_0 yields the primary field (i.e. the field that would exist if the upper medium were unbounded)

$$u_0 = \frac{1}{4\pi\varepsilon_1}\,\frac{f(t - R_1/v_1)}{R_1}\,, \tag{2.7}$$

where

$$R_1 = [x^2 + y^2 + (z - h)^2]^{1/2} \tag{2.8}$$

and where $f(t)$ denotes the moment of the dipole as a function of time ($f(t) = 0$ when $t < 0$). Maxwell's equations together with the boundary conditions (continuity of E_x, E_y, H_x and H_y at $z = 0$) lead to the following conditions for $u_1 = u_1(x, y, z, t)$ and

$u_2 = u_2(x, y, z, t)$. The functions u_1 and u_2, together with their first and second order partial derivatives, are assumed to be continuous in $z > 0$ and $z < 0$, respectively. In the appropriate half-spaces they satisfy the homogeneous wave equation

$$\Delta u_{1,2} - \frac{1}{v^2_{1,2}} \frac{\partial^2 u_{1,2}}{\partial t^2} = 0\,, \tag{2.9}$$

where $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$. The boundary conditions lead to

$$\lim_{z\to+0} \left(\frac{\partial u_0}{\partial z} + \frac{\partial u_1}{\partial z}\right) = \lim_{z\to-0} \frac{\partial u_2}{\partial z}\,, \tag{2.10}$$

$$\varepsilon_1 \lim_{z\to+0} (u_0 + u_1) = \varepsilon_2 \lim_{z\to-0} u_2\,. \tag{2.11}$$

Further $u_1(x, y, z, t) = 0$ and $u_2(x, y, z, t) = 0$ when $t < 0$.

3. The field in the air

A combination of a Laplace transform with respect to t and a two-dimensional Fourier transform with respect to x and y leads to an expression in closed form for the Hertzian vector in the air. Details of the procedure employed can be found in a paper by DE HOOP and FRANKENA [12]. The result is that u_1 can be written as the following convolution integral

$$u_1(x, y, z, t) = \begin{cases} 0 & (0 < t < R_2/v_1)\,, \\ \displaystyle\int_{R_2/v_1}^{t} f'(t-\tau)\, g_1(x,y,z,\tau)\, \mathrm{d}\tau & (R_2/v_1 < t < \infty), \end{cases} \tag{3.1}$$

where

$$R_2 = [x^2 + y^2 + (z + h)^2]^{1/2} \tag{3.2}$$

(R_2 = distance from the image of the source to the point of observation) and

$$g_1(x, y, z, \tau) = \frac{1}{2\pi^2 \varepsilon_1 R_2} \int_0^{\pi/2} \mathrm{Re} \left\{\frac{\varepsilon_2\gamma_1 - \varepsilon_1\gamma_2}{\varepsilon_2\gamma_1 + \varepsilon_1\gamma_2}\right\} \mathrm{d}\psi\,, \tag{3.3}$$

in which the following substitutions have to be made

$$\gamma_1 = (q^2 + 1/v_1^2 - p^2)^{1/2} \qquad (\mathrm{Re}\,\gamma_1 \geqq 0)\,, \tag{3.4}$$

$$\gamma_2 = (q^2 + 1/v_2^2 - p^2)^{1/2} \qquad (\mathrm{Re}\,\gamma_2 \geqq 0)\,, \tag{3.5}$$

$$p = (r/R_2^2)\tau + i[(z+h)/R_2^2](\tau^2 - R_2^2/v_1^2)^{1/2} \cos \psi \,, \tag{3.6}$$

$$q = (\tau^2/R_2^2 - 1/v_1^2)^{1/2} \sin \psi \,, \tag{3.7}$$

$$r = (x^2 + y^2)^{1/2}. \tag{3.8}$$

In (3.1) the prime denotes differentiation with respect to the argument of the function f. The integral on the right-hand side of (3.3) can easily be computed numerically.

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