Three-dimensional relativistic scattering of electromagnetic waves by an object in uniform translational motion

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A general, relativistic formalism is developed for the three-dimensional scattering of electromagnetic waves by an object that is in uniform translational motion with respect to a source of electromagnetic radiation. The theory applies to objects of arbitrary size, shape, and physical composition. In particular, the temporal frequency spectrum of the field detected by a receiver that is stationary with respect to the source is determined. Numerical results pertaining to the scattering of a time-harmonic plane wave by a small, uniformly moving particle are presented.

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1. INTRODUCTION

The relativistic theory of scattering of electromagnetic waves by obstacles in uniform translational motion has been the subject of investigation by several authors. For the scattering of two-dimensional waves by cylindrical obstacles,[1–3] as well as for three-dimensional scattering by obstacles of bounded extent,[4,5] expressions have been obtained for the scattered field as it is observed by a receiver that is stationary with respect to the source that illuminates the moving object. However, all these expressions apply to the case where the distance between the moving obstacle and the receiver is large at all times, in which case the far-field approximation for the scattered field can be used.

In the present paper, we develop the three-dimensional relativistic scattering of electromagnetic waves by a uniformly moving obstacle of bounded extent for any location of the receiver, stationary with respect to the source. As in Refs. 1–5, the Lorentz transformation is used to transform the field radiated by the source to the frame of reference in which the obstacle is at rest and the scattering problem is solved in this reference frame, after which the scattered field is transformed to the frame of reference in which the receiver is at rest. A Fourier transform of the latter result finally leads to the desired frequency content of the received signal (SI units are used throughout).

2. FORMULATION OF THE PROBLEM

We consider the scattering of electromagnetic waves by an obstacle that moves with uniform velocity $\mathbf{v}$ with respect to a source of electromagnetic radiation in free space. We adopt two inertial frames of reference K and K' (Fig. 1), where K is denoted as the laboratory frame and K' as the obstacle frame. The source is at rest with respect to K, while the obstacle is at rest with respect to K'. We assume that K and K' coincide at the instant $t = t' = 0$ and therefore, the space-time coordinates $\{\mathbf{r},t\}$ of K and $\{\mathbf{r}',t'\}$ of K' are mutually related by the Lorentz transformation[6]

$$\mathbf{r}' = (\mathbf{i}_{\|} \times \mathbf{r}) \times \mathbf{i}_{\|} + \gamma\{-\mathbf{v}t + (\mathbf{i}_{\|}\cdot\mathbf{r})\mathbf{i}_{\|}\} \tag{1}$$

and

$$t' = \gamma\{t - \mathbf{v}\cdot\mathbf{r}/c_0^2\}, \tag{2}$$

where c_0 is the speed of light *in vacuo* and $\mathbf{i}_{\|}$ and γ are given by

$$\mathbf{i}_{\|} = \mathbf{v}/(\mathbf{v}\cdot\mathbf{v})^{1/2} \tag{3}$$

and

$$\gamma = [1 - (\mathbf{v}\cdot\mathbf{v})/c_0^2]^{-1/2}, \tag{4}$$

respectively.

For notational simplicity, the electric field strength $\mathbf{E}$ and the magnetic field strength $\mathbf{H}$ are grouped in the column matrix $[F]$ according to

$$[F] = \begin{bmatrix}\mathbf{E}\\ \mathbf{H}\end{bmatrix}. \tag{5}$$

The incident field $[F^i]$ in K is defined as the field that would be present in the absence of the obstacle. The scattered field $[F^s]$ in K is introduced as the difference between the total

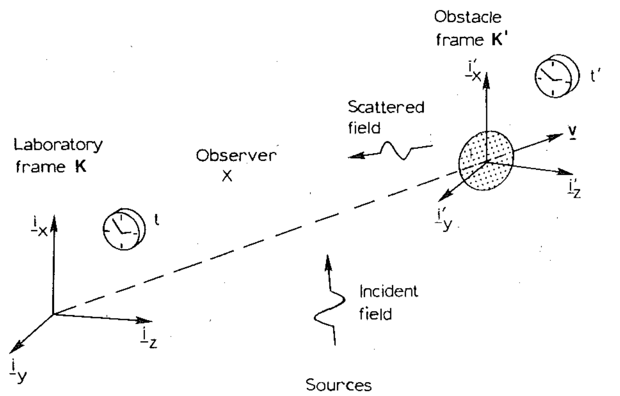

FIG. 1. Geometry of the scattering problem.

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field $[F]$ in K and $[F^i]$, i.e.,

$$[F^s] = [F] - [F^i]. \tag{6}$$

Similarly the incident, the total, and the scattered fields as observed in K' will be denoted by $[F^{i'}]$, $[F']$, and $[F^{s'}]$, respectively.

It is our purpose to arrive at an expression from which the scattered field $[F^s]$ in K can be calculated if the incident field $[F^i]$ (in K), the geometry and the electromagnetic properties of the obstacle (in K'), and its relative speed $\mathbf{v}$ with respect to K are known.

3. DERIVATION OF AN EXPRESSION FOR THE SCATTERED FIELD IN THE LABORATORY FRAME

In this section we derive a general expression for the scattered field in the laboratory frame that is suitable for our further considerations. To this aim we carry out a specific scheme of Lorentz transformations that is also at the root of the analysis in Refs. 1–5. The relevant steps (see also Fig. 2) are listed below.

A. The incident field $[F^i]$ is specified in the laboratory frame.

B. From $[F^i]$, the incident field $[F^{i'}]$ in the obstacle frame K' follows by the Lorentz transformation (see, e.g., Ref. 6),

$$[F^{i'}] = [\mathscr{L}(\mathbf{v})][F^i], \tag{7}$$

in which the square matrix $[\mathscr{L}(\mathbf{v})]$ can be written as

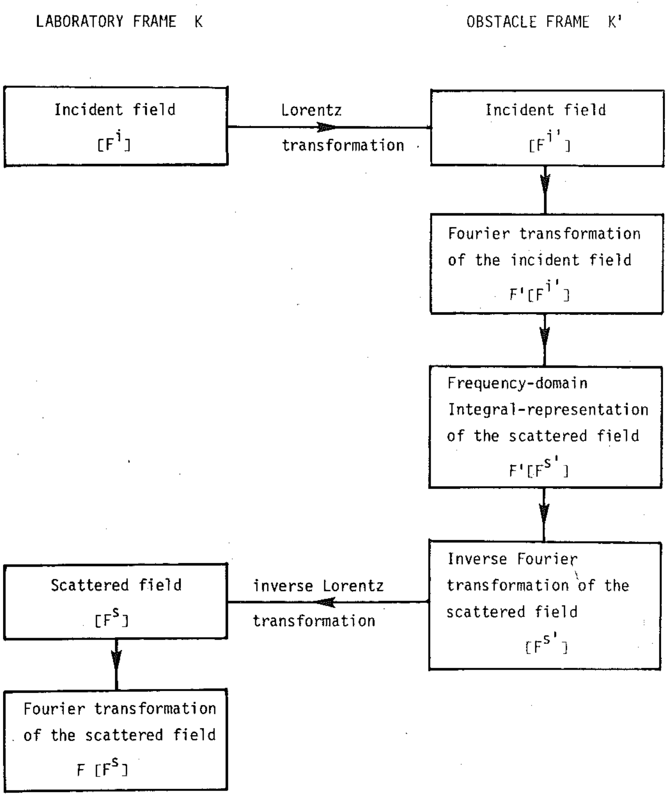


FIG. 2. Flow diagram illustrating the derivation of an expression for the scattered field in the laboratory frame.

$$[\mathscr{L}(\mathbf{v})] = \begin{bmatrix} [\mathscr{L}^{ee}(\mathbf{v})] \, [\mathscr{L}^{em}(\mathbf{v})] \\ [\mathscr{L}^{me}(\mathbf{v})] \, [\mathscr{L}^{mm}(\mathbf{v})] \end{bmatrix}, \tag{8}$$

with the submatrices defined through the relations

$$\begin{aligned} [\mathscr{L}^{ee}(\mathbf{v})][\mathbf{E}^i] &= (\mathbf{i}_\parallel \cdot \mathbf{E}^i)\mathbf{i}_\parallel + \gamma(\mathbf{i}_\parallel \times \mathbf{E}^i)\times \mathbf{i}_\parallel, \\ [\mathscr{L}^{em}(\mathbf{v})][\mathbf{H}^i] &= \mu_0 \gamma \mathbf{v} \times \mathbf{H}^i, \\ [\mathscr{L}^{me}(\mathbf{v})][\mathbf{E}^i] &= -\epsilon_0 \gamma \mathbf{v} \times \mathbf{E}^i, \\ [\mathscr{L}^{mm}(\mathbf{v})][\mathbf{H}^i] &= (\mathbf{i}_\parallel \cdot \mathbf{H}^i)\mathbf{i}_\parallel + \gamma(\mathbf{i}_\parallel \times \mathbf{H}^i)\times \mathbf{i}_\parallel, \end{aligned} \tag{9}$$

where $\mathbf{i}_\parallel$ is given by Eq. (3), γ is given by Eq. (4), and ϵ_0 and μ_0 are the permittivity and permeability of vacuum, respectively.

C. Since the scattering problem in K' will be formulated in the frequency domain, the next step is the application of the temporal Fourier transformation $\mathscr{F}'$ to $[F^{i'}]$; it is defined by the relation

$$\mathscr{F}'[F^{i'}] = \int_{-\infty}^{\infty} dt'[F^{i'}]\exp(i\omega' t'). \tag{10}$$

D. The scattered field in K' can be thought of as to be generated by contrast currents (polarization and magnetization currents) that are of the volume type for penetrable objects and of the surface type for impenetrable objects. The electromagnetic properties of the former are described in terms of constitutive relations (in the obstacle frame K') that express the contrast currents in terms of the field values in the obstacle, while for the latter the electromagnetic properties are described in terms of boundary conditions to be laid upon the limiting values of the field at the boundary surface of the obstacle (see, e.g., Ref. 7).

Let the domain occupied by the scattering object in K' be denoted by V' and let $\partial V'$ be the boundary surface of V'. For penetrable obstacles, we can now express the Fourier transform of the vector potential $[A']$, i.e., $\mathscr{F}'[A']$, in terms of contrast currents of the volume type $[J'_V]$ by

$$\mathscr{F}'[A'] = \int_{V'} dV(\boldsymbol{\rho}')G'\mathscr{F}'[J'_V], \tag{11}$$

in which $[A']$ and $[J'_V]$ can be written as

$$[A'] = \begin{bmatrix} \mathbf{A}^{e'} \\ \mathbf{A}^{m'} \end{bmatrix} \tag{12}$$

and

$$[J'_V] = \begin{bmatrix} \mathbf{J}_V^{e'} \\ \mathbf{J}_V^{m'} \end{bmatrix}, \tag{13}$$

where $\mathbf{A}^{e'}$ and $\mathbf{A}^{m'}$ are the volume vector potentials due to the polarization current of the volume type $\mathbf{J}_V^{e'}$ and the magnetization current of the volume type $\mathbf{J}_V^{m'}$, respectively. These currents are defined as

$$\mathbf{J}_V^{e'} = \mathbf{J}' + \partial_{t'}\mathbf{P}'$$

and (14)

$$\mathbf{J}_V^{m'} = \partial_{t'}\mu_0\mathbf{M}',$$

where $\mathbf{J}'$ is the current density, $\mathbf{P}'$ is the electric polarization, and $\mathbf{M}'$ is the magnetization inside V'. In (11), G' is the three-dimensional, free space Green's function, given by Ref. 7

$$G' = \exp[i\omega'|\mathbf{r}' - \boldsymbol{\rho}'|/c_0]/4\pi|\mathbf{r}' - \boldsymbol{\rho}'|. \tag{15}$$

Similarly, for impenetrable scatters, we have

$$\mathscr{F}'[A'] = \int_{\partial V'} dA\,(\rho')G'\mathscr{F}'[J'_S], \tag{16}$$

where the surface currents are defined as

$$\mathbf{J}_S^{e'} = -\mathbf{n}'\times\mathbf{H}' \tag{17}$$

and

$$\mathbf{J}_S^{m'} = \mathbf{n}'\times\mathbf{E}'.$$

The Fourier transform of the scattered field in K', $\mathscr{F}'[F^{s'}]$, can be expressed in terms of the vector potentials according to[7]

$$\mathscr{F}'[F^{s'}] = [D']\mathscr{F}'[A'], \tag{18}$$

where $\mathscr{F}'[A']$ is given by Eq. (11) for a penetrable scatterer and by Eq. (16) for an impenetrable scatterer and where the differential operator $[D']$ can be written as

$$[D'] = \begin{bmatrix} [D^{ee'}][D^{em'}] \\ [D^{me'}][D^{mm'}] \end{bmatrix}, \tag{19}$$

with the submatrices defined through the relations

$$\begin{aligned}
[D^{ee'}]\mathscr{F}'\mathbf{A}^{e'} &= i\omega'\mu_0\mathscr{F}'\mathbf{A}^{e'} \\
&\quad -(i\omega'\epsilon_0)^{-1}\nabla'\{\nabla'\cdot\mathscr{F}'\mathbf{A}^{e'}\}, \\
[D^{em'}]\mathscr{F}'\mathbf{A}^{m'} &= -\nabla'\times\mathscr{F}'\mathbf{A}^{m'}, \\
[D^{me'}]\mathscr{F}'\mathbf{A}^{e'} &= \nabla'\times\mathscr{F}'\mathbf{A}^{e'}, \\
[D^{mm'}]\mathscr{F}'\mathbf{A}^{m'} &= i\omega'\epsilon_0\mathscr{F}'\mathbf{A}^{m'} \\
&\quad -(i\omega'\mu_0)^{-1}\nabla'\{\nabla'\cdot\mathscr{F}'\mathbf{A}^{m'}\},
\end{aligned} \tag{20}$$

with $\nabla' = \mathbf{i}_{x'}\partial_{x'} + \mathbf{i}_{y'}\partial_{y'} + \mathbf{i}_{z'}\partial_{z'}$.

E. In order to calculate the scattered field $[F^s]$ in the laboratory frame K, we apply the inverse Lorentz transformation to $[F^{s'}]$. Now, $[F^{s'}]$ is recovered from $\mathscr{F}'[F^{s'}]$ by applying the inverse Fourier transformation $\mathscr{F}^{-1'}$:

$$[F^{s'}] = \mathscr{F}^{-1'}\{\mathscr{F}'[F^{s'}]\}, \tag{21}$$

which is defined by the relation

$$\mathscr{F}^{-1'}\{\mathscr{F}'[F^{s'}]\} = (2\pi)^{-1}\int_{-\infty}^{\infty} d\omega'\mathscr{F}'[F^{s'}]\exp(-i\omega't'). \tag{22}$$

F. Now, $[F^s]$ follows from $[F^{s'}]$ by applying the inverse Lorentz transformation

$$[F^s] = [\mathscr{L}^{-1}(\mathbf{v})][F^{s'}], \tag{23}$$

where $[\mathscr{L}^{-1}(\mathbf{v})]$ follows from (8) and (9) through

$$[\mathscr{L}^{-1}(\mathbf{v})] = [\mathscr{L}(-\mathbf{v})]. \tag{24}$$

G. The temporal frequency spectrum of the scattered field in K is finally found from the Fourier transformation

$$\mathscr{F}[F^s] = \int_{-\infty}^{\infty} dt\,[F^s]\exp(i\omega t). \tag{25}$$

From Eqs. (23)–(25), it follows that the temporal frequency spectrum of the scattered field in K can be written as

$$\mathscr{F}[F^s] = \mathscr{F}[\mathscr{L}^{-1}][F^{s'}]. \tag{26}$$

The transformations occurring in (26) are linear and we now use this property to exhibit the structure of $\mathscr{F}[F^s]$ more explicitly. In principle, the approach presented here runs along the same lines as followed in Ref. 5, be it that we have not restricted our considerations to the far-field approximation of the scattered field in K'. Of course, the latter approximation follows as a special case from our expressions.

Since the transformations $\mathscr{F}$ and $[\mathscr{L}^{-1}]$ are linear and $[\mathscr{L}^{-1}]$ does not depend on the time coordinate t, they can be interchanged. To perform the Fourier transformation $\mathscr{F}$ of the scattered field $[F^{s'}]$, this field has to be specified as a function of the time coordinate t. This is accomplished by first expressing, with the aid of (21) and (18), $[F^{s'}]$ in terms of the space-time coordinates $\{\mathbf{r}',t'\}$ in K', introducing the vector potentials $[A']$. Equation (26) can now be written as

$$\mathscr{F}[F^s] = [\mathscr{L}^{-1}]\mathscr{F}\mathscr{F}^{-1'}[D']\mathscr{F}'[A']. \tag{27}$$

Substitution of (18) and (22) in (27) then results in

$$\begin{aligned}
\mathscr{F}[F^s] = (2\pi)^{-1}\int_{-\infty}^{\infty} d\omega'[\mathscr{L}^{-1}]\mathscr{F}[D'] \\
\times\exp(-i\omega't')\mathscr{F}'[A'],
\end{aligned} \tag{28}$$

where the integration with respect to ω' has been interchanged with $\mathscr{F}$ and $[\mathscr{L}^{-1}]$. We have to keep in mind that, in order to perform the Fourier transformation $\mathscr{F}$, all space-time coordinates $\{\mathbf{r}',t'\}$ occurring in the terms to the right of the operator $\mathscr{F}$ in (28) have to be expressed in terms of the space-time cordinates $\{\mathbf{r},t\}$ by the Lorentz transformation (1) and (2).

It can be shown that $\mathscr{F}$ and $[D']$ can be interchanged, too, in the following way:

$$\begin{aligned}
&\mathscr{F}[D']\exp(-i\omega't')\mathscr{F}'[A'] \\
&\quad = [\mathscr{D}]\mathscr{F}\exp(-i\omega't')\mathscr{F}'[A'],
\end{aligned} \tag{29}$$

where $[\mathscr{D}]$ follows from $[D']$ given in (19) and (20) by replacing ∇' by $(\omega'/c_0)\mathbf{d}$, where $\mathbf{d}$ is given by

$$\mathbf{d} = (c_0/\omega')\nabla_\perp + is\mathbf{i}_\parallel, \tag{30}$$

with

$$\nabla_\perp = -\mathbf{i}_\parallel\times(\mathbf{i}_\parallel\times\nabla) = \nabla - \mathbf{i}_\parallel(\mathbf{i}_\parallel\cdot\nabla) \tag{31}$$

and

$$s = (\omega - \gamma\omega')/\beta\gamma\omega', \tag{32}$$

where γ is given by (4) and β is given by

$$\beta = |\mathbf{v}|/c_0. \tag{33}$$

Since in Eq. (29) $G'\exp(-i\omega't')$ is the only term to the right of $\mathscr{F}$ that needs expression in terms of $\{\mathbf{r},t\}$ [cf. (11)], we only have to determine

$$\mathscr{G} = \mathscr{F}\{G'\exp(-i\omega't')\}, \tag{34}$$

where G' is given by (15). The relevant function $\mathscr{G}$ is obtained as[10]

$$\begin{aligned}
\mathscr{G} &= (-i/4\gamma|\mathbf{v}|)\exp(-i\omega's\rho'_\parallel/c_0)\exp[i\omega'\gamma(\beta+s)r_\parallel/c_0] \\
&\quad\times\begin{bmatrix} H_0^{(1)}[\phi(1-s^2)^{1/2}] & (|s|<1) \\ (-2i/\pi)K_0[\phi(s^2-1)^{1/2}] & (|s|>1) \end{bmatrix}
\end{aligned} \tag{35}$$

[for this purpose, 10.1.1 from Ref. 9 has to be used in (34), after which the result can be found on p. 56 of Ref. 10.] where $H_0^{(1)}$ is the Hankel function of the first kind and order zero, K_0 is a modified Bessel function of the second kind and order zero, while

$$
\begin{aligned}
\rho'_{\parallel} &= \mathbf{i}_{\parallel}\cdot\boldsymbol{\rho}',\\
r_{\parallel} &= \mathbf{i}_{\parallel}\cdot\mathbf{r},\\
\boldsymbol{\rho}'_{\perp} &= \boldsymbol{\rho}' - \rho'_{\parallel}\mathbf{i}_{\parallel},\\
\mathbf{r}_{\perp} &= \mathbf{r} - r_{\parallel}\mathbf{i}_{\parallel},\\
\phi &= \omega'|\mathbf{r}_{\perp} - \boldsymbol{\rho}'_{\perp}|/c_0.
\end{aligned}
\tag{36}
$$

Finally, combining (26)–(36), the temporal frequency spectrum of the scattered field in K,$\mathscr{F}[F^s]$, can be written as

$$
\mathscr{F}[F^s] = (2\pi)^{-1}\int_{-\infty}^{\infty} d\omega'[\mathscr{L}^{-1}][\mathscr{D}] \times \int_{V'} dV(\boldsymbol{\rho}')\mathscr{G}\mathscr{F}'[J'_V]
\tag{37}
$$

for penetrable scatterers and as

$$
\mathscr{F}[F^s] = (2\pi)^{-1}\int_{-\infty}^{\infty} d\omega'[\mathscr{L}^{-1}][\mathscr{D}] \times \int_{\partial V'} dA(\boldsymbol{\rho}')\mathscr{G}\mathscr{F}'[J'_s]
\tag{38}
$$

for impenetrable scatterers.

Through (37) and (38) the temporal frequency spectrum of the scattered field in the laboratory frame is expressed in terms of volume or surface sources in the obstacle frame that are located either in the interior region or on the boundary of the obstacle, respectively. The strength of these sources can be determined by solving the scattering problem in the obstacle frame, i.e., for an obstacle at rest that is illuminated by $[F^{i'}]$. Depending upon the geometry and the electromagnetic properties of the obstacle, computational as well as analytical methods can be used for this purpose.

The Green's function occurring in (37) and (38), as given by (35), has a singularity at $|s| = 1$. This kind of singularity has already been noted and discussed by de Zutter.[5] When the distance between observer and moving obstacle is large at all times ($|\mathbf{r}_{\perp}|\rightarrow\infty$), asymptotic expressions for $H_0^{(1)}$ and K_0 can be used in (35) and, in that case, Eqs. (37) and (38) can be reduced to an expression already presented in Ref. 5.

4. PLANE-WAVE SCATTERING BY A SMALL OBSTACLE IN UNIFORM, TRANSLATIONAL MOTION

We shall now apply the results from Sec. 3 to the scattering of a sinusoidal uniform plane wave with angular frequency Ω by a uniformly moving obstacle with spatial dimensions that are small compared to the wavelength of the incident field. The incident field in K is now given as

$$
[F^i] = [f^i]\cos(\Omega t - \mathbf{k}\cdot\mathbf{r}),
\tag{39}
$$

where the wave vector $\mathbf{k}$ is given by

$$
\mathbf{k} = \mathbf{u}\Omega/c_0
\tag{40}
$$

($\mathbf{u}$ is the direction of propagation of the incident plane wave), and $[f^i]$ by

$$
[f^i] = \begin{bmatrix}\mathbf{e}^i\\ \mathbf{h}^i\end{bmatrix}.
\tag{41}
$$

Here $\mathbf{e}^i$ and $\mathbf{h}^i$ are the plane-wave amplitudes of the electric and the magnetic field strengths, respectively. The incident field in K' is also a plane wave, written as

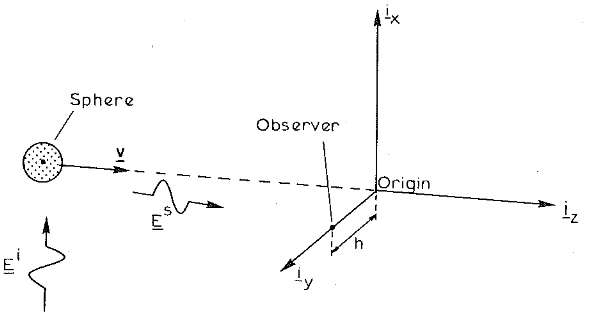


FIG. 3. Geometry of the scattering problem for which numerical results are presented in Figs. 4–6. The incident electric field is directed along the y axis.

$$
[F^{i'}] = [f^{i'}]\cos(\Omega't' - \mathbf{k}'\cdot\mathbf{r}').
\tag{42}
$$

Expressions for $[f^{i'}]$, $\mathbf{k}'$, and Ω' are given in the Appendix. Since further, the obstacle is assumed to be very small, we can in the limit $|\boldsymbol{\rho}'|\rightarrow 0$, use the dipole approximation (see, e.g., Ref. 8) in (37) and (38):

$$
\left.\begin{aligned}
&\int_{V'} dV(\boldsymbol{\rho}')\mathscr{G}\mathscr{F}'[J'_V]\\
&\int_{\partial V'} dV(\boldsymbol{\rho}')\mathscr{G}\mathscr{F}'[J'_s]
\end{aligned}\right\}
= -i\Omega'\begin{bmatrix}\mathbf{p}'\\ \mu_0\mathbf{m}'\end{bmatrix}\pi\{\delta(\omega'-\Omega') + \delta(\omega'+\Omega')\}\mathscr{G}_{\boldsymbol{\rho}'=0},
\tag{43}
$$

where $\mathscr{G}_{\boldsymbol{\rho}'=\mathbf{0}}$ follows from Eq. (35) by substituting $\boldsymbol{\rho}' = \mathbf{0}$. In (43), the electric dipole moment $\mathbf{p}'$ and the magnetic dipole

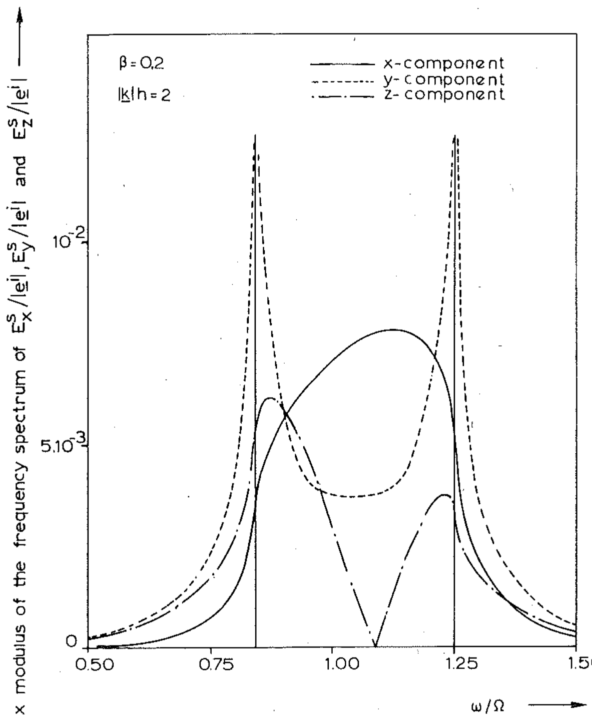


FIG. 4. The normalized x, y, and z component of the spectrum of the scattered electric field as a function of the normalized frequency for $\beta = 0.2$ and $\mathbf{k}|h| = 2$. The other parameters are $\Pi^e = \Pi^m = 10^{-2}|\mathbf{k}|^{-3}I$, $\mathbf{v} = 0.2c\mathbf{i}_z$, $\mathbf{u} = \mathbf{i}_x$, $\mathbf{e}^i = |\mathbf{e}^i|\mathbf{i}_y$.

moment $\mathbf{m}'$ are given by

$$\mathbf{p}' = \epsilon_0 \Pi^{e'} \cdot \mathbf{e}^{i'} \tag{44}$$

and

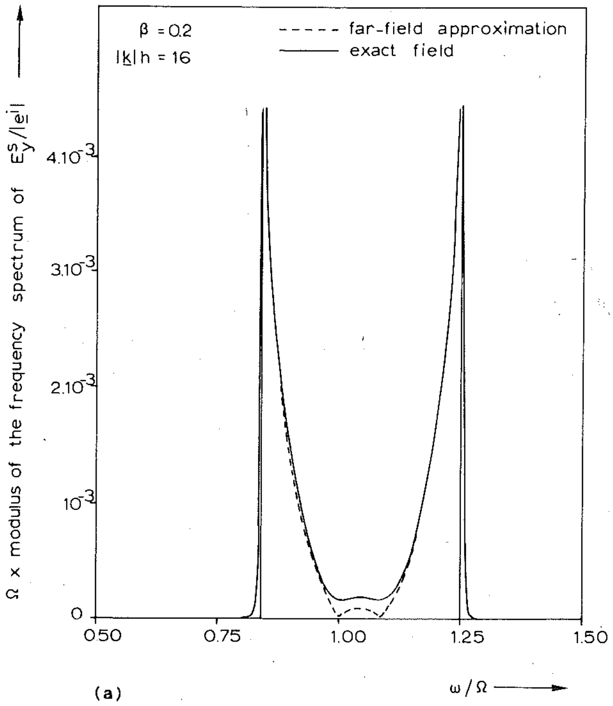
$$\mathbf{m}' = \Pi^{m'} \cdot \mathbf{h}^{i'}, \text{ respectively.}$$

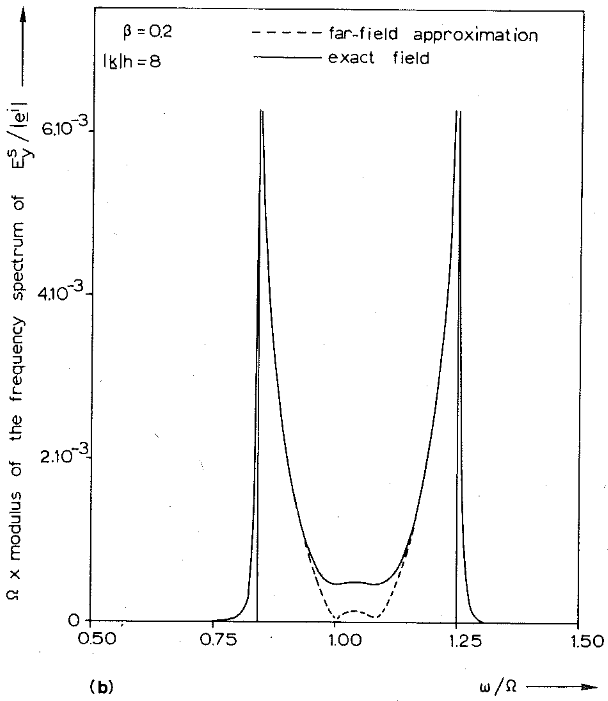
The electric polarizability tensor $\Pi^{e'}$ and the magnetic polarizability tensor $\Pi^{m'}$ reflect the geometry and the electromagnetic properties of the obstacle.

After substitution of Eq. (43) in (37) and (38), we now have an explicit expression from which the temporal frequency spectrum of $[F^s]$ can be calculated as soon as $[F^i]$ and the electric and magnetic polarizability tensors $\Pi^{e'}$ and $\Pi^{m'}$ are specified. The differentiations occurring in $[\mathscr{D}]$ can be performed explicitly with the aid of the well-known differentiation formulas for $H_0^{(1)}$ and K_0.[9]

5. NUMERICAL RESULTS

In this section we present some numerical results for the scattering of a time-harmonic, uniform plane wave with frequency Ω by a small, uniformly moving, homogeneous, and isotropic sphere. For a sphere Π^e and Π^m are given by Ref. 11 (p. 259)





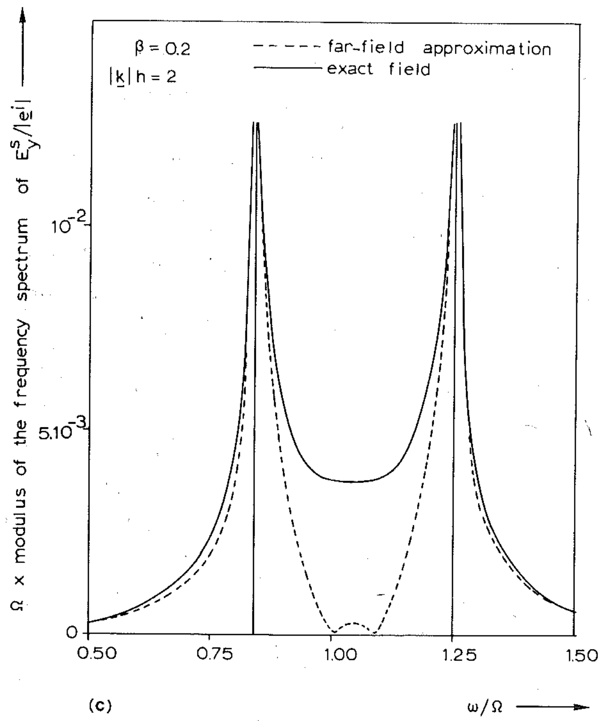


FIG. 5. Comparison between the y component of the scattered electric field calculated with the aid of the far-field approximation and calculated with the exact field expressions for $|\mathbf{k}|h = 16$ (a), $|\mathbf{k}|h = 8$ (b), and $|\mathbf{k}|h = 2$ (c). The other parameters are $\Pi^e = \Pi^m = 10^{-2}|\mathbf{k}|^{-3} I$, $\beta = 0.2$, $\mathbf{v} = 0.2c\mathbf{i}_z$, $\mathbf{u} = \mathbf{i}_x$, $\mathbf{e}^i = |\mathbf{e}^i|\mathbf{i}_y$.

$$\Pi^e = 4\pi a^3 \frac{\epsilon_r - 1}{\epsilon_r + 2} I \tag{45}$$

and

$$\Pi^m = 4\pi a^3 \frac{\mu_r - 1}{\mu_r + 2} I, \tag{46}$$

where I is the unit tensor of rank two, a is the radius of the sphere, and ϵ_r and μ_r are the relative permittivity and permeability of the sphere, respectively.

The numerical results presented in this section apply to the case $\Pi^e = \Pi^m = 10^{-2}|\mathbf{k}|^{-3}I$, where the wave vector of the incident wave, $\mathbf{k}$, is defined by Eq. (40). The sphere is moving uniformly in the direction of increasing z and therefore, we can write

$$\mathbf{v} = \beta c_0 \mathbf{i}_z. \tag{47}$$

The direction of propagation of the incident wave is assumed to be given by

$$\mathbf{u} = \mathbf{i}_x. \tag{48}$$

Furthermore, the incident electric field is taken to be directed along the y axis, i.e.,

$$\mathbf{e}^i = |\mathbf{e}^i|\mathbf{i}_y, \tag{49}$$

while the incident magnetic field is given by

$$\mathbf{h}^i = (\epsilon_0/\mu_0)^{1/2}\mathbf{u}\times\mathbf{e}^i. \tag{50}$$

The observer is located on the positive y axis of the laboratory frame at a distance h from the origin 0 (see Fig. 3).

In Fig. 4, the frequency spectrum of the scattered electric field is shown as a function of the normalized frequency ω/Ω for $|\mathbf{k}|h = 2$. It is observed that only the y component has a singularity at $|s| = \pm 1$ $[\omega/\Omega = (1 \pm \beta)\gamma^2]$.

Figure 5 shows for several values of $|\mathbf{k}|h$, the difference between the actual field values and the far-field approximation.[5] The dashed curve is obtained from the latter approximation, while the solid curve is obtained from (37), (43), and (44). It follows that the far-field approximation is quite good at $|\mathbf{k}|h = 8$ and $|\mathbf{k}|h = 16$, while significant deviations occur at $|\mathbf{k}|h = 2$. The largest differences occur around those frequencies where the major contribution to the spectrum originates from those time values during which the scatterer passes the observer. Since the distance between the scatterer and observer is small at that time, the far-field approximation is then inaccurate. It was already stated in Ref. 5 that, for larger values of $|\mathbf{k}|h$, the scattered field decays very rapidly for ω/Ω outside the region $((1-\beta)\gamma^2, (1+\beta)\gamma^2)$; Fig. 5(a) illustrates this. For small values of $|\mathbf{k}|h$, however, this is no longer true, as Fig. 5(c) ($|\mathbf{k}|h = 2$) and Fig. 6 ($|\mathbf{k}|h = 0.5$) show.

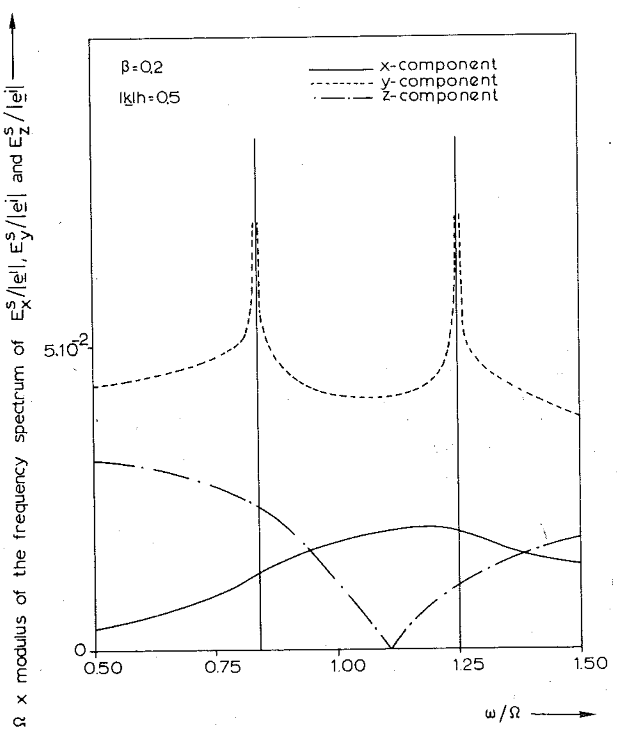


FIG. 6. The normalized x, y, and z components of the scattered electric field as a function of the normalized frequency for $\beta = 0.2$ and $|\mathbf{k}|h = 0.5$. The other parameters are $\Pi^e = \Pi^m = 10^{-2}|\mathbf{k}|^{-3}I$, $v = 0.2c\mathbf{i}_z$, $\mathbf{u} = \mathbf{i}_x$, $\mathbf{e}^i = |\mathbf{e}^i|\mathbf{i}_y$.

CONCLUSION

In this paper, an integral-equation formalism is developed for the three-dimensional, relativistic scattering of electromagnetic waves by an object that is in translational motion with respect to a source of electromagnetic radiation. The theory applies to objects or arbitrary size, shape, and physical composition and the field expressions that are obtained are valid for arbitrary positions of the observer with respect to the moving obstacle. In this respect, the formalism is more general than the ones published up to now. Also, an attempt has been made to make the presentation of the transformation schemes involved more transparent.

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APPENDIX. LORENTZ TRANSFORMATION OF A UNIFORM PLANE WAVE WITH SINUSOIDAL TIME DEPENDENCE

A uniform plane wave with angular frequency Ω, wave vector $\mathbf{k}$, and plane-wave amplitude $[f^i]$ in the laboratory frame K, as given by (39), is also a uniform plane wave in the obstacle frame K', but now with angular frequency Ω', wave vector $\mathbf{k}'$, and plane-wave amplitude $[f^{i'}]$. The quantities Ω', $\mathbf{k}'$, Ω, and $\mathbf{k}$ are interrelated through[3]

$$\Omega' = \gamma(\Omega - \mathbf{v}\cdot\mathbf{k}) \tag{A1}$$

and

$$\mathbf{k}' = (\mathbf{i}_\parallel \times \mathbf{k})\times \mathbf{i}_\parallel + \gamma\{-\mathbf{v}\Omega/c_0 + (\mathbf{i}_\parallel\cdot\mathbf{k})\mathbf{i}_\parallel\}, \tag{A2}$$

where $\mathbf{i}_\parallel$ and γ are given by (3) and (4), respectively. The relations between $[f^i]$ and $[f^{i'}]$ can be obtained from (7)–(9) by replacing $[F^i]$ by $[f^i]$ and $[F^{i'}]$ by $[f^{i'}]$.

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