RECIPROCITY - AN INSTRUMENTAL APPROACH TO INTEGRAL RESERVOIR MONITORING

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Synopsis:

- The remote sensing/monitoring problem
- Reciprocity for electric/electronic circuits
- Reciprocity for quasi-stationary electric currents
- Reciprocity for diffusive electromagnetic fields
- Reciprocity for electromagnetic wavefields
- Reciprocity for acoustic waves in fluids
- Reciprocity for elastic waves in solids
- Reciprocity for acousto/elastic waves in fluid-filled porous solids
- Sensitivity analysis for integral monitoring

Constituents of the **remote sensing problem** (e.g., reservoir monitoring problem):

- **Configuration** to be sensed/monitored
- **Physical (wave)field** as **sensing agent**
- **Electr(on)ically operated sources** and **receivers**
- **Signal processing** to reconstruct configurational parameters from measured data
- **Visualization** of reconstructed configuration

Physical sensing agents in remote sensing/monitoring:

- **(Quasi-)stationary electric currents**
- **Quasi-static magnetic fields**
- **Diffusive electromagnetic fields** (electric displacement current negligeable with respect to conduction current)
- **Electromagnetic wavefields**

$\Longrightarrow$ sensing agents **directly** **electr(on)ically controllable**

Remote sensing problem (physical sensing agents - 1)

Physical sensing agents in remote sensing/monitoring:

- **Acoustic waves in fluids**
- **Elastic waves in solids (condensed matter)**
- **Acousto-elastic waves in fluid-filled porous solids**

$\Longrightarrow$ sensing agents **electr(on)ically controllable** **via** **electromechanical transduction**

Remote sensing problem (physical sensing agents - 2)

Reconstruction procedure in remote sensing/monitoring:

- Reformulate **Actual configuration** as its **contrast** with respect to (computationally generated or previously reconstructed) **Background configuration**
- **Construct** the **contrast electr(on)ic data**
- Relate **contrast electr(on)ic data** to **contrast in constitutive parameters** of the media in the configuration
- **Reconstruct contrast in constitutive parameters** from the constructed **contrast data** (e.g., via optimization procedures)

Reconstruction procedure in remote sensing/monitoring

The **quantity** that **interrelates** the **instrumental responses** in the **Actual & Background states** is furnished by the (wave)field **reciprocity theorems** of the **time-convolution type**:

- **Kirchhoff/Tellegen (electr(on)ic circuits & quasi-stationary electric currents)**
- **Lorentz (electromagnetic (wave)fields)**
- **Rayleigh (acoustic waves in fluids)**
- **Betti/Rayleigh (elastic waves in solids/condensed matter)**

For **electric/electronic circuits** the interaction quantity for **time-convolution reciprocity** is, for each accessible circuit port,

$$\overset{A \rightleftarrows B}{\{V, I\}}(t) \overset{\text{def}}{=} V^A(t) \overset{(t)}{*} I^B(t) - V^B(t) \overset{(t)}{*} I^A(t)$$

where

V = **voltage** across the port

I = **electric current** oriented into the port

t = time

$\overset{(t)}{*}$ = time convolution

Reciprocity quantity for electric/electronic circuits

For **quasi-stationary electric currents** the local interaction quantity for **time-convolution reciprocity** is

$$\{\overset{A\rightleftarrows B}{\Phi, J_k}\}(\boldsymbol{x},t) \overset{\text{def}}{=} \left[\Phi^A(\boldsymbol{x},t) \overset{(t)}{*} J_k^B(\boldsymbol{x},t) - \Phi^B(\boldsymbol{x},t) \overset{(t)}{*} J_k^A(\boldsymbol{x},t)\right]$$

where

Φ = **electric potential**

J_k = **volume density of electric current**

$\boldsymbol{x}$ = position vector

t = time

$\overset{(t)}{*}$ = time convolution

Reciprocity quantity for quasi-stationary electric currents

For **electromagnetic (wave)fields** the local interaction quantity for **time-convolution reciprocity** is

$$\varepsilon_{m,r,p} \{ \overset{A \rightleftarrows B}{E_r, H_p} \} (\boldsymbol{x}, t) \stackrel{\text{def}}{=} \varepsilon_{m,r,p} \left[E_r^A(\boldsymbol{x}, t) \overset{(t)}{*} H_p^B(\boldsymbol{x}, t) - E_r^B(\boldsymbol{x}, t) \overset{(t)}{*} H_p^A(\boldsymbol{x}, t) \right]$$

where

E_r = **electric field strength**

H_p = **magnetic field strength**

$\boldsymbol{x}$ = position vector

t = time

$\overset{(t)}{*}$ = time convolution

$\varepsilon_{m,r,p}$ = completely antisymmetric unit tensor of rank three

Reciprocity quantity for electromagnetic (wave)fields

For **acoustic waves in fluids** the local interaction quantity for **time-convolution reciprocity** is

$$\{p, v_r\}^{A \rightleftarrows B} (\boldsymbol{x}, t) \stackrel{\text{def}}{=} \left[p^A(\boldsymbol{x}, t) \overset{(t)}{*} v_r^B(\boldsymbol{x}, t) - p^B(\boldsymbol{x}, t) \overset{(t)}{*} v_r^A(\boldsymbol{x}, t) \right]$$

where

$$p = \textbf{acoustic pressure}$$

$$v_r = \textbf{particle velocity}$$

$$\boldsymbol{x} = \text{position vector}$$

$$t = \text{time}$$

$$\overset{(t)}{*} = \text{time convolution}$$

Reciprocity quantity for acoustic waves in fluids

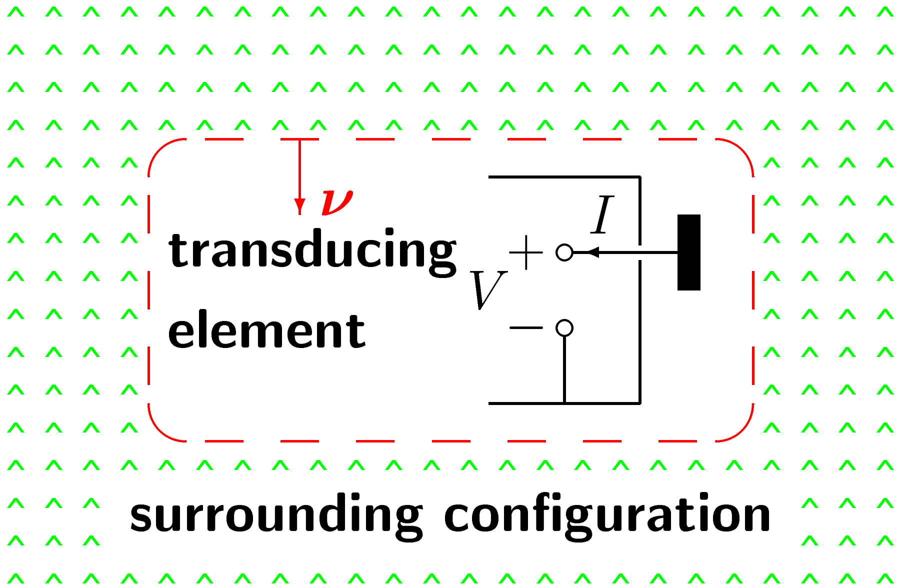
For **elastic waves in solids** the local interaction quantity for **time-convolution reciprocity** is

$$\Delta^{+}_{m,r,p,q}\left\{-\overset{A\rightleftarrows B}{\tau_{p,q}, v_r}\right\}(\boldsymbol{x},t) \overset{\text{def}}{=} \Delta^{+}_{k,r,p,q}$$
$$\left[-\tau^{A}_{p,q}(\boldsymbol{x},t) \overset{(t)}{*} v^{B}_{r}(\boldsymbol{x},t) + \tau^{B}_{p,q}(\boldsymbol{x},t) \overset{(t)}{*} v^{A}_{r}(\boldsymbol{x},t)\right]$$

where

$$\tau_{p,q} = \text{dynamic stress}$$
$$v_r = \text{particle velocity}$$
$$\boldsymbol{x} = \text{position vector}$$
$$t = \text{time}$$
$$\overset{(t)}{*} = \text{time convolution}$$
$$\Delta^{+}_{m,r,p,q} = \text{symmetric unit tensor of rank four}$$

Reciprocity quantity for elastic waves in solids



- Each **(wave)field interaction quantity** is a **vectorial quantity** whose **flux** across the closed **boundary surface** of the **transducing element** generates the interaction in the electr(on)ic **data** $\overset{A \rightleftarrows B}{\{V, I\}}$ ($\boldsymbol{\nu}$ = unit vector along the normal)

For **quasi-stationary electric currents** the instrumental interaction quantity is

$$\overset{A\rightleftarrows B}{\{V, I\}} = \int_{\partial\mathcal{T}} \nu_k \,\overset{A\rightleftarrows B}{\{\Phi, J_k\}}\,\mathrm{d}A$$

$$\overset{K/T}{=} -\int_{\mathcal{D}_\rho} J_r^B \overset{(t)}{*} (\rho_{k,r}^A - \rho_{r,k}^B) \overset{(t)}{*} J_k^A \mathrm{d}V$$

where

$\partial\mathcal{T}$ = boundary surface of transducing element

$\overset{K/T}{=}$ = **Kirchhoff/Tellegen**

$\rho_{k,r}$ = resistivity relaxation function

$\mathcal{D}_\rho = \mathrm{supp}(\rho_{k,r}^A - \rho_{r,k}^B)$

Instrumental interaction (electric currents)

For **diffusive electromagnetic fields** the instrumental interaction quantity is

$$\overset{A\rightleftarrows B}{\{V,I\}} = \int_{\partial\mathcal{T}} \nu_m \varepsilon_{m,r,p} \overset{A\rightleftarrows B}{\{E_r, H_p\}} \,\mathrm{d}A$$
$$\overset{L}{=} \int_{\mathcal{D}_\sigma} E_k^B \overset{(t)}{*} (\sigma_{k,r}^A - \sigma_{r,k}^B) \overset{(t)}{*} E_r^A \mathrm{d}V -$$
$$\partial_t \int_{\mathcal{D}_\mu} H_j^B \overset{(t)}{*} (\mu_{j,p}^A - \mu_{p,j}^B) \overset{(t)}{*} H_p^A \mathrm{d}V$$

where

$\partial\mathcal{T}$ = boundary surface of transducing element

$\overset{L}{=}$ = **Lorentz**

$\sigma_{k,r}$ = conductivity relaxation function

$\mu_{j,p}$ = permeability relaxation function

$\mathcal{D}_\sigma = \mathrm{supp}(\sigma_{k,r}^A - \sigma_{r,k}^B)$

$\mathcal{D}_\mu = \mathrm{supp}(\mu_{j,p}^A - \mu_{p,j}^B)$

For **electromagnetic wavefields** the instrumental interaction quantity is

$$\overset{A\rightleftarrows B}{\{V, I\}} = \int_{\partial\mathcal{T}} \nu_m \varepsilon_{m,r,p} \overset{A\rightleftarrows B}{\{E_r, H_p\}} \,\mathrm{d}A$$

$$\overset{L}{=} \partial_t \int_{\mathcal{D}_\epsilon} E_k^B \overset{(t)}{*} (\epsilon_{k,r}^A - \epsilon_{r,k}^B) \overset{(t)}{*} E_r^A \mathrm{d}V -$$

$$\partial_t \int_{\mathcal{D}_\mu} H_j^B \overset{(t)}{*} (\mu_{j,p}^A - \mu_{p,j}^B) \overset{(t)}{*} H_p^A \mathrm{d}V$$

where

$\partial\mathcal{T}$ = boundary surface of transducing element

$\overset{L}{=}$ = **Lorentz**

$\epsilon_{k,r}$ = permittivity relaxation function

$\mu_{j,p}$ = permeability relaxation function

$\mathcal{D}_\epsilon = \mathrm{supp}(\epsilon_{k,r}^A - \epsilon_{r,k}^B)$

$\mathcal{D}_\mu = \mathrm{supp}(\mu_{j,p}^A - \mu_{p,j}^B)$

Instrumental interaction (EM wavefields)

For **acoustic wavefields in fluids** the instrumental interaction quantity is

$$\overset{A\rightleftarrows B}{\{V, I\}} = \int_{\partial\mathcal{T}} \nu_m \overset{A\rightleftarrows B}{\{p, v_m\}}\,\mathrm{d}A$$
$$\overset{R}{=} \partial_t \int_{\mathcal{D}_\kappa} p^B \overset{(t)}{*} (\kappa^A - \kappa^B) \overset{(t)}{*} p^A \mathrm{d}V -$$
$$\partial_t \int_{\mathcal{D}_\rho} v_k^B \overset{(t)}{*} (\rho_{k,r}^A - \rho_{r,k}^B) \overset{(t)}{*} v_r^A \mathrm{d}V$$

where

$\partial\mathcal{T}$ = boundary surface of transducing element

$\overset{R}{=}$ = **Rayleigh**

κ = compliance relaxation function

$\rho_{k,r}$ = inertia relaxation function

$\mathcal{D}_\kappa = \mathrm{supp}(\kappa^A - \kappa^B)$

$\mathcal{D}_\rho = \mathrm{supp}(\rho_{k,r}^A - \rho_{r,k}^B)$

Instrumental interaction (acoustic wavefields in fluids)

For **elastic wavefields in solids** the instrumental interaction quantity is

$$\overset{A\rightleftarrows B}{\{V,I\}} = \int_{\partial\mathcal{T}} \nu_m \Delta^{+}_{m,r,p,q}\, \overset{A\rightleftarrows B}{\{-\tau_{p,q}, v_r\}}\,\mathrm{d}A$$
$$\overset{B/R}{=} \partial_t \int_{\mathcal{D}_S} \tau^B_{i,j} \overset{(t)}{*} (S^A_{i,j,p,q} - S^B_{p,q,i,j}) \overset{(t)}{*} \tau^A_{p,q}\mathrm{d}V$$
$$-\partial_t \int_{\mathcal{D}_\rho} v^B_k \overset{(t)}{*} (\rho^A_{k,r} - \rho^B_{r,k}) \overset{(t)}{*} v^A_r \mathrm{d}V$$

where

$\partial\mathcal{T}$ = boundary surface of transducing element

$\overset{B/R}{=}$ = **Betti/Rayleigh**

$S_{i,j,p,q}$ = compliance relaxation function

$\rho_{k,r}$ = inertia relaxation function

$\mathcal{D}_S = \mathrm{supp}(S^A_{i,j,p,q} - S^B_{p,q,i,j})$

$\mathcal{D}_\rho = \mathrm{supp}(\rho^A_{k,r} - \rho^B_{r,k})$

Instrumental interaction (elastic wavefields in solids)

The different expressions for $\overset{A \rightleftarrows B}{\{V, I\}}$, used in a multiple source/multiple receiver sensing configuration, provide the basis for:

- **Inversion procedures** (via optimization techniques) as to the constitutive parameters sensed by the sensing agents
- Sensing agent **configurational sensitivity analysis** through the first-order, low-contrast approximation: **Actual sensing field** $\simeq$ **Background field**
- **Identification of constituting material** via **material** $\Longleftrightarrow$ **constitutive parameter incidence matrix**