Pulsed EM field response of a thin, high-contrast, finely layered structure with dielectric and conductive properties

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Synopsis:

- Motivation
- Formulation of the problem
- The thin-layer, high-contrast boundary condition
- Plane-wave reflection against and transmission across a thin, high-contrast layer of infinite extent
- Numerical results for an incident wave of rectangular pulse shape (E-polarization)
- Numerical results for an incident wave of rectangular pulse shape (H-polarization)

Examples of thin, high-contrast, finely layered structures in Electronics and Telecommunication Engineering:

- Aircraft radar radomes
- Integrated optical devices
- Micro- and Nano-Electronic devices

Difficulties with computational discretization of the Maxwell EM field equations:

$\Longrightarrow$ **FINENESS OF THE GRIDDING/MESHING**

'Global' type conditions:

- Harrington & Mautz (1975) $\Longrightarrow$ based on contrast volume source EM field representations and associated Green's functions
- Koh & Sarabandi (2005) $\Longrightarrow$ based on Lorentz's reciprocity theorem and Rumsey's EM field reaction concept

'Local' type conditions:

- Collin (1960), Senior (1981) $\Longrightarrow$ based on EM field integration across the layer

'Moving average' type conditions (acoustics and elastodynamics):

- Schoenberg & Muir (1989), Hovem (1995) $\Longrightarrow$ based on moving averages perpendicular to the layering

All of these: $\Longrightarrow$ FREQUENCY-DOMAIN CONDITIONS

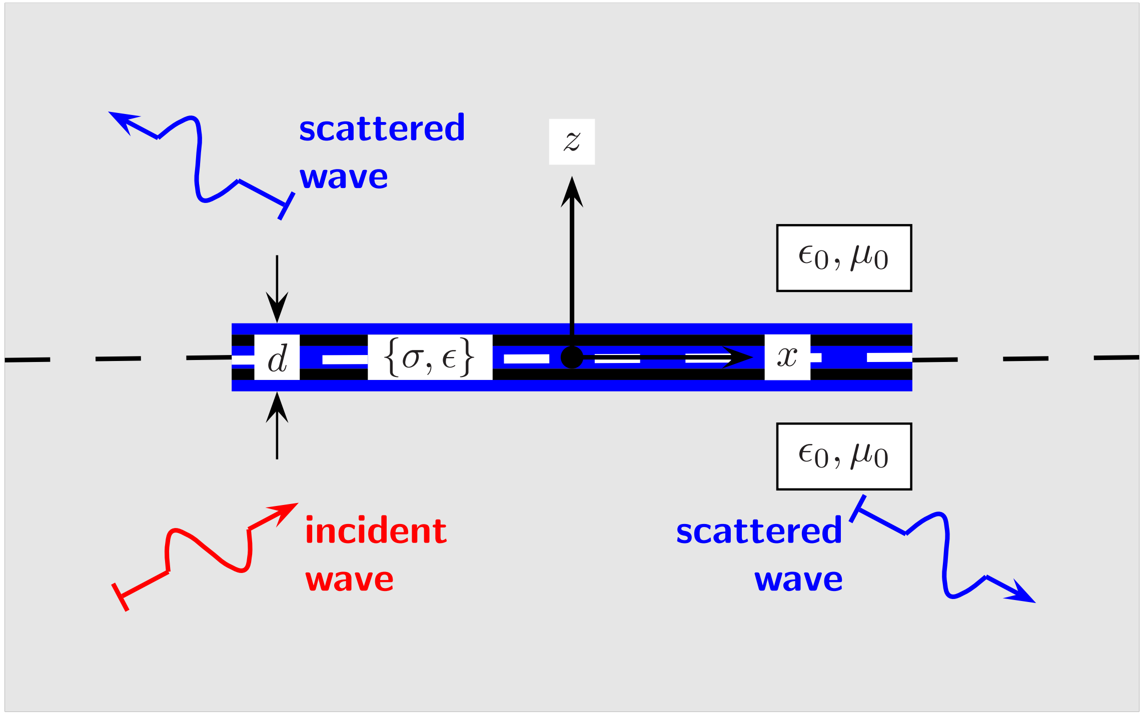
Approximate thin-layer interface boundary conditions in the literature

Present (time-domain) approach $\Longrightarrow$ based on:

- Volume-integrated time-domain Maxwell field equations
- Contrast-source scattering description
- Local field continuity considerations across the layer
- Lumped-element capacitance and conductance description of the contrast sheet electric current density
- Time-domain circulation and flux integral form of Maxwell field equations + compatibility relations
- 'Edge condition' on contrast sheet electric current density
- Time-domain uniqueness considerations via time Laplace transform methods [De Hoop (2003)]

The present time-domain approach

Thin, high-contrast layer ($\mathcal{D} = \{(x,y) \in \mathbb{R}^2, -d/2 < z < d/2\}$):



Layer conductance:

- $G_\mathrm{L} = \int_{z=-d/2}^{d/2} \sigma(z)\mathrm{d}z = O(1)$ as $d \downarrow 0$

Layer capacitance:

- $C_\mathrm{L} = \int_{z=-d/2}^{d/2} \epsilon(z)\mathrm{d}z = O(1)$ as $d \downarrow 0$

Cartesian coordinate position vector: • $\boldsymbol{r} = x\boldsymbol{i}_x + y\boldsymbol{i}_y + z\boldsymbol{i}_x \in \mathbb{R}^3$

Time coordinate: • $t \in \mathbb{R}$

Thin, high-contrast layer configuration and local lumped elements

Volume-integrated Maxwell field equations in sourcefree domain Ω with boundary surface $\partial\Omega$:

- $\int_{\partial\Omega} \boldsymbol{\nu}\times\boldsymbol{H}\,\mathrm{d}A = \int_{\Omega} (\epsilon\partial_t\boldsymbol{E} + \sigma\boldsymbol{E})\,\mathrm{d}V$
- $\int_{\partial\Omega} \boldsymbol{\nu}\times\boldsymbol{E}\,\mathrm{d}A = -\int_{\Omega} \mu_0\partial_t\boldsymbol{H}\,\mathrm{d}V$

Ω = 'pillbox' perpendicularly crossing the layer $\Longrightarrow$

interface boundary conditions:

- $\boldsymbol{i}_z\times\boldsymbol{E}(x,y,d/2,t) - \boldsymbol{i}_z\times\boldsymbol{E}(x,y,-d/2,t) = O(d)$

  for $(x,y)\in\Sigma$ as $d\downarrow 0$
- $\boldsymbol{i}_z\times\boldsymbol{H}(x,y,d/2,t) - \boldsymbol{i}_z\times\boldsymbol{H}(x,y,-d/2,t) = (G_{\mathrm{L}} + C_{\mathrm{L}}\partial_t)\boldsymbol{E}(x,y,0,t) + O(d)$

  for $(x,y)\in\Sigma$ as $d\downarrow 0$

Scattering description:

- $\boldsymbol{E}^{\rm s}(\boldsymbol{r},t) = \boldsymbol{E}(\boldsymbol{r},t) - \boldsymbol{E}^{\rm i}(\boldsymbol{r},t)$ • $\boldsymbol{H}^{\rm s}(\boldsymbol{r},t) = \boldsymbol{H}(\boldsymbol{r},t) - \boldsymbol{H}^{\rm i}(\boldsymbol{r},t)$ for $\boldsymbol{r} \in \mathbb{R}^3$

Volume-integrated Maxwell field equations ($\mathcal{D} \subset \Omega$):

- $\displaystyle\int_{\partial\Omega} \boldsymbol{\nu}^{\rm out} \times \boldsymbol{H}^{\rm s} {\rm d}A = \int_{\Omega} \epsilon_0 \partial_t \boldsymbol{E}^{\rm s} + \int_{\mathcal{D}} \boldsymbol{J}^{\rm s} {\rm d}V$
- $\displaystyle\int_{\partial\Omega} \boldsymbol{\nu}^{\rm out} \times \boldsymbol{E}^{\rm s} {\rm d}A = -\int_{\Omega} \mu_0 \partial_t \boldsymbol{H}^{\rm s} {\rm d}V$

Volume source density of electric contrast current:

- $\boldsymbol{J}^{\rm s} = (\epsilon - \epsilon_0)\partial_t \boldsymbol{E} + \sigma \boldsymbol{E}$ with $\text{supp}(\boldsymbol{J}^{\rm s}) = \mathcal{D}$

Contrast sheet electric current density:

- $\boldsymbol{J}^{\rm s}_{\rm sheet} = (G_{\rm L} + C_{\rm L}\partial_t)\boldsymbol{E}(x,y,0,t)$ for $(x,y) \in \Sigma$

Local interface boundary conditions (scattering description):

- $\boldsymbol{i}_z \times \boldsymbol{E}^{\mathrm{s}}(x,y,d/2,t) - \boldsymbol{i}_z \times \boldsymbol{E}^{\mathrm{s}}(x,y,-d/2,t) = O(d)$

  for $(x,y) \in \Sigma$ as $d \downarrow 0$

- $\boldsymbol{i}_z \times \boldsymbol{H}^{\mathrm{s}}(x,y,d/2,t) - \boldsymbol{i}_z \times \boldsymbol{H}^{\mathrm{s}}(x,y,-d/2,t) = \boldsymbol{J}^{\mathrm{s}}_{\mathrm{sheet}} + O(d)$

  for $(x,y) \in \Sigma$ as $d \downarrow 0$

Evidently: $\boldsymbol{i}_z \cdot \boldsymbol{J}^{\mathrm{s}}_{\mathrm{sheet}} = 0$ $\Longrightarrow$

Scattered field $\subset$ Class of 'even' EM fields w.r.t. $\{z = 0\}$:

- $\{E^{\mathrm{s}}_x, E^{\mathrm{s}}_y, H^{\mathrm{s}}_z\}(x,y,-z,t) = \{E^{\mathrm{s}}_x, E^{\mathrm{s}}_y, H^{\mathrm{s}}_z\}(x,y,z,t)$ for all $\boldsymbol{r} \in \mathbb{R}^3$
- $\{E^{\mathrm{s}}_z, H^{\mathrm{s}}_x, H^{\mathrm{s}}_y\}(x,y,-z,t) = -\{E^{\mathrm{s}}_z, H^{\mathrm{s}}_x, H^{\mathrm{s}}_y\}(x,y,z,t)$ for all $\boldsymbol{r} \in \mathbb{R}^3$

$\Longrightarrow$ **REDUCTION IN COMPUTATION TIME**

Local interface boundary conditions (scattering description)

Circulation/Flux integral form of Maxwell's field equations

($\mathcal{S}$ = bounded two-sided surface, $\mathcal{C}$ = oriented boundary curve of $\mathcal{S}$):

- $$\oint_{\mathcal{C}} \boldsymbol{H}^{\mathrm{s}} \cdot \boldsymbol{\tau} \mathrm{d}s = \int_{\mathcal{S}} (\epsilon_0 \partial_t \boldsymbol{E}^{\mathrm{s}} + \boldsymbol{J}^{\mathrm{s}}) \cdot \boldsymbol{\nu} \mathrm{d}A$$
- $$\oint_{\mathcal{C}} \boldsymbol{E}^{\mathrm{s}} \cdot \boldsymbol{\tau} \mathrm{d}s = - \int_{\mathcal{S}} \mu_0 \partial_t \boldsymbol{H}^{\mathrm{s}} \cdot \boldsymbol{\nu} \mathrm{d}A$$

EM field compatibility relations:

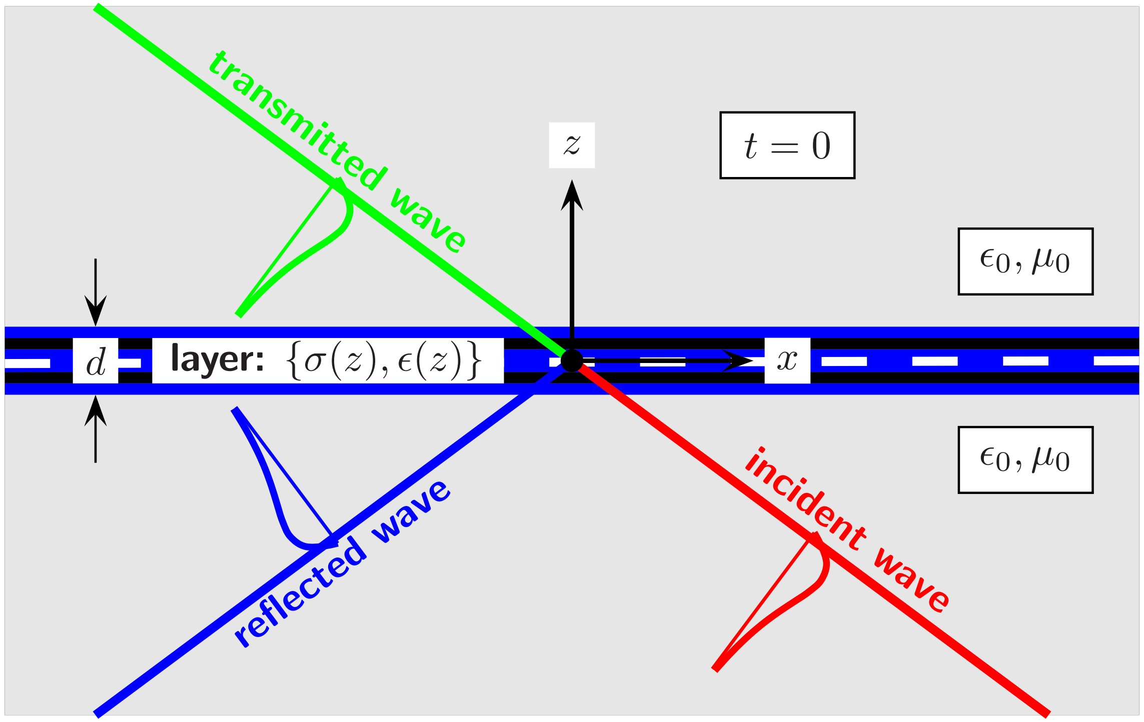
- $$\int_{\textbf{closed surface}} (\epsilon_0 \partial_t \boldsymbol{E}^{\mathrm{s}} + \boldsymbol{J}^{\mathrm{s}}) \cdot \boldsymbol{\nu}^{\mathrm{out}} \mathrm{d}A = 0$$
- $$\int_{\textbf{closed surface}} \mu_0 \partial_t \boldsymbol{H}^{\mathrm{s}} \cdot \boldsymbol{\nu}^{\mathrm{out}} \mathrm{d}A = 0$$

$\Longrightarrow$ **'Edge condition' on $\boldsymbol{J}^{\mathrm{s}}_{\text{sheet}}$:**

- $\boldsymbol{\nu}_{\Sigma} \cdot \boldsymbol{J}^{\mathrm{s}}_{\text{sheet}} = 0$ for $(x, y) \in \partial\Sigma$

The circulation/flux integral form of Maxwell's field equations + Compatibility relations

Thin, high-contrast layer of infinite extent



- $\mathcal{D}^- = \{(x, y) \in \mathbb{R}^2, -\infty < z < 0\}$ **(incident + reflected wave)**
- $\mathcal{D}^+ = \{(x, y) \in \mathbb{R}^2, 0 < z < \infty\}$ **(transmitted wave)**

Plane pulsed E-polarized field ($\alpha_x^2 + \alpha_z^2 = 1$)

Incident field:

- $\{H_x^{\rm i}, E_y^{\rm i}, H_z^{\rm i}\} = \{-\alpha_z Y_0, 1, \alpha_x Y_0\} \times E_0^{\rm i}[t - (\alpha_x x + \alpha_z z)/c_0]$ for $\boldsymbol{r} \in \mathbb{R}^3$

Scattered field:

- $\{H_x^{\rm s}, E_y^{\rm s}, H_z^{\rm s}\} = \{\mp\alpha_z Y_0, 1, \alpha_x Y_0\}\, E_0^{\rm s}[t - (\alpha_x x \pm \alpha_z z)/c_0]$ for $\boldsymbol{r} \in \mathcal{D}^{\pm}$

Medium parameters:

- $c_0 = (\epsilon_0 \mu_0)^{-1/2} =$ electromagnetic wave speed in background medium
- $Y_0 = (\epsilon_0/\mu_0)^{1/2} =$ plane-wave admittance of background medium

Time-domain scattering coefficient:

- $E_0^{\rm s}(t) = S^E(t) \overset{(t)}{*} E_0^{\rm i}(t)$ • $\overset{(t)}{*} =$ time convolution

Plane pulsed E-polarized incident and scattered fields

Plane pulsed H-polarized field ($\alpha_x^2 + \alpha_z^2 = 1$)

Incident field:

- $\{E_x^{\rm i}, H_y^{\rm i}, E_z^{\rm i}\} = \{\alpha_z Z_0, 1, -\alpha_x Z_0\}\, H_0^{\rm i}[t - (\alpha_x x + \alpha_z z)/c_0]$ for $\boldsymbol{r} \in \mathbb{R}^3$

Scattered field:

- $\{E_x^{\rm s}, H_y^{\rm s}, E_z^{\rm s}\} = \pm\{\pm\alpha_z Z_0, 1, -\alpha_x Z_0\}\, H_0^{\rm s}[t - (\alpha_x x \pm \alpha_z z)/c_0]$ for $\boldsymbol{r} \in \mathcal{D}^{\pm}$

Medium parameters:

- $c_0 = (\epsilon_0 \mu_0)^{-1/2} =$ electromagnetic wave speed in background medium
- $Z_0 = (\mu_0/\epsilon_0)^{1/2} =$ plane-wave impedance of background medium

Time-domain scattering coefficient:

- $H_0^{\rm s}(t) = S^H(t) \overset{(t)}{*} H_0^{\rm i}(t)$
- $\overset{(t)}{*} =$ time convolution

Plane pulsed H-polarized incident and scattered fields

Via time Laplace transformation:

- $\{\hat{E}_0^{\mathrm{i}}, \hat{H}_0^{\mathrm{i}}\}(s) = \int_{t=0}^{\infty} \exp(-st)\,\{E_0^{\mathrm{i}}, H_0^{\mathrm{i}}\}(t)\,\mathrm{d}t$ for $s \in \mathbb{C},\ \mathrm{Re}(s) > 0$

s-domain scattering coefficients:

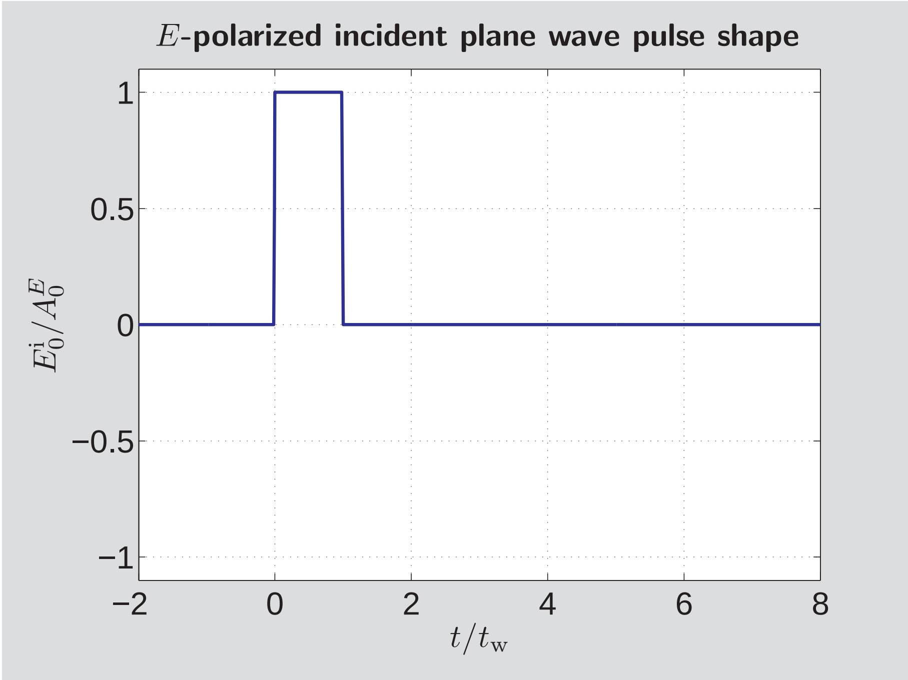
- $\hat{S}^E(s) = -1 + \dfrac{2\alpha_z Y_0}{2\alpha_z Y_0 + G_{\mathrm{L}} + sC_{\mathrm{L}}}$
- $\hat{S}^H(s) = -1 + \dfrac{2}{2 + \alpha_z Z_0 (G_{\mathrm{L}} + sC_{\mathrm{L}})}$

Time-domain scattering coefficients

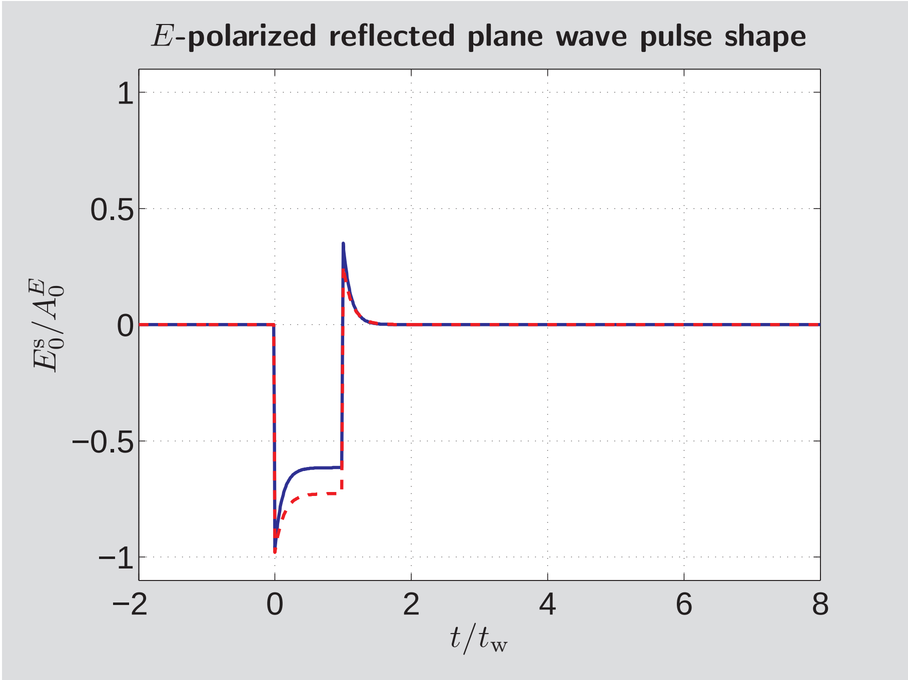
($\delta(t)$ = Dirac delta distribution, $H(t)$ = Heaviside unit step function):

- $S^E(t) = -\delta(t) + T^E \exp(-\beta^E t) H(t)$
  - $T^E = 2\alpha_z Y_0 / C_{\mathrm{L}}$
  - $\beta^E = T^E + G_{\mathrm{L}}/C_{\mathrm{L}}$
- $S^H(t) = -\delta(t) + T^H \exp(-\beta^H t) H(t)$
  - $T^H = 2/\alpha_z Z_0 C_{\mathrm{L}}$
  - $\beta^H = T^H + G_{\mathrm{L}}/C_{\mathrm{L}}$,

E-polarized incident pulse shape (A_0^E = amplitude, t_w = pulse time width):



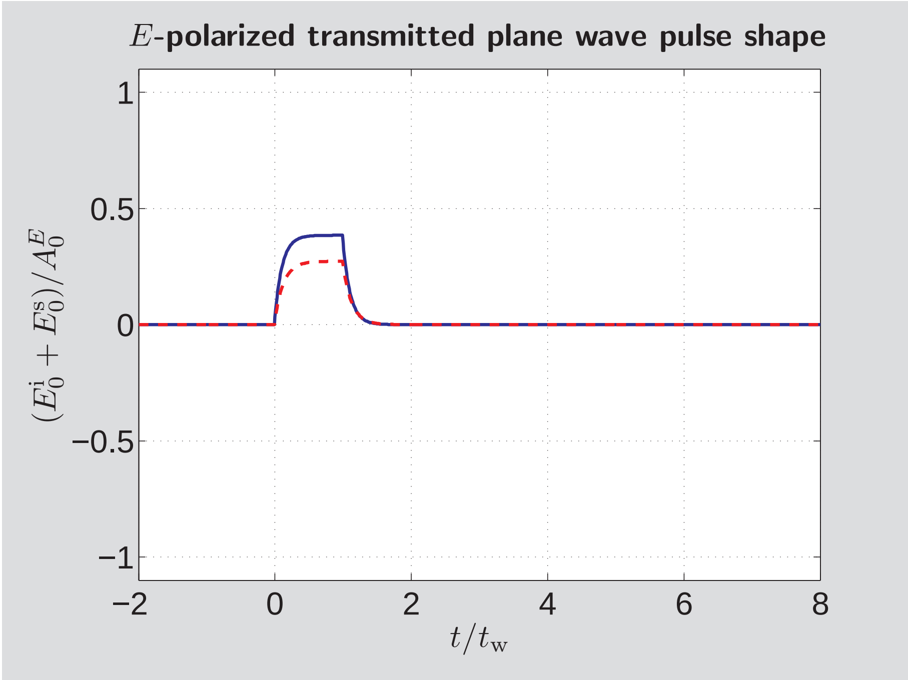
E-polarized reflected pulse shape
(A_0^E = amplitude, t_w = pulse time width incident wave):



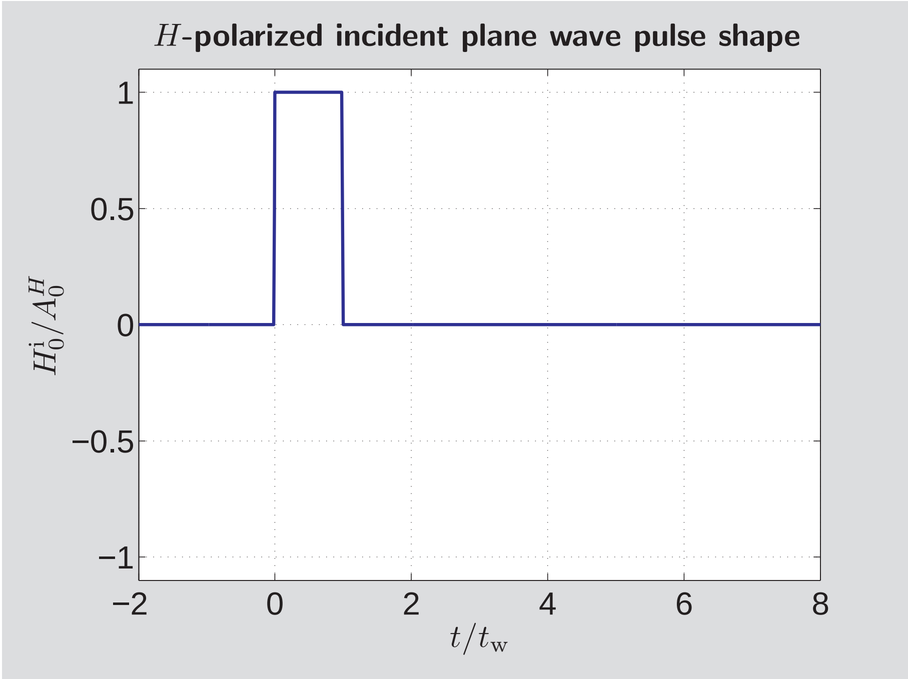
Numerical results (2) (incident wave: rectangular pulse)

E-polarized transmitted pulse shape

(A_0^E = amplitude, t_w = pulse time width incident wave):

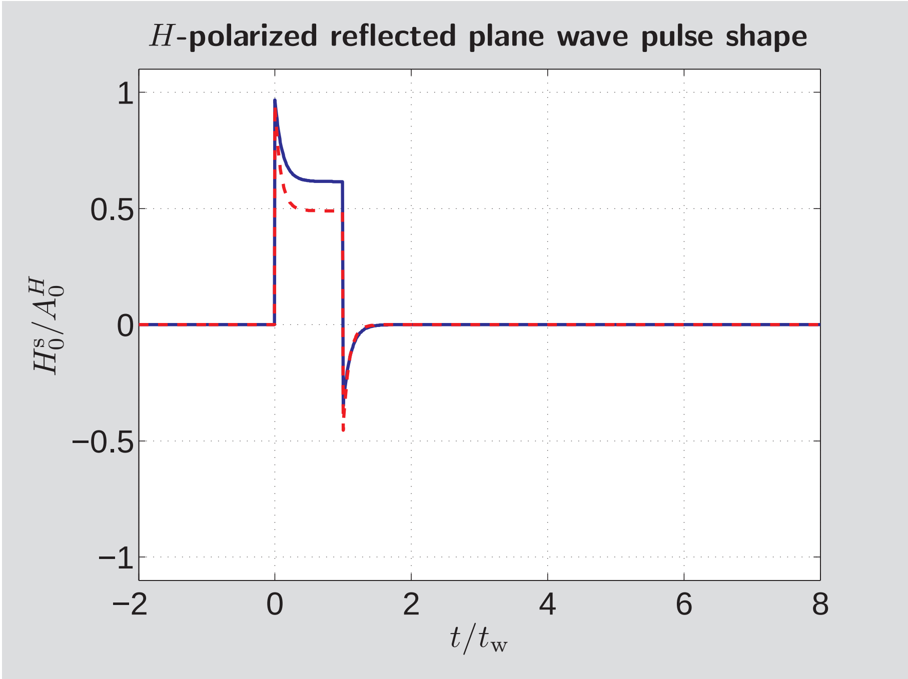


H-polarized incident pulse shape
(A_0^H = amplitude, t_w = pulse time width):



Numerical results (incident wave: rectangular pulse)

H-polarized reflected pulse shape (A_0^H = amplitude, t_w = pulse time width incident wave):



Numerical results (incident wave: rectangular pulse)

H-polarized transmitted pulse shape (A_0^H = amplitude, t_w = pulse time width incident wave):