Time-domain analysis of the pulsed EM field in planarly layered configurations

principles and software implementation

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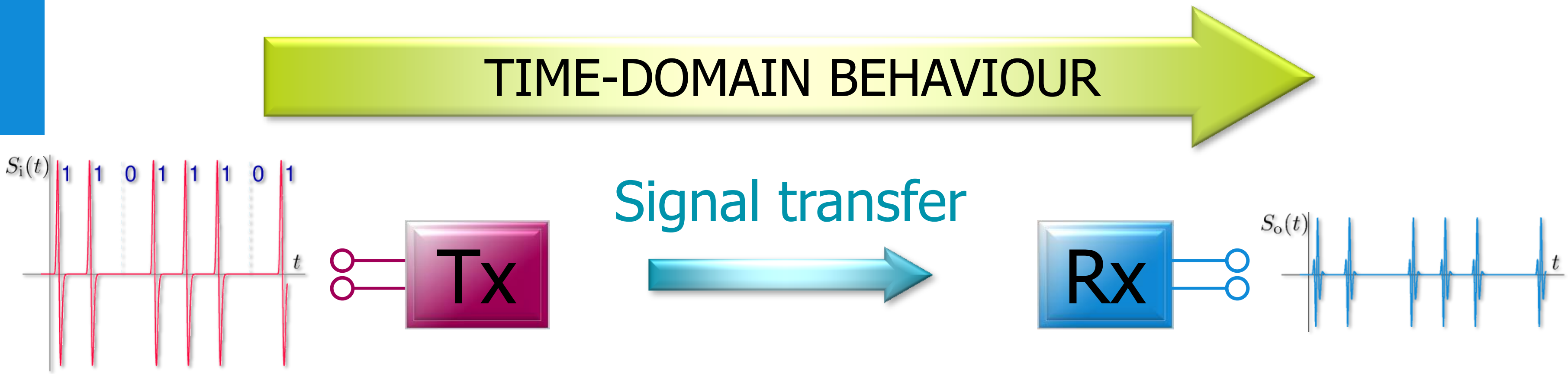
Digital wireless signal transfer



- Typical wireless signal transfer scenario
- Assume a digital input → train of coded pulses
- Output signal: train of distorted pulses

Time-domain analysis

Objective

- Principles
- Software implementation

TD analysis of the pulsed electromagnetic (EM) field signal transfer in planarly layered configurations

- **Instrument:** the Cagniard–deHoop method

- **Focus:**
  - abilities
  - strength
  - implementation

A MODIFICATION OF CAGNIARD'S METHOD FOR SOLVING SEISMIC PULSE PROBLEMS

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Summary

A modification of Cagniard's method for solving seismic pulse problems is given. In order to give a clear picture of our method, two simple problems are solved, viz. the determination of the scalar cylindrical wave generated by an impulsive line source and the scalar spherical wave generated by an impulsive point source.

§ 1. *Introduction.* The application of Cagniard's [1]) method in obtaining exact solutions of three-dimensional seismic pulse problems leads to complicated expressions for the components of the displacement vector in the elastic solid. This is partly due to the fact that in a homogeneous, isotropic, elastic solid two types of waves, travelling with different velocities, occur. In order to give a clear picture of Cagniard's method, Dix [2]) applied it to a simple problem in scalar wave propagation, viz. the determination of the spherical wave generated by an impulsive point source located in a homogeneous, isotropic, unbounded medium. Even in this simple problem (the solution of which can also be obtained by less complicated methods) quite a number of transformations of complex contour integrals are involved.

In the present paper it is shown that Cagniard's method can be simplified considerably if the corresponding modification for two-dimensional problems as developed by the present author [3,4]) is taken as a guidance. Again, the aforementioned point source problem will be considered; for reference, also the solution of the corresponding line source problem will be given.

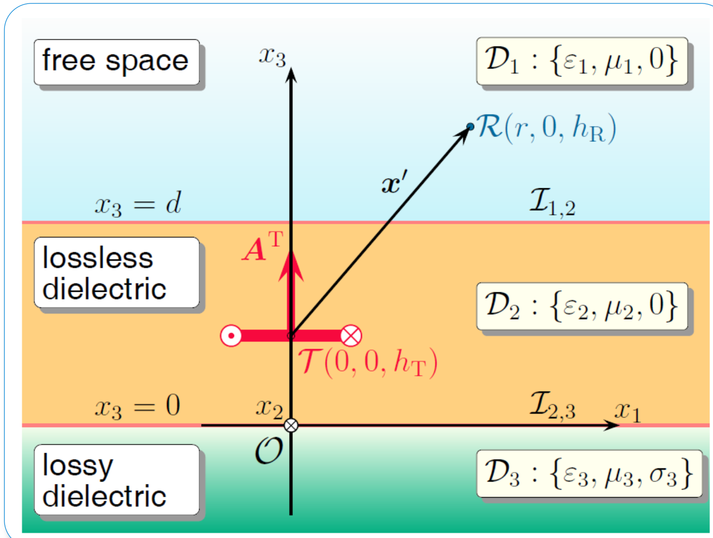
It is remarked that the resulting method is also simpler than

Synopsis

- Model configurations
- Main requisites for the TD analysis → emphasis on the steps in the Cagniard–deHoop method
- Illustrative results
- Implementation aspects
  (seldom, if at all, discussed in the literature)

Model configuration: 'IC integrated antenna'

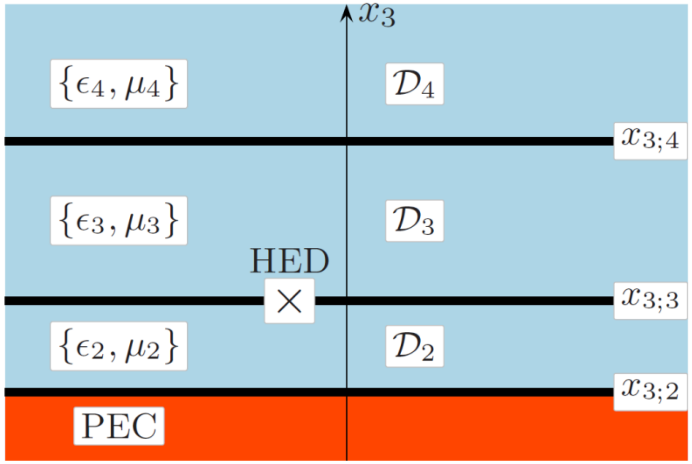
- Source: pulsed electric current carrying loop in $\mathcal{D}_2$, aligned with the interfaces
- maxdiam(Loop) $< c_2 t_\mathrm{w}$, $c_2 = (\varepsilon_2 \mu_2)^{-1/2}$ and $t_\mathrm{w} =$ pulse time width
- Loop area $\boldsymbol{A}^\mathrm{T} = A^\mathrm{T} \boldsymbol{i}_3$
- Excitation: pulsed electric current $I^\mathrm{T}(t)$



Domain	ε_r	μ_r	σ (S/m)
$\mathcal{D}_1$	1	1	0
$\mathcal{D}_2$	3.9	1	0
$\mathcal{D}_3$	11.7	1	10^{-3}

Model configuration: ‘lossless, grounded stack’

- Excitation: Horizontal Electric Dipole (HED)



Domain	ε_r	μ_r	σ (S/m)
$\mathcal{D}_2$	2	2	0
$\mathcal{D}_3$	4	1	0
$\mathcal{D}_4$	1	1	0

Prerequisites: integral transforms

Causal function!

$$f(t) \xrightarrow{\text{LT}} \hat{f}(\boldsymbol{x}, s)$$

- Time: Laplace transform

$$\hat{f}(\boldsymbol{x}, s) = \int_{t=0}^{\infty} f(\boldsymbol{x}, t) \exp(-st)\mathrm{d}t, \text{ for } s \in \mathbb{R},\ s > 0$$

- Space: (scaled) Fourier transform
  - direct

$$\hat{f}(\boldsymbol{x}, s) \overset{\text{FT}}{\longmapsto} \tilde{f}(\alpha_1, \alpha_2, x_3, s)$$

$$\tilde{f}(\alpha_1, \alpha_2, x_3, s) = \int_{x_1=-\infty}^{\infty} \int_{x_2=-\infty}^{\infty} \hat{f}(x_1, x_2, x_3, s) \exp\left[s\left(\alpha_1 x_1 + \alpha_2 x_2\right)\right] \mathrm{d}x_1 \mathrm{d}x_2$$

with $s\alpha_1, s\alpha_2 \in \mathbb{C},\ \mathrm{Re}(s\alpha_1) = \mathrm{Re}(s\alpha_2) = 0$

$\alpha_{1,2} \in \mathbb{I}$

Prerequisites: integral transforms

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$$\hat{f}(\boldsymbol{x}, s) = \int_{t=0}^{\infty} f(\boldsymbol{x}, t) \exp(-st) \mathrm{d}t, \text{ for } s \in \mathbb{R}, \ s > 0$$

- Space: (scaled) Fourier transform
  - inverse

$$\tilde{f}(\alpha_1, \alpha_2, x_3, s) \xmapsto{\text{FT}^{-1}} \hat{f}(\boldsymbol{x}, s)$$

$$\hat{f}(\boldsymbol{x}, s) = \left(\frac{s}{2\pi \mathrm{i}}\right)^2 \int_{\alpha_2 \in \mathbb{I}} \int_{\alpha_1 \in \mathbb{I}} \tilde{f}(\alpha_1, \alpha_2, x_3, s) \exp\left[-s\left(\alpha_1 x_1 + \alpha_2 x_2\right)\right] \mathrm{d}\alpha_1 \mathrm{d}\alpha_2$$

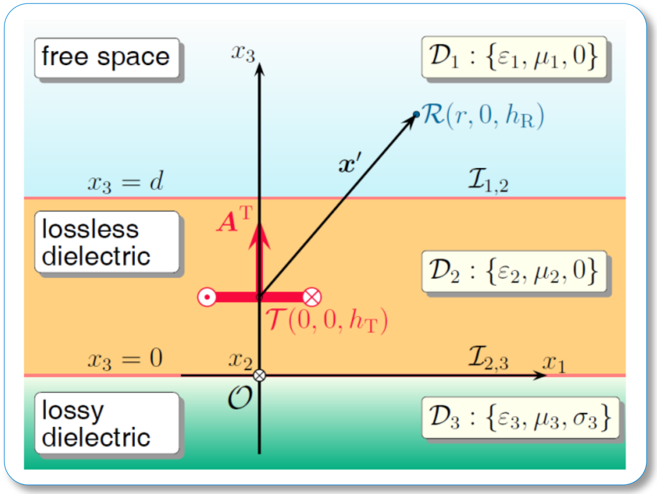
Field equations

- Time-domain EM field equations
- Laplace transform $t \Rightarrow s$; condition: causal excitation
- Fourier transform $\{x_1, x_2\} \Rightarrow \{\alpha_1, \alpha_2\}$
- Solve for the spectral EM field quantities

Field equations: example

- Inside each sub-domain of continuity $\mathcal{D}_i,\ i=1,2,3$

$$\boldsymbol{E}(\boldsymbol{x}',t) = -\mu\,(\boldsymbol{i}_1\partial_2 - \boldsymbol{i}_2\partial_1)\,\partial_t u(\boldsymbol{x}',t)$$
$$\boldsymbol{H}(\boldsymbol{x}',t) = \boldsymbol{\nabla}\left[\partial_3 u(\boldsymbol{x}',t)\right] - \boldsymbol{i}_3\,\mu\varepsilon\,\partial_t^2 u(\boldsymbol{x}',t) - \boldsymbol{i}_3\,\mu\sigma\,\partial_t u(\boldsymbol{x}',t)$$



with:

- $\boldsymbol{E}(\boldsymbol{x}',t)$ = electric field strength
- $\boldsymbol{H}(\boldsymbol{x}',t)$= magnetic field strength
- $u(\boldsymbol{x}',t)$ = potential function → dissipative wave equation:

  $$\boldsymbol{\nabla}^2 u - c_i^{-2}\partial_t^2 u - \sigma_i\mu_i\,\partial_t u = 0, \text{ for } i=1,2,3$$

- $c_i = (\varepsilon_i\mu_i)^{-1/2}$, for $i=1,2,3$ = wave speeds

Field equations: example

- Time-domain wave equation ➡ $\xrightarrow{\text{LT}}$ + $\xrightarrow{\text{FT}}$ ➡ spectral wave equation

$$\partial_3^2 \tilde{u} - s^2 \left[\gamma_i^2 + s^{-1}\mu_i \sigma_i\right] \tilde{u} = 0, \quad \text{for } i = 1, 2, 3.$$

in which

$$\gamma_i(\alpha_1, \alpha_2) = \left[c_i^{-2} - (\alpha_1^2 + \alpha_2^2)\right]^{1/2}, \quad \text{with } \mathrm{Re}(\gamma_i) \geqslant 0, \; i = 1, 2, 3$$

is the propagation coefficient

⬇

- Solve the spectral wave equation

Determination of the TD quantities

1. $\xrightarrow{\text{FT}^{-1}}$ is applied; a change of variables $\{\alpha_1, \alpha_2\} \Longrightarrow \{\omega, q\}$, with $\omega \in \mathbb{C},\ q \in \mathbb{R}$, is used

2. The Laplace inversion, via the Cagniard–deHoop method: a deformation of the Bromwich contour via parametrisation ⇓

   an expression ⟺ the Laplace transform of a real function ⇒ the Laplace inversion by inspection

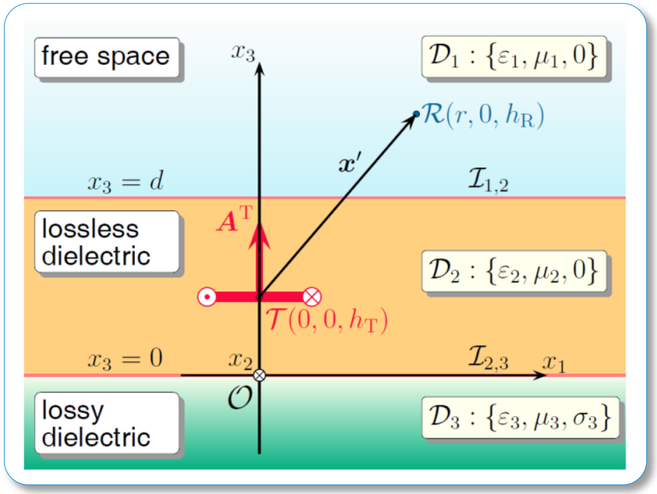
Solution = Σ { generalised-ray TD constituents }

Post-processing

- The radiated field values can be compared with EMI specifications for assessing the EMC of devices
- Reciprocity → The signal intercepted by a (nearby located) receiving antenna
- The field quantities are available with arbitrary spatial and temporal resolutions → TD equivalents of the radiation patterns

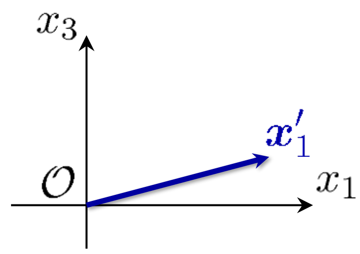
Illustrative numerical experiments: 'IC integrated antenna'

- **Loop:** area $A^{\mathrm{T}} = 0.0314\,\mathrm{mm}^2$, located at $h^{\mathrm{T}} = 2\mu\mathrm{m}$
- **Dielectric slab thickness:** $d = 7\mu\mathrm{m}$
- **Feeding pulse:** normalised, time-differentiated, power exponential pulse, with $\nu = 5$ and $t_{0\mathrm{x}} = 0.1\,\mathrm{ns}$

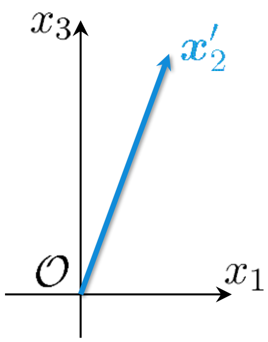


Illustrative numerical experiments: 'IC integrated antenna'

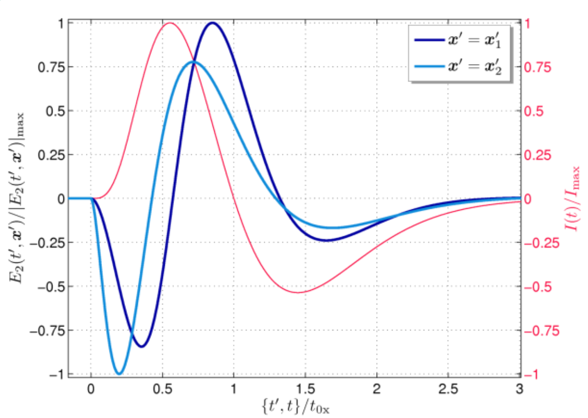
- Accurate
- Excellent temporal and spatial resolution

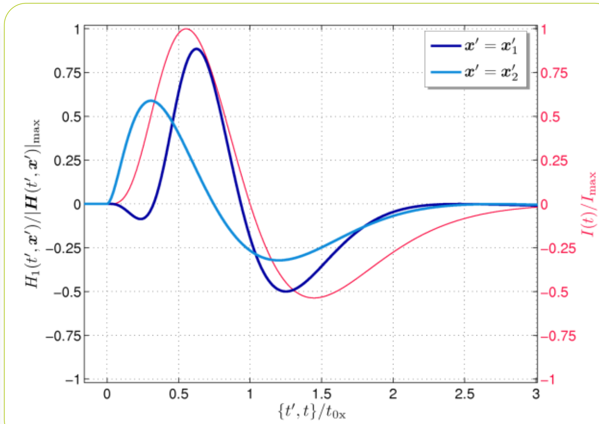


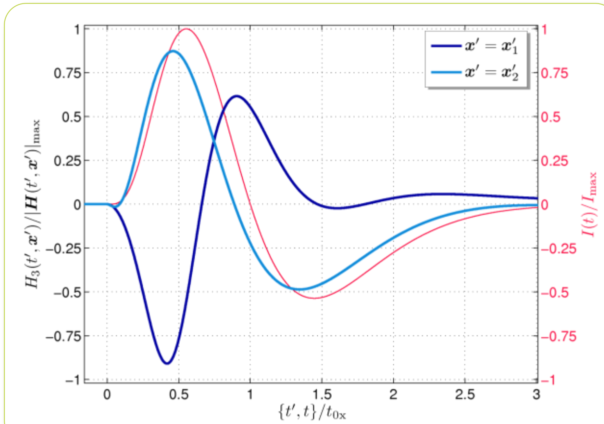
$$x'_1 = 4i_1 + 1i_3 \text{ (mm)}$$



$$x'_2 = 4i_1 + 10i_3 \text{ (mm)}$$

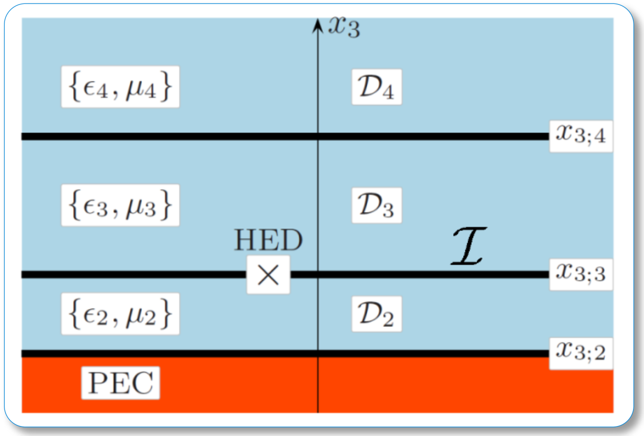


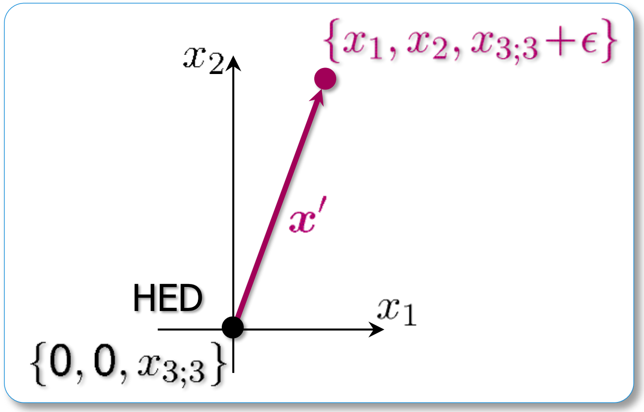




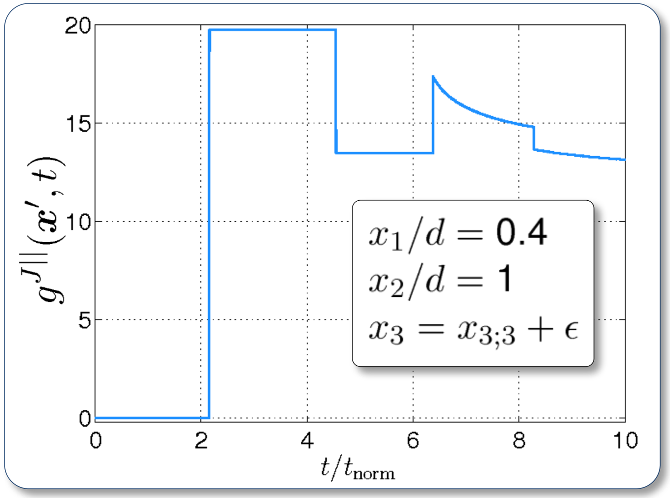
Illustrative numerical experiments: 'lossless, grounded stack'

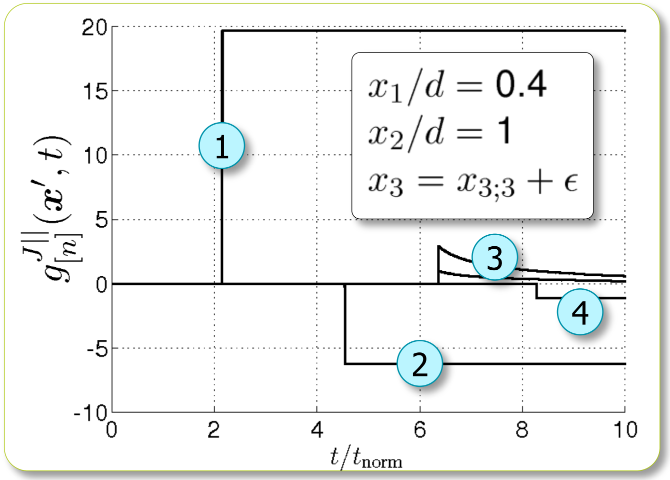
- **Layer thicknesses:** $x_{3;3} - x_{3;2} = d$ and $x_{3;4} - x_{3;3} = 1.5d$
- **Field observed:** just above the interface $\mathcal{I}$, at $\{x_1/d, x_2/d\} = \{0.4, 1\}$
- **Examined quantities:** the step responses $g^{J||}(\boldsymbol{x}', t)$ and $g^{Q||}(\boldsymbol{x}', t)$ $= \boldsymbol{\Sigma}\, \{ g^{Q||}_{[n]}(\boldsymbol{x}', t), g^{J||}_{[n]}(\boldsymbol{x}', t) \}$
- **Examined time window:** $\{0 \leqslant t/t_{\mathrm{norm}} \leqslant 10\}$, with $t_{\mathrm{norm}} = d/c_0$

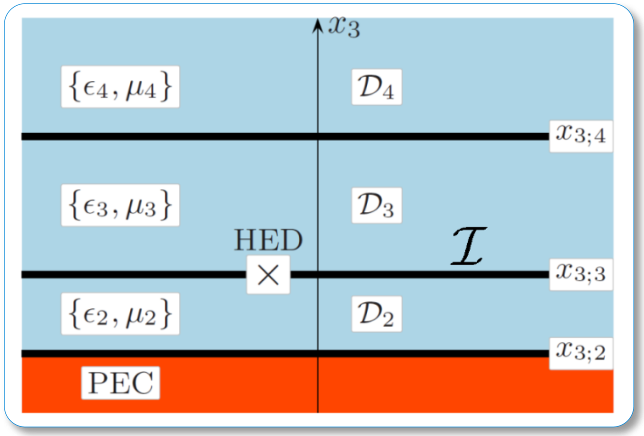


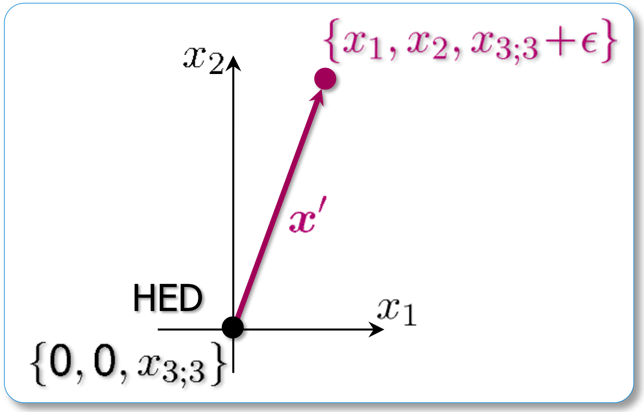


Illustrative numerical experiments: ‘lossless, grounded stack’









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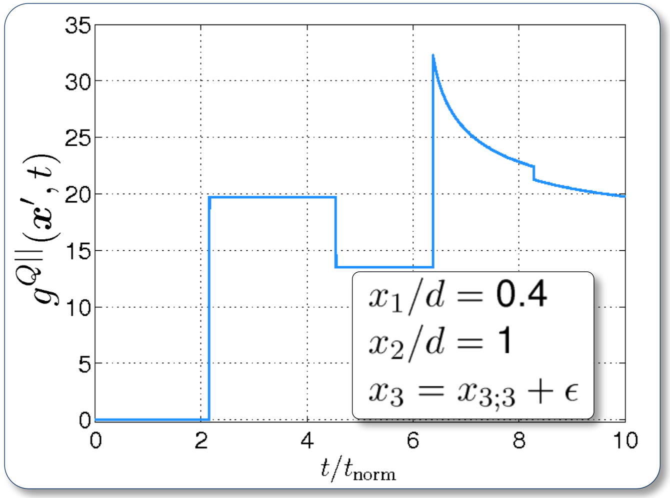

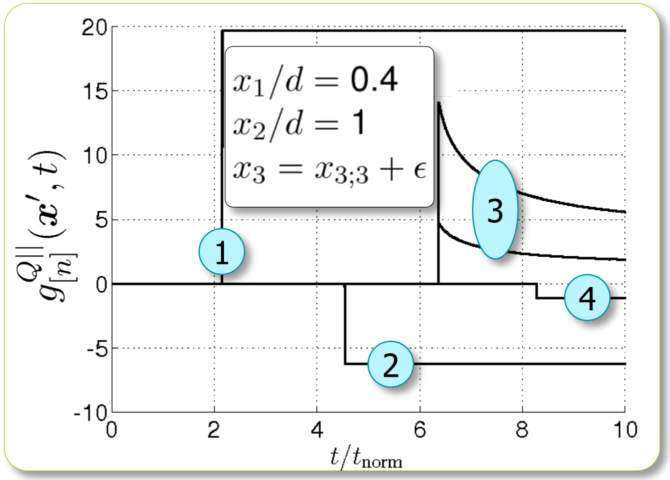

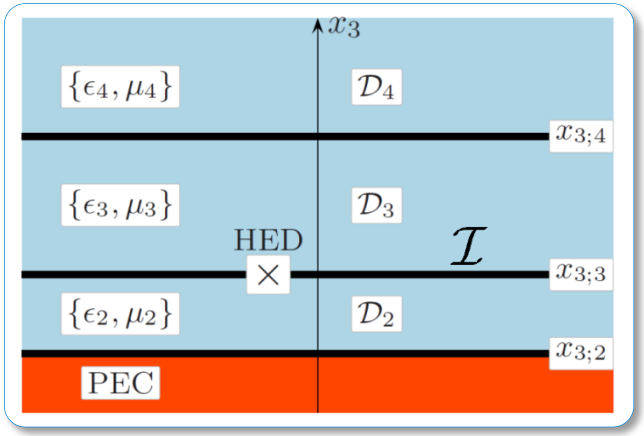

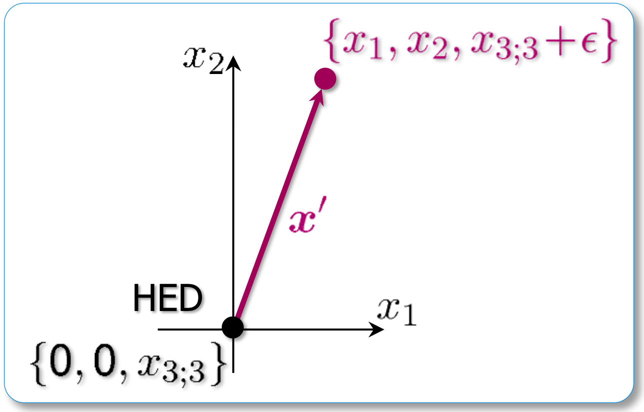

Implementation aspects

Effective programming: ingredients yield a (logical) tree-like structure

starts at the exciting source

↓

terminates at the end of a specified time window

Implementation aspects

- Determination of the arrival times → numerical solution
  - ! time consuming
- The Cagniard–deHoop path must be constructed numerically
  - ! time consuming → effective solvers + interpolation
- Triple/quadruple integrals → numerical quadrature
  - ! effective quadrature solutions
  - – pre-computed data
  - – trapezoidal rule + parallelisation

Conclusion

A computational framework based on the Cagniard–deHoop method is:

- Accurate: within the examined time window, the field expression is exact
- Expedient: field values at any location outside the direct geometry of the device under consideration
- Effective: a complete computational tool in the realm of the design of microelectronic circuits