Pulse shape distortion in closed-waveguide axial modal signal transfer

an analytic time-domain study

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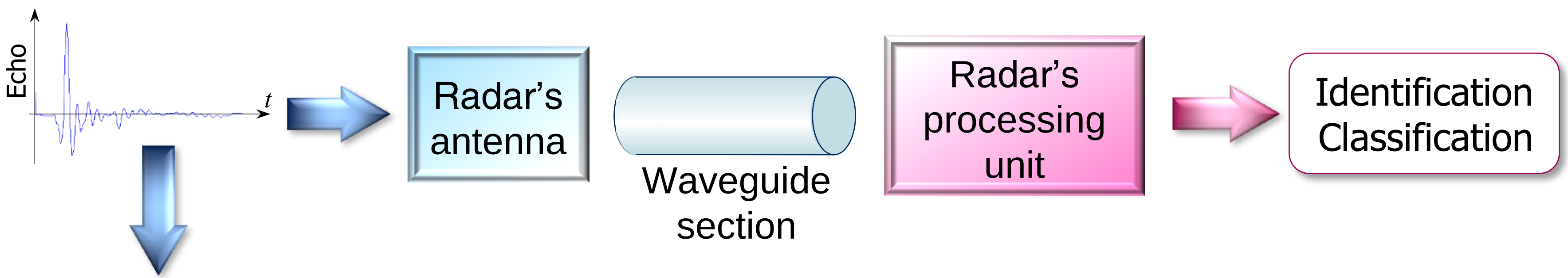
Pulsed radar scenario

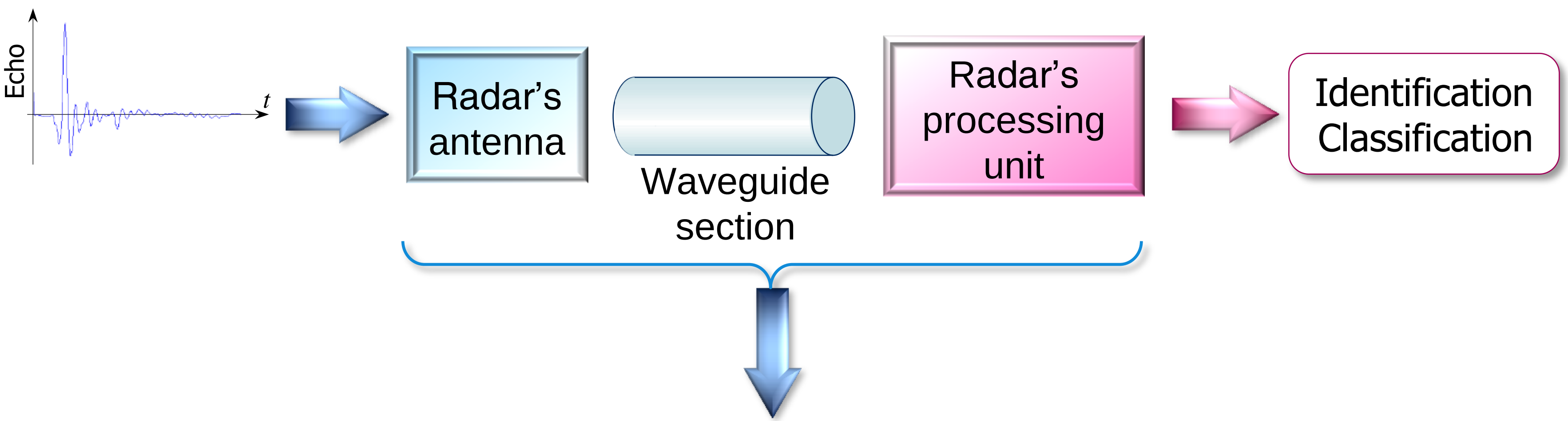


$$f(t)H(t) \overset{\text{LT}}{\longmapsto} \hat{f}(s) = \int_{t=0}^{\infty} f(t)\exp(-st)\mathrm{d}t$$

$$s \in \mathbb{C},\ \mathrm{Re}(s) > 0$$

- Causal echo ➡ via the time Laplace transform

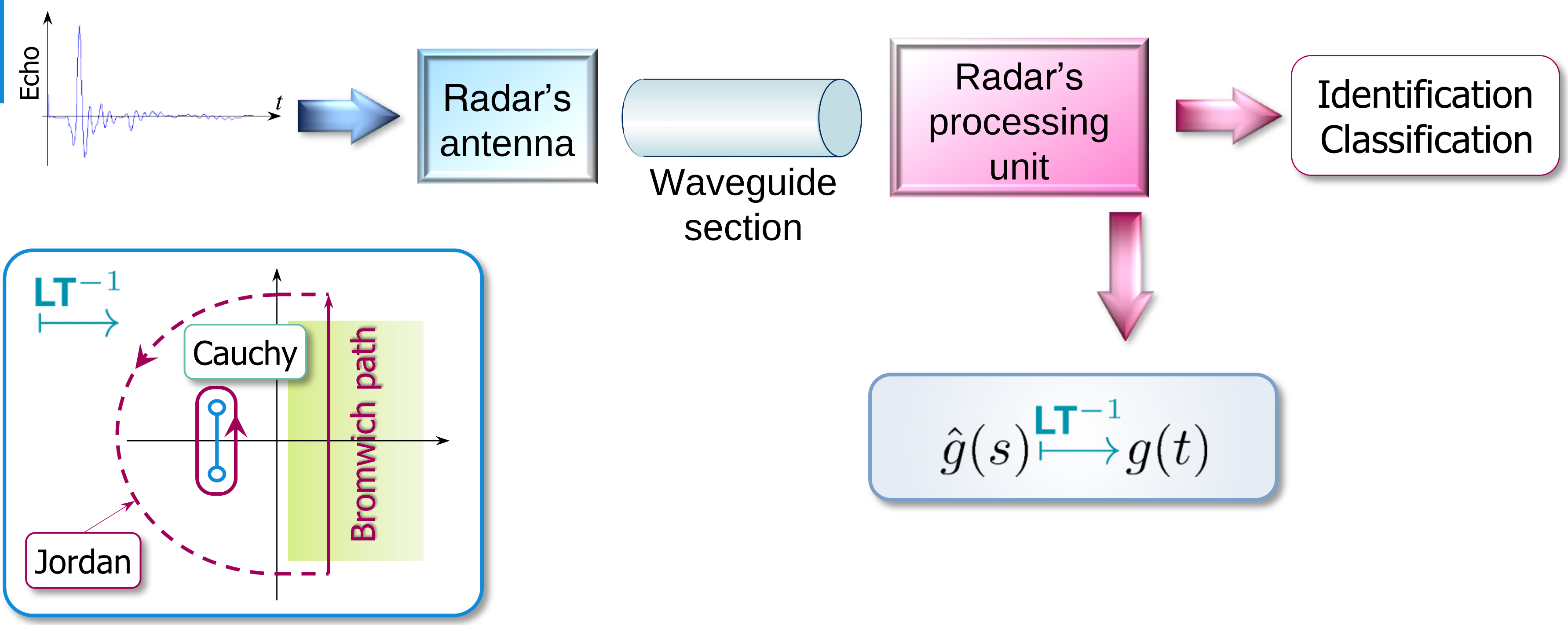
Pulsed radar scenario



$$\hat{g}(s) = \hat{f}(s)\hat{T}(s)$$

- Linear, time-invariant, passive systems → causal response

Pulsed radar scenario



- The dispersion properties of the transfer path ➡ **determinant for the detection ability**

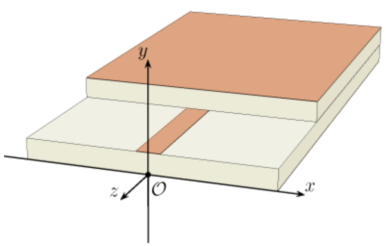
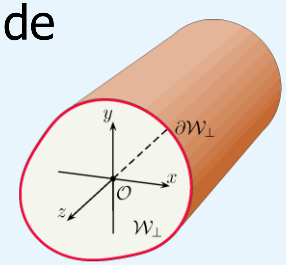
Synopsis

- Propagation / propagators
- Time-domain modal expansion in closed waveguides
- Illustrative numerical experiments
- Conclusions

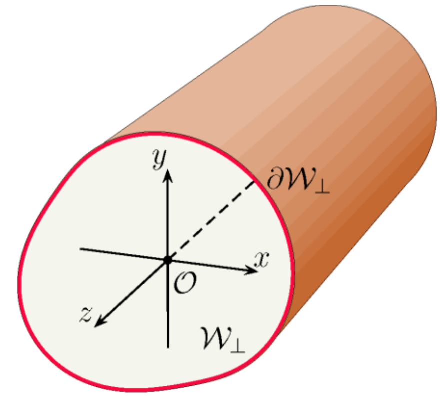
Prerequisites

- Material parameters:
  - $\varepsilon > 0$ = electric permittivity
  - $\sigma \geqslant 0$ = electric conductivity
  - $\mu > 0$ = magnetic permeability
  - $\kappa \geqslant 0$ = linear magnetic loss coefficient
  - $c = (\varepsilon\mu)^{-1/2}$ = wave speed
- Instrument: the time Laplace transform $f(t)H(t) \xmapsto{\text{LT}} \hat{f}(s)$
- Impedances:
  - $\hat{\zeta} = \mu(s + \kappa/\mu)$ = longitudinal impedance of the medium
  - $\hat{\eta} = \varepsilon(s + \sigma/\varepsilon)$ = transverse admittance of the medium

Propagation / propagators

Propagation path	**Propagator**
Free space	$\dfrac{\exp(-\gamma R)}{4\pi R}$ R = radial distance $\gamma = s/c_0$ = propagation coeff.
Stripline 	$\exp(\mp\gamma\|z\|)$ $\|z\|$ = axial distance $\gamma = \dfrac{1}{c}\left[(s+\alpha)(s+\beta)\right]^{1/2}$ Material dispersion Boltzmann relaxation coefficients $\alpha \geqslant 0$, $\beta \geqslant 0$
Closed waveguide 	$\exp(\mp\gamma_n\|z\|)$ $\gamma_n = \left(\gamma^2 + \kappa_n^2\right)^{1/2}$ modal eigenvalue Modal geometrical dispersion

Closed waveguides



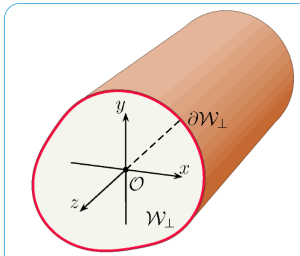
$\mathcal{W} = \mathcal{W}_\perp \times \mathbb{R}$ with
$\mathcal{W}_\perp \subset \mathbb{R}^2$ + $\partial\mathcal{W}_\perp$

- Time Laplace transformed Maxwell eqs.

$$\partial \times \hat{\boldsymbol{H}} - \hat{\eta}\hat{\boldsymbol{E}} = \hat{\boldsymbol{J}},$$
$$\partial \times \hat{\boldsymbol{E}} + \hat{\zeta}\hat{\boldsymbol{H}} = -\hat{\boldsymbol{K}},$$

  - ∂ = spatial differential operator
  - $\hat{\boldsymbol{E}}$ = electric field strength
  - $\hat{\boldsymbol{H}}$ = magnetic field strength
  - $\hat{\boldsymbol{J}}$ = volume source density of electric current
  - $\hat{\boldsymbol{K}}$ = volume source density of magnetic current
- Geometric vectorial decomposition
  - $\parallel$ = parallel to waveguide axis
  - $\perp$ = perpendicular to waveguide axis

Closed waveguides



$\mathcal{W} = \mathcal{W}_\perp \times \mathbb{R}$ with $\mathcal{W}_\perp \subset \mathbb{R}^2 + \partial\mathcal{W}_\perp$

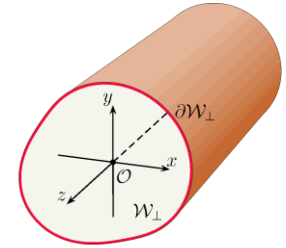
- Modal decomposition → modal constituents:
  - transverse magnetic (TM) modes → in terms of $E_{||}$
  - transverse electric (TE) modes → in terms of $H_{||}$
- Time Laplace-transformed scalar wave equations

$$\left(\boldsymbol{\partial}_\perp \cdot \boldsymbol{\partial}_\perp + \partial_{||}\partial_{||} - \hat{\eta}\hat{\zeta}\right)\left\{\hat{E}_{||}, \hat{H}_{||}\right\} = -\left\{\hat{Q}^{\mathrm{E}}_{||}, \hat{Q}^{\mathrm{H}}_{||}\right\}$$

- Time Laplace-transformed Green's function of the dissipative scalar wave equation

$$\left[\boldsymbol{\partial}_\perp \cdot \boldsymbol{\partial}_\perp + \partial_{||}\partial_{||} - c^{-2}(s+\alpha)(s+\beta)\right]\hat{G}$$
$$= -\delta\left(\boldsymbol{x}_\perp, \boldsymbol{x}'_\perp, \boldsymbol{x}_{||} - \boldsymbol{x}'_{||}\right)$$

Modal decomposition



$\mathcal{W} = \mathcal{W}_{\perp} \times \mathbb{R}$ with

$\mathcal{W}_{\perp} \subset \mathbb{R}^2 + \partial\mathcal{W}_{\perp}$

- Modal expansion functions ⇒ solutions of the transverse Helmholtz equation

$$\left(\partial_{||}\partial_{||} + \kappa_n^2\right)\varphi_n = 0, \text{ for } \boldsymbol{x}_{\perp} \in \mathcal{W}_{\perp}$$

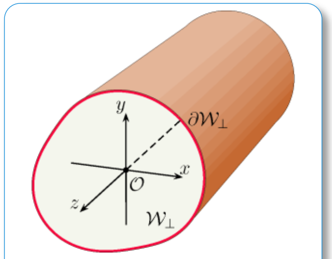
  - Dirichlet B.C.-s on $\mathcal{W}_{\perp}$ ⇒ TM-modes
  - Neumann B.C.-s on $\mathcal{W}_{\perp}$ ⇒ TE-modes
  - normalisation:

$$\delta\left(\boldsymbol{x}_{\perp}, \boldsymbol{x}'_{\perp}\right) = \sum_n N_n^{-1}\varphi_n\left(\boldsymbol{x}_{\perp}\right)\varphi_n\left(\boldsymbol{x}'_{\perp}\right)$$

with

$$N_n = \int_{\mathcal{W}_{\perp}} \varphi_n\left(\boldsymbol{x}_{\perp}\right)\varphi_n\left(\boldsymbol{x}_{\perp}\right)\mathrm{d}A$$

Modal decomposition



$\mathcal{W} = \mathcal{W}_\perp \times \mathbb{R}$ with

$\mathcal{W}_\perp \subset \mathbb{R}^2 + \partial\mathcal{W}_\perp$

- Spectral Green's function

$$\hat{G} = \sum_n N_n^{-1} \varphi_n(\boldsymbol{x}_\perp)\, \varphi_n(\boldsymbol{x}'_\perp)\, \hat{g}_n\left(\boldsymbol{x}_\parallel - \boldsymbol{x}'_\parallel, s\right)$$

in which

$$\left(\partial_\parallel \partial_\parallel - \gamma_n^2\right) \hat{g}_n\left(\boldsymbol{x}_\parallel - \boldsymbol{x}'_\parallel, s\right) = -\delta\left(\boldsymbol{x}_\parallel - \boldsymbol{x}'_\parallel\right)$$

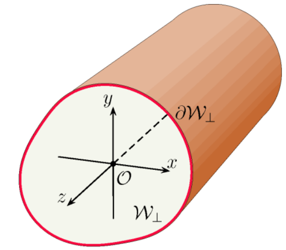
- Modal characteristic

$$\gamma_n^2 = \frac{(s+\alpha)(s+\beta)}{c^2} + \kappa_n^2 = \frac{\left[s + (\alpha+\beta)/2\right]^2}{c^2} + \Omega_n^2$$

with

$$\Omega_n^2 = \kappa_n^2 - \frac{c^{-2}(\alpha-\beta)^2}{4}$$

Modal decomposition



$\mathcal{W} = \mathcal{W}_\perp \times \mathbb{R}$ with
$\mathcal{W}_\perp \subset \mathbb{R}^2 + \partial\mathcal{W}_\perp$

- Time-domain Green's function

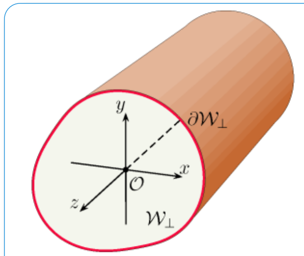
$$G_n = \sum_n N_n^{-1} \varphi_n(\boldsymbol{x}_\perp)\, \varphi_n(\boldsymbol{x}'_\perp)\, g_n\left(\boldsymbol{x}_\parallel - \boldsymbol{x}'_\parallel, t\right)$$

with

$$g_n\left(\boldsymbol{x}_\parallel - \boldsymbol{x}'_\parallel, t\right) = \frac{c}{2} \exp\left[-\frac{(\alpha+\beta)t}{2}\right] J_0\left[c\Omega_n \left(t^2 - \left|\boldsymbol{x}_\parallel - \boldsymbol{x}'_\parallel\right|^2 / c^2\right)^{1/2}\right]$$

$$H\left[t - \left|\boldsymbol{x}_\parallel - \boldsymbol{x}'_\parallel\right| / c\right] \quad \text{for } \Omega_n^2 > 0$$

Oscillatory behaviour



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Modal decomposition

- Time-domain Green's function

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$$H\left[t - \left|\boldsymbol{x}_\| - \boldsymbol{x}'_\|\right| / c\right] \quad \text{for } \Omega_n^2 < 0$$

Monotonic behaviour

Illustrative numerical experiments

- Analysed quantity:

$$P_n\left(\boldsymbol{x}_{\|}-\boldsymbol{x}'_{\|},t\right)=g_n\left(\boldsymbol{x}_{\|}-\boldsymbol{x}'_{\|},t\right)\overset{(t)}{*}S(t)$$

with $S(t)$ = feeding signal

- Normalisation:
  - time: $t_{\mathrm{norm}}=t\,c\Omega_{n,\mathrm{ref}}$, with $\Omega^2_{n,\mathrm{ref}}>0$ sufficiently large
  - space: $|\boldsymbol{x}_{\|}-\boldsymbol{x}'_{\|}|_{\mathrm{norm}}=|\boldsymbol{x}_{\|}-\boldsymbol{x}'_{\|}|\Omega_{n,\mathrm{ref}}$
  - analysed value: $P_n\left(\boldsymbol{x}_{\|}-\boldsymbol{x}'_{\|},t\right)$ to its peak absolute value
- Medium inside the waveguide:
  - vacuum, $\{\alpha,\beta\}=\{0,0\}$
  - Rogers RO4003™, $\{\alpha,\beta\}=\{0.0027,0\}$

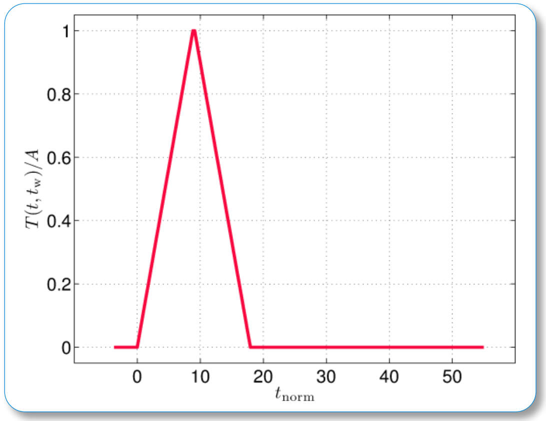
Illustrative numerical experiments

- Feeding signal: triangular pulse

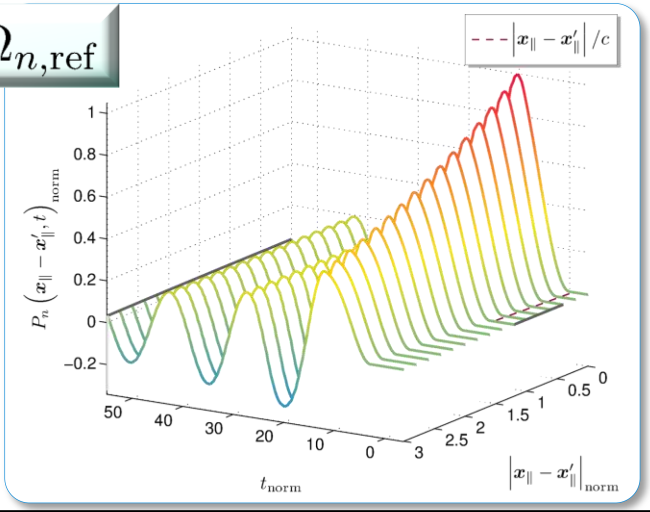
$$T(t,t_{\mathrm{w}}) = A\begin{cases} t/t_{\mathrm{w}}\,H(t) & \text{for } t \leqslant t_{\mathrm{w}} \\ 2 - t/t_{\mathrm{w}} & \text{for } t_{\mathrm{w}} < t \leqslant 2t_{\mathrm{w}} \\ 0 & \text{for } t > 2t_{\mathrm{w}} \end{cases}$$

with $2t_{\mathrm{w}}$ = the pulse width

- Medium: vacuum



$\Omega_n^2 > 0,\ \Omega_n < \Omega_{n,\mathrm{ref}}$



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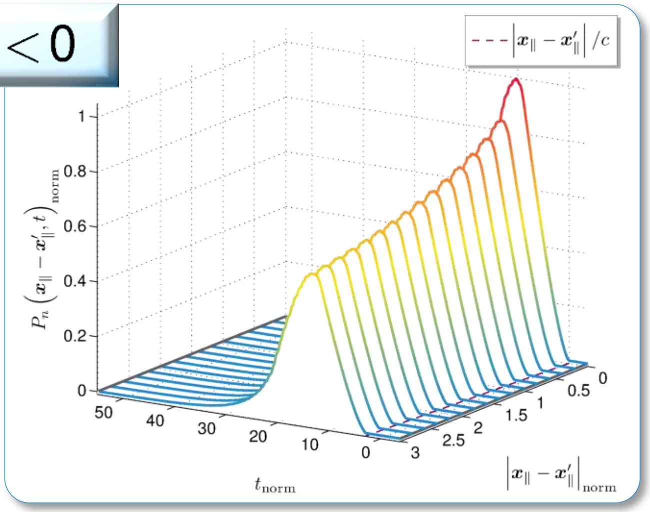
- **Medium:** Rogers RO4003™

$\Omega_n^2 < 0$



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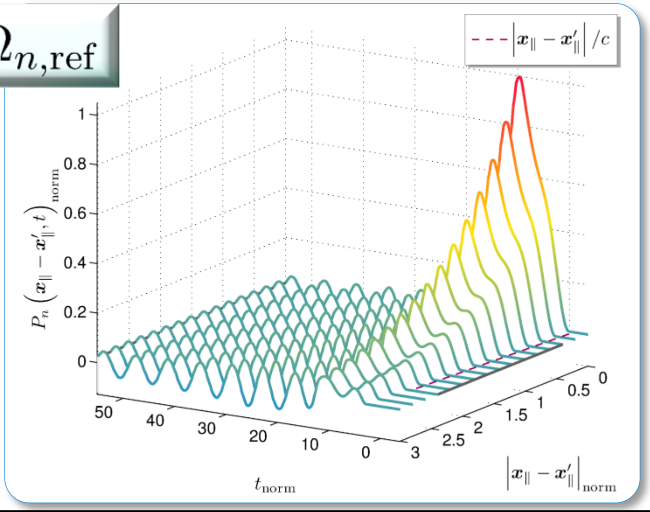
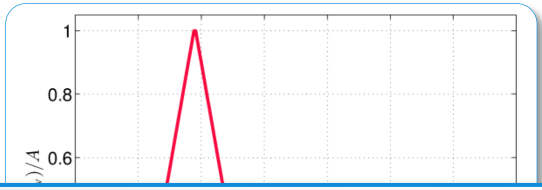




Relevance:
TD pulsed signal transfer ⟷ FD analyses correspondences

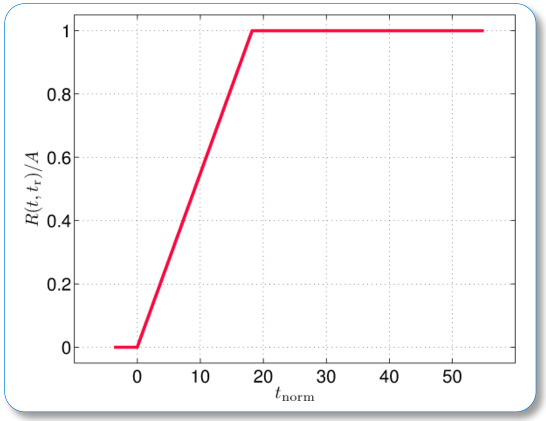
Illustrative numerical experiments

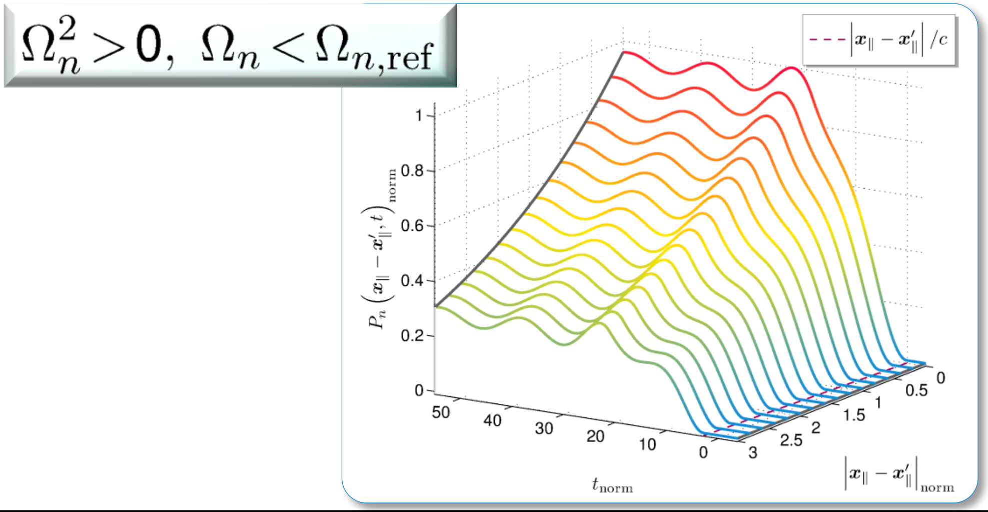
- **Feeding signal:** clipped ramp function

$$R(t, t_{\rm r}) = A \begin{cases} t/t_{\rm r}\, H(t) & \text{for } t \leqslant t_{\rm r} \\ 1 & \text{for } t > t_{\rm r} \end{cases}$$

with $t_{\rm r}$ = pulse rise time

- **Medium:** vacuum





Illustrative numerical experiments

- **Feeding signal:** clipped ramp fu

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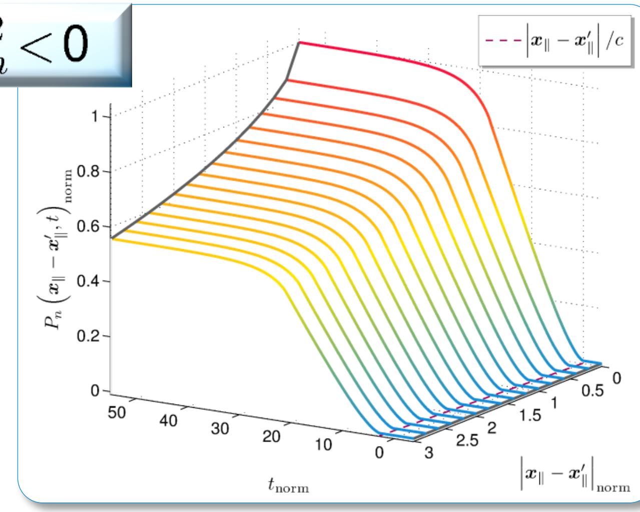
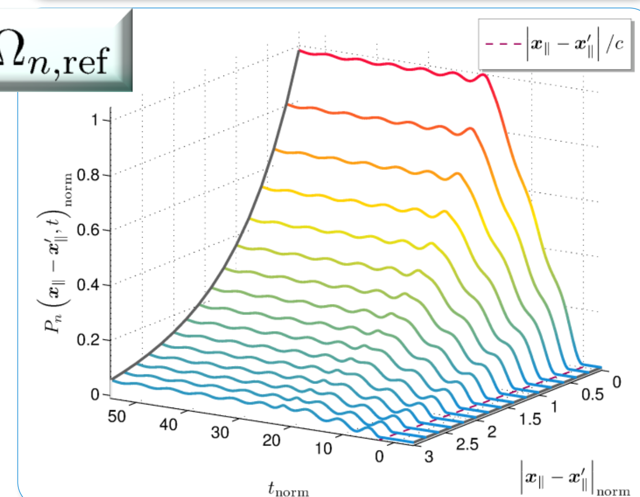
with t_{r} = pulse rise time

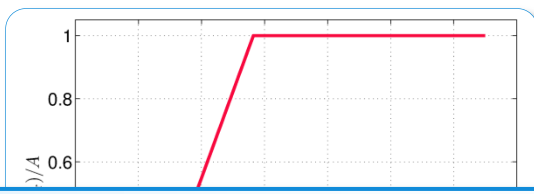
- **Medium:** Rogers RO4003™

$\Omega_n^2 < 0$



$\Omega_n^2 > 0,\ \Omega_n < \Omega_{n,\mathrm{ref}}$





Relevance:
TD reconstruction of the pulse shapes of monostatic or multistatic radar returns

Conclusions

- Examined: influence of material- and geometry-induced dispersion of a pulsed signal in a closed waveguide
- General applicability: effective tools for constructing a deconvolution algorithm that filters out the pertaining dispersion
- Practical application: RADAR → reconstruction of a target's dimensions and shape from the irradiation by a field with dedicated pulse time shape