Elastodynamic wavefield modeling in configurations containing thin, highly contrasting layers

by

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Presentation given at:

Schlumberger-Doll Research, Cambridge MA, USA, 2014 September 09

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Synopsis:

- Wavefield physics
  - Observer in spacetime
  - Array-structured wavefield and source quantities
- Acoustic, elastic, EM wavefields
- High-contrast, thin layer saltus conditions
- (Visco-)elastic layer in elastic embedding
- TD model configurations

General framework

Array-structured Wavefield Physics in Affine $(N+1)$-spacetime

- Wave phenomena : spacetime phenomena that
  - occur throughout the universe
  - are carriers of
    - information
    - energy
    - momentum

Wavefield content interaction scheme		
wavefield content item	interaction	with
information	$\Leftrightarrow$	observer
energy	$\Leftrightarrow$	thermodynamic systems
momentum	$\Leftrightarrow$	(quantum-)mechanical systems

Observer $\mathcal{O}$:

- interprets spacetime as an affine space $\mathbb{R}^{\boldsymbol{N}} \times \mathbb{R}$ (Weyl, ~~Einstein~~) ;
- registers an **EVENT** at
  - position $\boldsymbol{x} = x_m = \{x_1, \ldots, x_N\} \in \mathbb{R}^{\boldsymbol{N}}$ in its
  - evolution with time $t \in \mathbb{R}$
- structures wavefield and source quantities as **ARRAYS** of dimension $p = \{0, 1, 2, \ldots\}$ and arraylength $\boldsymbol{N}$

General wavefield and source quantities (arrays, arraylength=N)		
	2d	1d
intensive field	$H_{m,k}$	E_k
flow of energy	$S_m = H_{m,k} E_k$	
extensive field	$B_{i,j}$	D_k
flow of momentum	$G_i = B_{i,j} D_j$	
volume source densities	$K_{i,j}$	J_k

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* DeHoop, Abubakar, Habashy, JASA, Vol. 126 (2009), pp.1095–1100.

Orthogonal decomposition 2d arrays:		
• $\sigma_{i,j} = [\sigma_{i,j}]^{-} + [\sigma_{i,j}]^{+} = [\sigma_{i,j}]^{-} + [\sigma_{i,j}]^{\delta} + [\sigma_{i,j}]^{\Delta}$		
constituent	symbol	definition
anti-symmetrical part	$[\sigma_{i,j}]^{-}$	$(\sigma_{i,j} - \sigma_{j,i})/2$
symmetrical part	$[\sigma_{i,j}]^{+}$	$(\sigma_{i,j} + \sigma_{j,i})/2$
omnidirectional part	$[\sigma_{i,j}]^{\delta}$	$(\sigma_{k,k}/3)\ \delta_{i,j}$
deviatoric part	$[\sigma_{i,j}]^{\Delta}$	$[\sigma_{i,j}]^{+} - [\sigma_{i,j}]^{\delta}$

Wave equations

- $\partial_{\boldsymbol{x}} \begin{bmatrix} \text{intensive} \\ \text{field} \end{bmatrix} + \partial_t \begin{bmatrix} \text{extensive} \\ \text{field} \end{bmatrix} = \begin{bmatrix} \text{volume source} \\ \text{densities} \end{bmatrix}$

Constitutive relations

- $\begin{bmatrix} \text{extensive} \\ \text{field} \end{bmatrix} = \underbrace{\begin{bmatrix} \text{constitutive} \\ \text{operator} \end{bmatrix}}\overset{(t)}{*} \begin{bmatrix} \text{intensive} \\ \text{field} \end{bmatrix}$

- local
- linear
- passive
- time invariant
- causal

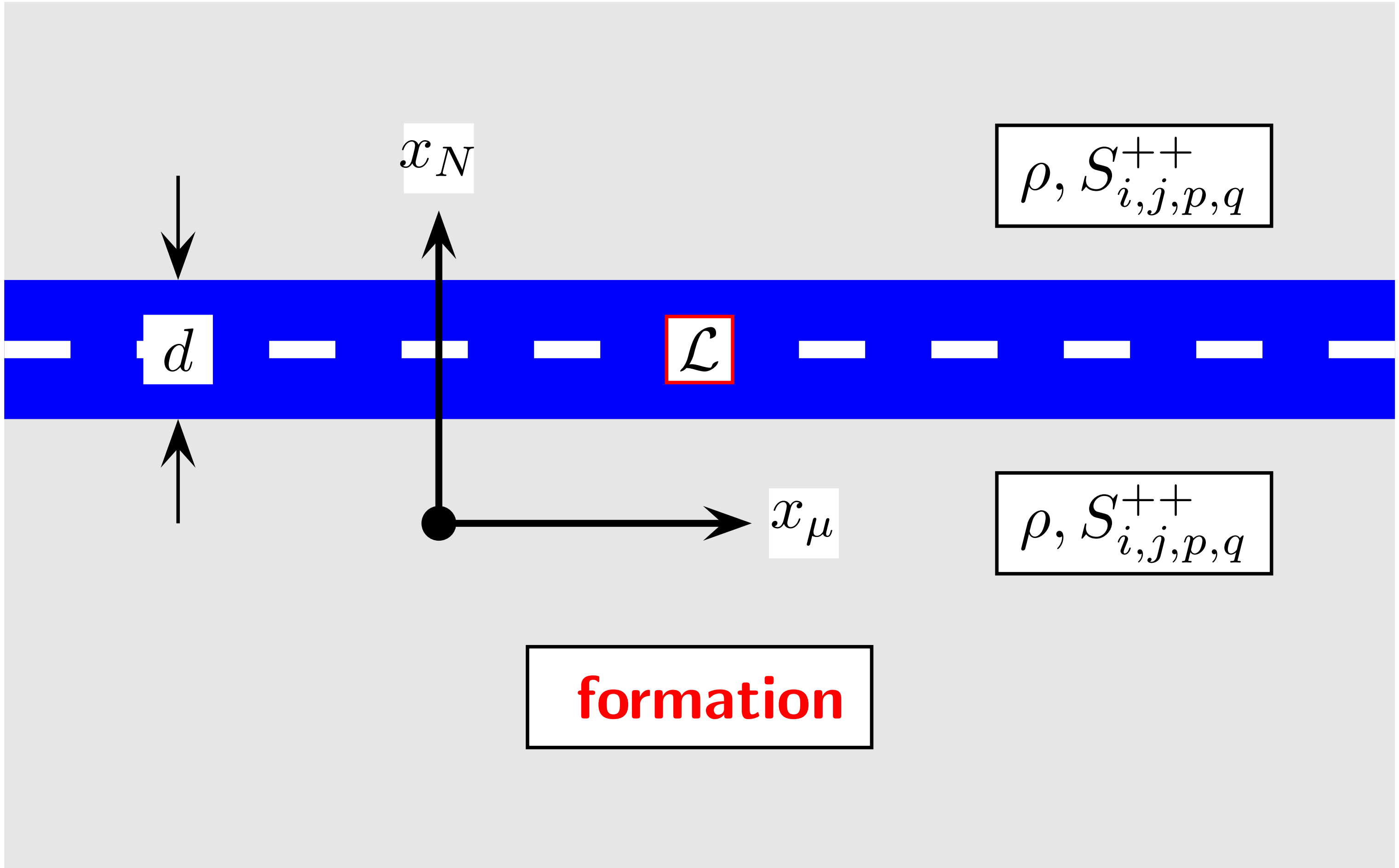
$\Downarrow$ $\Downarrow$

uniquely solvable initial-value problem (= time evolution in space)

Acoustic wavefield and source quantities (arrays, arraylength=3)		
	2d	1d
intensive field	$p\ \delta_{m,k}$	v_k
flow of energy	$S_m = p\ \delta_{m,k} v_k$	
extensive field	$\theta\ \delta_{i,j}$	Φ_k
flow of momentum	$G_i = \theta\ \delta_{i,j} \Phi_j$	
volume source densities	$q\ \delta_{i,j}$	f_k

Elastic wavefield and source quantities (arrays, arraylength=3)		
	2d	1d
intensive field	$-\tau^{+}_{m,k}$	v_k
flow of energy	$S_m = -\tau^{+}_{m,k} v_k$	
extensive field	$d^{+}_{i,j}$	Φ_k
flow of momentum	$G_i = d^{+}_{i,j} \Phi_j$	
volume source densities	$h^{+}_{i,j}$	f_k

EM wavefield and source quantities (arrays, arraylength=3)		
	2d (magnetic)	1d (electric)
intensive field	$H^{-}_{m,k}$	E_k
flow of energy	$S_m = H^{-}_{m,k} E_k$	
extensive field	$B^{-}_{i,j}$	D_k
flow of momentum	$G_i = B^{-}_{i,j} D_j$	
volume source densities	$K^{-}_{i,j}$	J_k



Configuration with high-contrast, thin (visco-)elastic layer $\mathcal{L}$

Elastodynamic quantities	
Wavefield quantities	
$\tau^{+}_{i,j} = \tau^{+}_{j,i}$	dynamic stress
v_k	particle velocity
$d^{+}_{i,j} = d^{+}_{j,i}$	dynamic strain
Φ_k	mass flow density
Source quantities	
f_k	volume source density of force
$h^{+}_{i,j} = h^{+}_{j,i}$	volume source density of deformation rate
Constitutive coefficients	
ρ	volume density of mass
$S^{++}_{i,j,p,q} = S^{++}_{i,j,q,p} = S^{++}_{j,i,q,p} = S^{++}_{j,i,p,q} = S^{++}_{p,q,i,j}$	elastic compliance (instantaneous reaction + relaxation)

Elastic wavefield and source quantities (1)

Elastic wavefield and source quantities (arrays, arraylength=3)		
	2d	1d
intensive field	$-\tau^{+}_{m,k} = -\tau^{+}_{k,m}$	v_k
flow of energy	$S_m = -\tau^{+}_{m,k} v_k$	
extensive field	$d^{+}_{i,j} = d^{+}_{j,i}$	Φ_k
flow of momentum	$G_i = d^{+}_{i,j} \Phi_j$	
volume source densities	$h^{+}_{i,j} = h^{+}_{j,i}$	f_k

Elastic wavefield and source quantities (2)

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Elastic wave equations

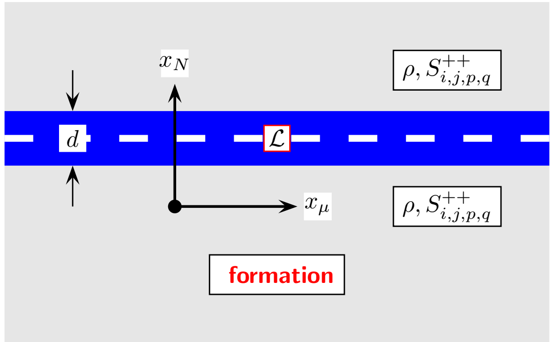
- $-\partial_m \tau^+_{m,k} + \rho\, \partial_t \Phi_k = f_k$
- $[\partial_i v_j]^+ - \partial_t d^+_{i,j} = h^+_{i,j}$

Elastodynamic constitutive relations

- $\Phi_k = \rho \; v_k$
- $d^+_{i,j} = S^{++}_{i,j,p,q} \overset{(t)}{*} \tau^+_{p,q}$ $\quad (\overset{(t)}{*} =$ time convolution$)$

Planar layer in embedding

- $\mathcal{L} = \{x_m \in \mathbb{R}^3; x_N \in \mathbb{R}, x_N^- < x_N < x_N^+ = x_N^- + d, x_\mu \in \mathbb{R}^2, \mu \neq N\}$
  (d=thickness, N = 'normal')



High-contrast, thin-layer assumptions

- $\rho\, d = O(1)$ as $d \downarrow 0$,
- $S_{i,j,p,q}\, d = O(1)$ as $d \downarrow 0$

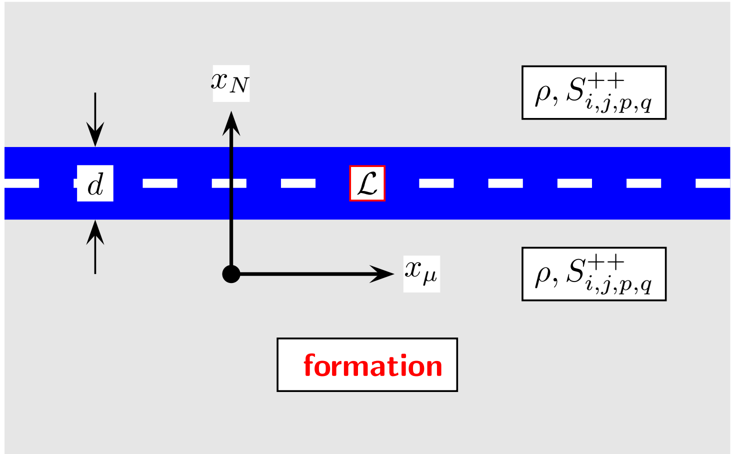
From wave propagation inside layer to saltus conditions across layer

PROCEDURE

- Integrate wave field equations + constitutive relations across $\mathcal{L}$
- Replace integrals with their trapezoidal-rule values
- Assign averaged values over $\mathcal{L}$ to centerplane $\mathcal{L}$
- Take limit $d \downarrow 0$ under the
- high-contrast, thin-layer assumptions:

$$\rho\, d = O(1) \text{ as } d \downarrow 0$$
$$S^{++}_{i,j,p,q}\, d = O(1) \text{ as } d \downarrow 0$$

(volume constitution $\longrightarrow$ area constitution)



Saltus conditions:

- $-\,[\tau^{+}_{N,k}]|^{+}_{-} + \rho\, d\, \partial_t v_k = 0$
- $[v_j]|^{+}_{-} - S_{N,p,q}\, d \overset{(t)}{*} \partial_t \tau^{+}_{p,q} = 0$

Elastic layer saltus conditions

OBJECTIVE

- Retrieve from scattered wave information about the constitution of the layer

Time-domain analytic model configurations (ready for start-up)	
configuration	tool
plane-wave reflection/transmission	Laplace transform
cylindrical-wave reflection/transmission	Cagniard-dH2D
spherical-wave reflection/transmission	Cagniard-dH3D
plane-wave scattering by half-layer (edge effect)	Wiener-Hopf + Cagniard-dH2D
viscous-fluid layer (κ=compressibility, ν=shear viscosity)	

- $S^{++}_{i,j,p,q} = (\kappa/3)\,\delta_{i,j}\delta_{p,q}\,\delta(t) + [(\delta_{i,p}\delta_{j,q} + \delta_{j,p}\delta_{i,q})/2 - \delta_{i,j}\delta_{p,q}/3]\,\nu\,H(t)$

TD analytic model configurations (ready for start-up)

THE END

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THANK YOU

FOR YOUR ATTENTION