TD radiation properties of array antennas composed of pulsed electric-current excited elements

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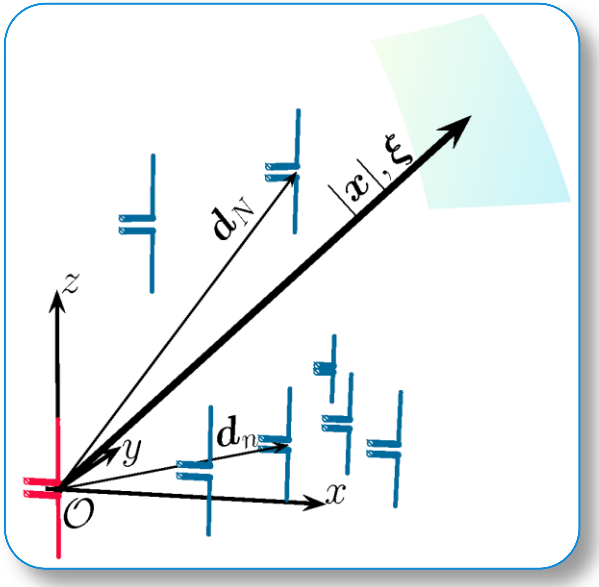
Investigated configuration



- **Elements:** $N+1$, identical, mutually translationally shifted, pulsed electric-current excited elements
- **Medium:** μ_0, ε_0, $c_0 = \sqrt{\mu_0 \varepsilon_0}$
- **Excitation:** $\boldsymbol{J}_n(\boldsymbol{x}, t) = I_n^{\mathrm{G}}(t) \overset{(t)}{*} \boldsymbol{J}_n^{\delta}(\boldsymbol{x}, t)$

Dirac delta pulse excitation

Electric current

$$\boldsymbol{J}_n^{\delta}(\boldsymbol{x}, t) = \boldsymbol{J}_0^{\delta}(\boldsymbol{x} + \boldsymbol{d}_n, t), \quad \text{for } n = 1, \ldots, N$$

Radiated field

- Radiated field ← electri

$$(\boldsymbol{\nabla}\cdot\boldsymbol{\nabla})\boldsymbol{A} - c_0^{-2}\partial_t^2\boldsymbol{A} =$$

- Scalar wave equation

$$(\boldsymbol{\nabla}\cdot\boldsymbol{\nabla})G - c_0^{-2}\partial_t^2 G = -\delta(\boldsymbol{x},t)$$

Green's function $G(\boldsymbol{x},t) = \dfrac{\delta(t-|\boldsymbol{x}|/c_0)}{4\pi|\boldsymbol{x}|}$ for $\boldsymbol{x}\neq\mathbf{0}$

$$\boldsymbol{A}(\boldsymbol{x},t) = \sum_{n=0}^{N}\boldsymbol{A}_n(\boldsymbol{x},t) = \sum_{n=0}^{N} G(\boldsymbol{x},t)\overset{(\boldsymbol{x})}{*}\overset{(t)}{*}\boldsymbol{J}_n(\boldsymbol{x},t)$$ for $\boldsymbol{x}\in\mathbb{R}^3$

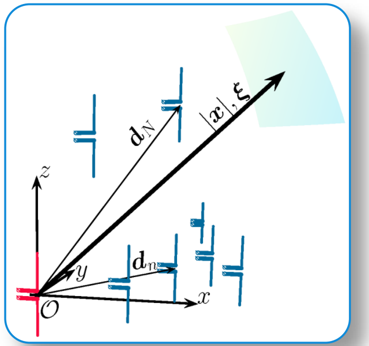
- From Maxwell's equations

$$\boldsymbol{E} = -\mu_0\partial_t\boldsymbol{A} + \epsilon_0^{-1}\mathsf{I}_t\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{A})$$

$$\boldsymbol{H} = \boldsymbol{\nabla}\times\boldsymbol{A}$$

where $\mathsf{I}_t = \displaystyle\int_{\tau=-\infty}^{t} f(\tau)\,\mathrm{d}\tau$

TD far-field radiation characteristics



- TD far-field expression

$$\{\boldsymbol{A},\boldsymbol{E},\boldsymbol{H}\}(\boldsymbol{x},t) = \frac{\{\boldsymbol{A}^{\infty},\boldsymbol{E}^{\infty},\boldsymbol{H}^{\infty}\}(\boldsymbol{\xi},t-|\boldsymbol{x}|c_0^{-1})}{4\pi|\boldsymbol{x}|}\left[1+O(|\boldsymbol{x}|^{-1})\right]$$

as $|\boldsymbol{x}| \to \infty$

- $\{\boldsymbol{A}^{\infty},\boldsymbol{E}^{\infty},\boldsymbol{H}^{\infty}\}(\boldsymbol{\xi},t)$ are interrelated by

$$\boldsymbol{E}^{\infty} = -\mu_0[\partial_t\boldsymbol{A}^{\infty} - \boldsymbol{\xi}(\boldsymbol{\xi}\cdot\partial_t\boldsymbol{A}^{\infty})]$$

$$\boldsymbol{H}^{\infty} = -c_0^{-1}\boldsymbol{\xi}\times\partial_t\boldsymbol{A}^{\infty}$$

Beam shaping and beam steering

- Excitation: time-shifted versions of the "0" pulse $I_n^{\mathrm{G}}(t) = I_0^{\mathrm{G}}(t - T_n)$, with T_n the relevant time delays

$$\boldsymbol{A}^{\infty} = \sum_{n=0}^{N} \boldsymbol{A}_n^{\infty}$$

$$= \sum_{n=0}^{N} \left\{ I_0^{\mathrm{G}}(t - T_n) \overset{(t)}{*} \int_{\mathcal{D}_n} \boldsymbol{J}_0^{\delta}[\boldsymbol{x}', t + c_0^{-1} \boldsymbol{\xi} \cdot (\boldsymbol{x}' + \boldsymbol{d}_n)] \mathrm{d}V(\boldsymbol{x}') \right\}$$

- Constructive interference occurs if

$$T_n = c_0^{-1} \boldsymbol{\xi}_{\mathrm{st}} \cdot \boldsymbol{d}_n, \text{ for } n = 1, 2, 3, \ldots, N$$

Area density of radiated energy

$$W^{\mathrm{rad}} = \int_{\boldsymbol{\xi}\cdot\boldsymbol{\xi}=1} \boldsymbol{\Phi}^{\mathrm{rad}}(\boldsymbol{\xi})\cdot\boldsymbol{\xi}\,\mathrm{d}\Omega$$

Poynting theorem

Area density of radiated energy

$$\boldsymbol{\Phi}^{\mathrm{rad}}(\boldsymbol{\xi}) = \frac{1}{16\pi^2}\int_{t\in\mathbb{R}} \boldsymbol{E}^{\infty}(\boldsymbol{\xi},t)\times\boldsymbol{H}^{\infty}(\boldsymbol{\xi},t)\mathrm{d}t$$

- For the examined case:

$$\boldsymbol{\Phi}^{\mathrm{rad}}(\boldsymbol{\xi}) = c_0^{-2}\boldsymbol{\xi}\int_{t\in\mathbb{R}} \left[\partial_t\boldsymbol{A}^{\infty}(\boldsymbol{\xi},t)\cdot\partial_t\boldsymbol{A}^{\infty}(\boldsymbol{\xi},t)\right]\mathrm{d}t$$

Numerical examples

- Examined quantity:

$$D_{\mathrm{dB}}(\boldsymbol{\xi}) = 10\log_{10}\left[\boldsymbol{\Phi}^{\mathrm{rad}}(\boldsymbol{\xi})\cdot\boldsymbol{\xi}/4\pi W^{\mathrm{rad}}\right]$$

- Electric current pulse $I_0^{\mathrm{G}}(t)$: normalised power-exponential pulse

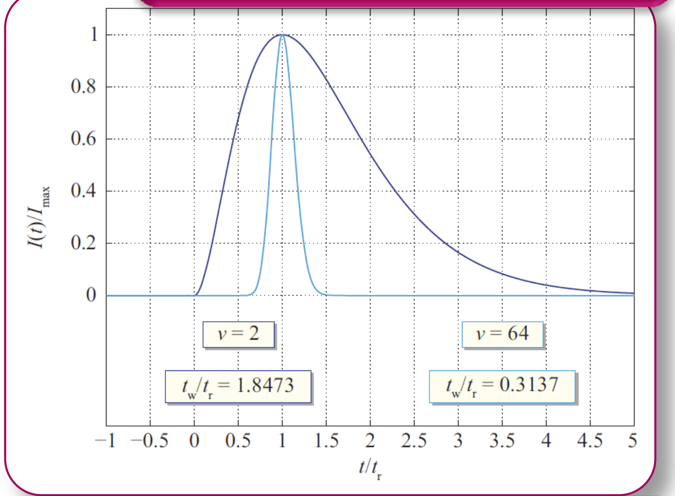
$$I(t) = I_{\max}\,(t/t_{\mathrm{r}})^{\nu}\exp\left[-\nu\,(t/t_{\mathrm{r}}-1)\right]H(t)$$

- Radiator: rhombic antenna

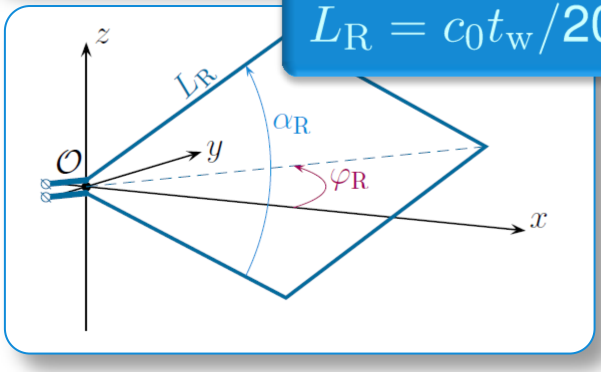
$$\boldsymbol{\Phi}^{\mathrm{rad}}(\boldsymbol{\xi}) = c_0^{-2}\boldsymbol{\xi}\int_{t\in\mathbb{R}}\left[\partial_t\boldsymbol{A}^{\infty}(\boldsymbol{\xi},t)\cdot\partial_t\boldsymbol{A}^{\infty}(\boldsymbol{\xi},t)\right]\mathrm{d}t$$

analytic!

$$t_{\mathrm{w}} = t_{\mathrm{r}}\frac{\Gamma(\nu+1)\exp(\nu)}{\nu^{\nu+1}}$$

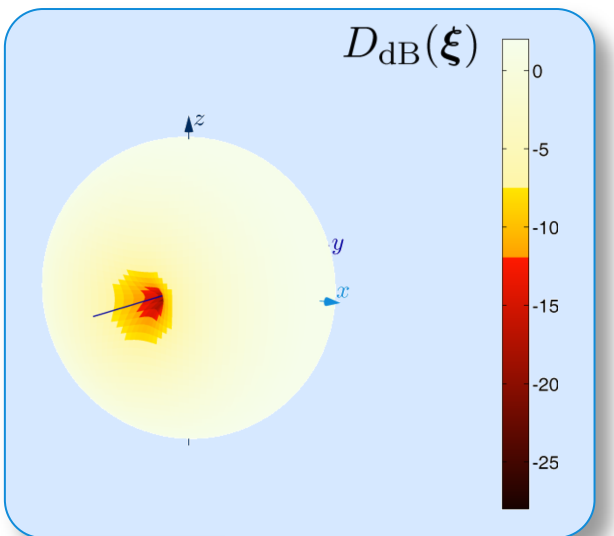


$$L_{\mathrm{R}} = c_0 t_{\mathrm{w}}/20$$

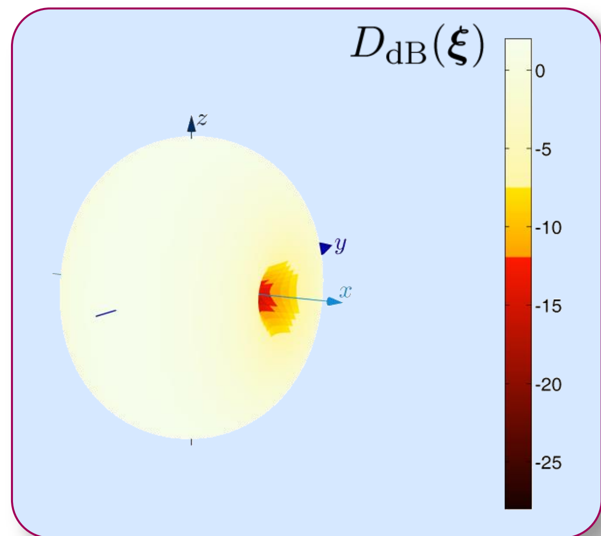


Numerical examples: single radiator

- Shape: ‘doughnut’ – characteristic for dipoles



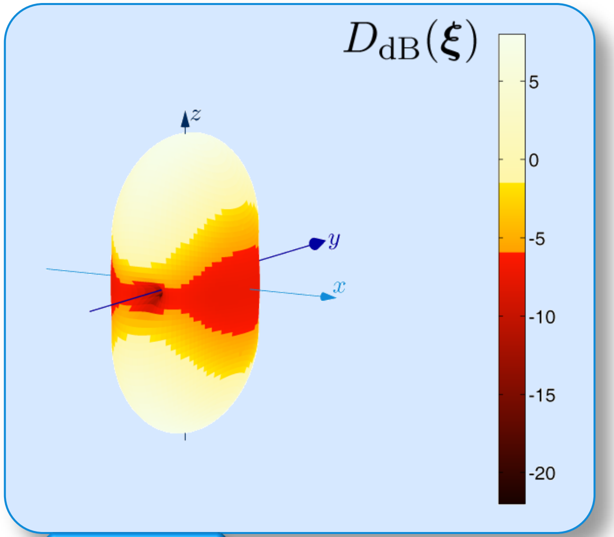
$\varphi_{\mathrm{R}} = 0°$



$\varphi_{\mathrm{R}} = 90°$

Numerical examples: ×8 linear array along $\mathcal{O}x$, broadside beam scan

- **Spacing:** $40\sqrt{2}L_{\mathrm{R}}$
- **Observations:** beam narrowing, no side-lobes



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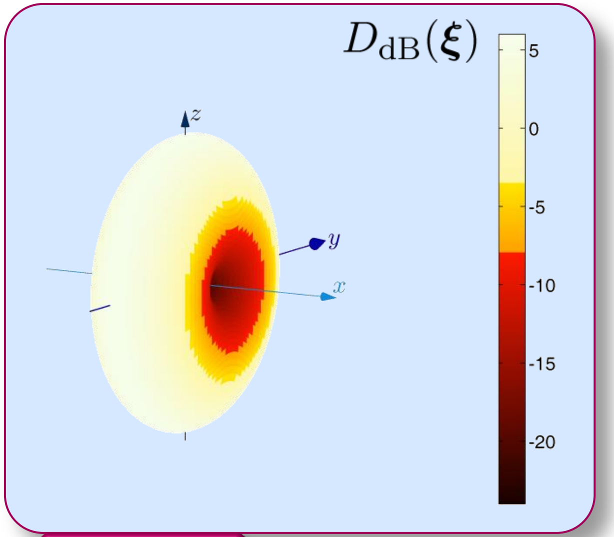
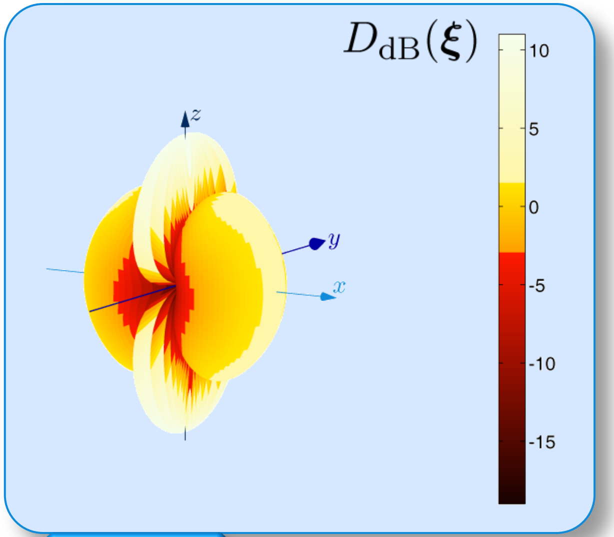


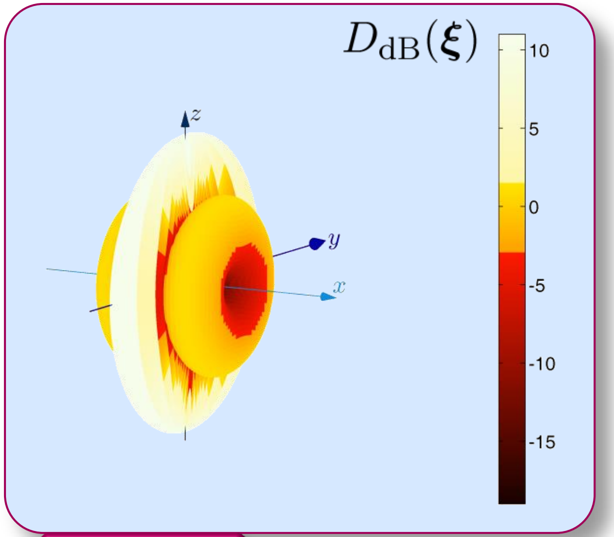
$\varphi_{\mathrm{R}} = 90°$

Numerical examples: ×8 linear array along $\mathcal{O}x$, broadside beam scan

- **Spacing:** $40\sqrt{2}L_{\mathrm{R}} = \times 10\left(4\sqrt{2}L_{\mathrm{R}}\right)$
- **Observations:** further beam narrowing, smooth side-lobes, no grating lobes



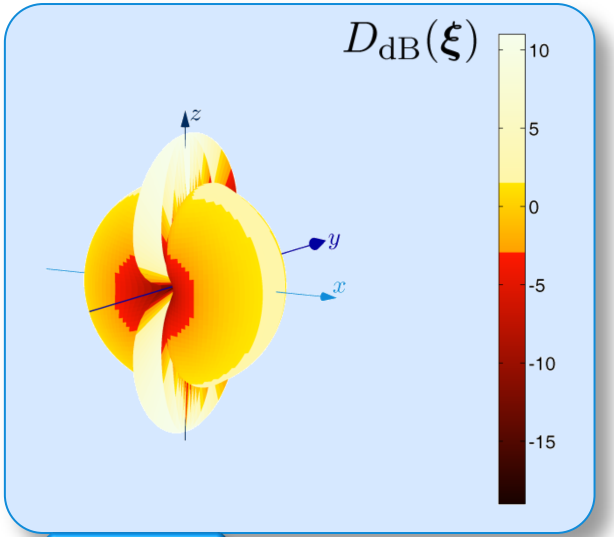
$\varphi_{\mathrm{R}} = 0°$



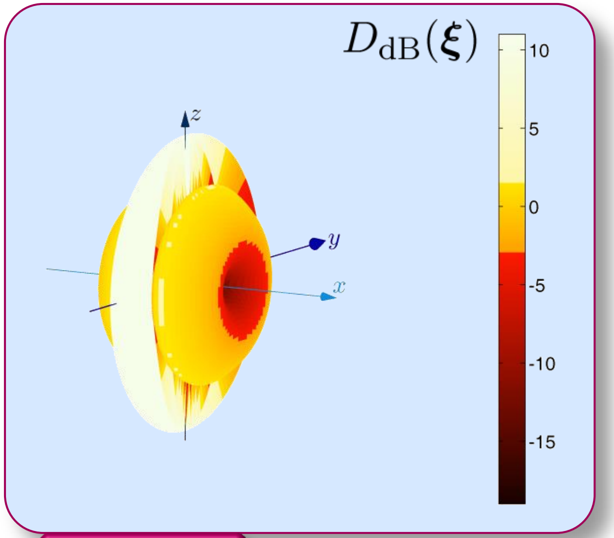
$\varphi_{\mathrm{R}} = 90°$

Numerical examples: ×8 linear array along $\mathcal{O}x$, broadside beam scan

- **Spacing:** $80\sqrt{2}L_{\mathrm{R}} = \times 20\left(4\sqrt{2}L_{\mathrm{R}}\right)$
- **Observations:** further beam narrowing, smooth side-lobes, still no grating lobes!



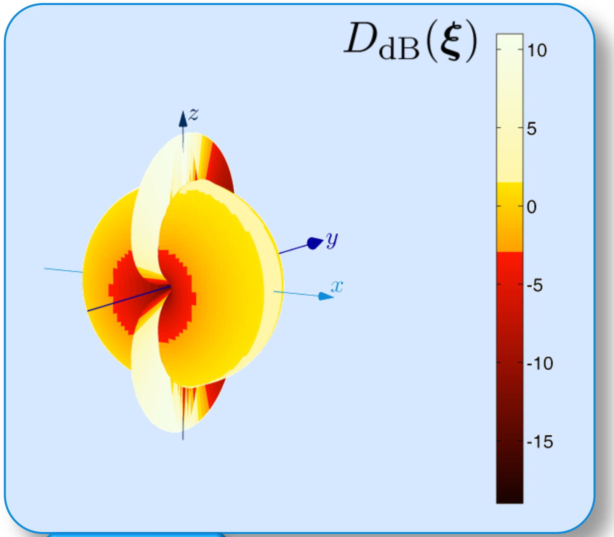
$\varphi_{\mathrm{R}} = 0^\circ$



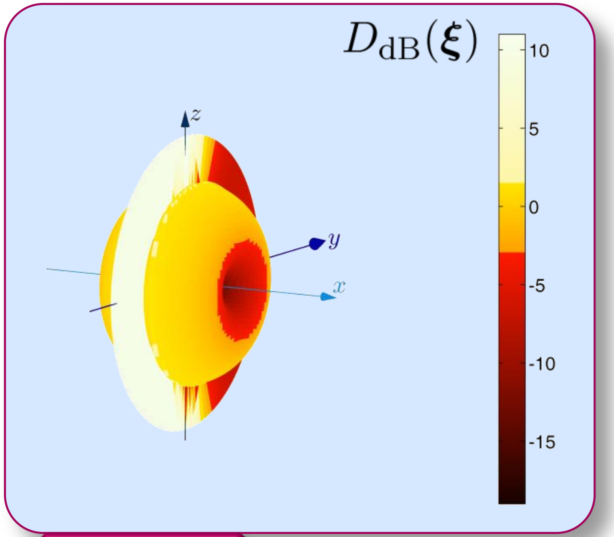
$\varphi_{\mathrm{R}} = 90^\circ$

Numerical examples: ×8 linear array along $\mathcal{O}x$, broadside beam scan

- Spacing: $160\sqrt{2}L_{\mathrm{R}} = \times 40\left(4\sqrt{2}L_{\mathrm{R}}\right)$
- Observations: further beam narrowing, smooth side-lobes, and still no grating lobes!!!



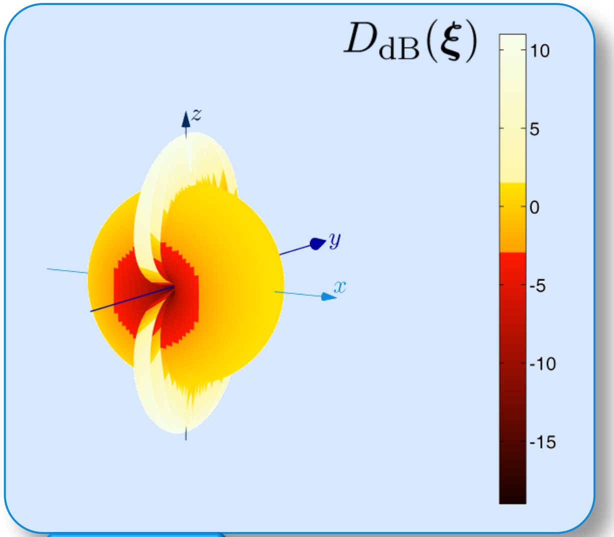
$\varphi_{\mathrm{R}} = 0°$



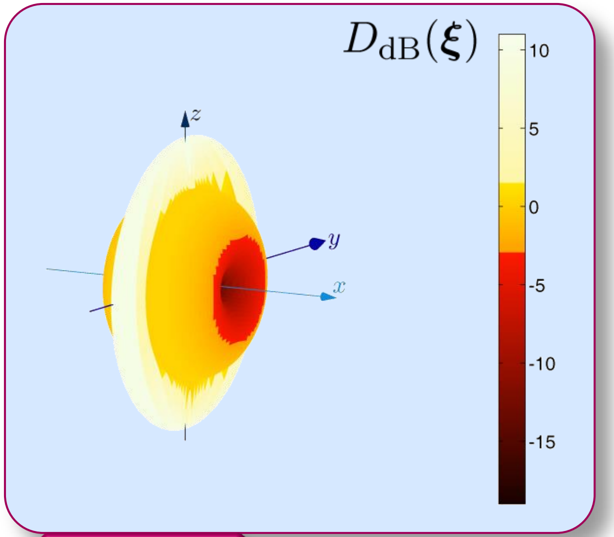
$\varphi_{\mathrm{R}} = 90°$

Numerical examples: ×8 linear array along $\mathcal{O}x$, broadside beam scan

- **Placement:** $\{57, 8, 1\}$–CDS, $L_{\mathrm{array}} = \times 10\left(4\sqrt{2}L_{\mathrm{R}}\right)$
- **Observations:** similar radiation pattern



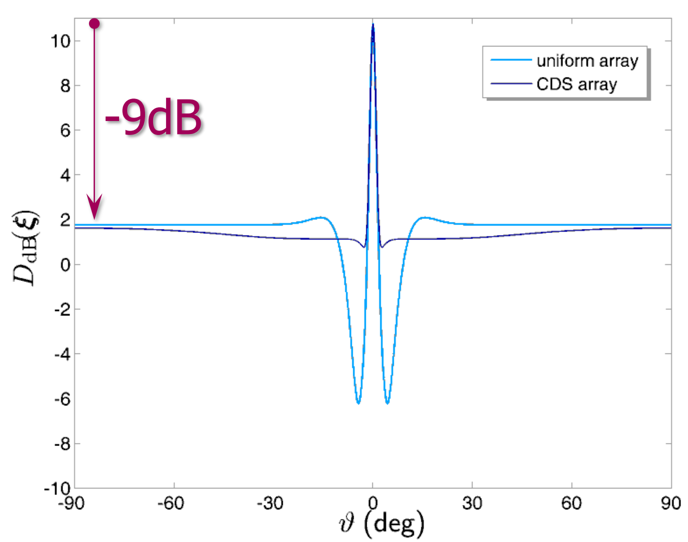
$\varphi_{\mathrm{R}} = 0°$



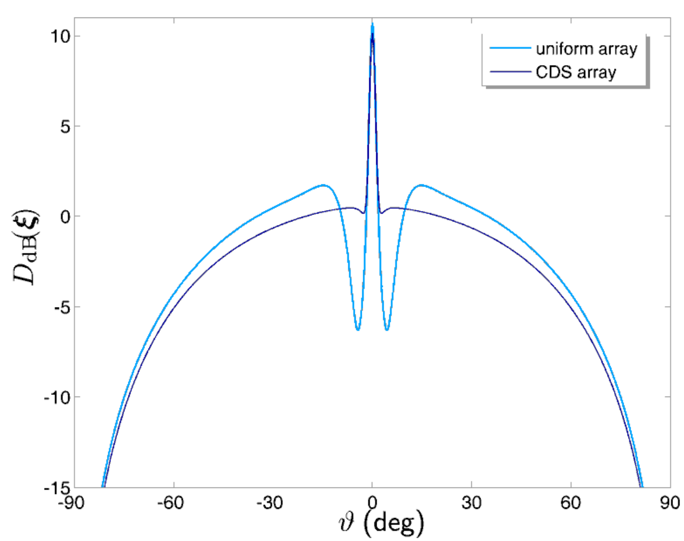
$\varphi_{\mathrm{R}} = 90°$

Numerical examples: ×8 linear array along $\mathcal{O}x$, broadside beam scan

- **Placement:** $\{57, 8, 1\}$–CDS, $L_{\text{array}} = \times 10\left(4\sqrt{2}L_{\text{R}}\right)$
- **Observations:** much smoother side-lobes



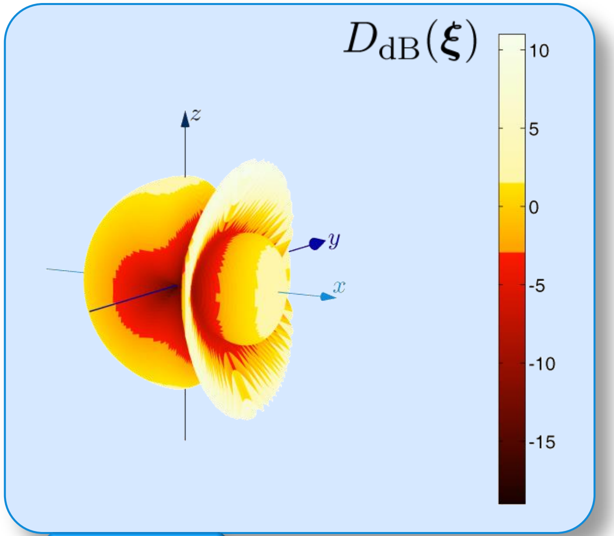
$\varphi_{\text{R}} = 0°$



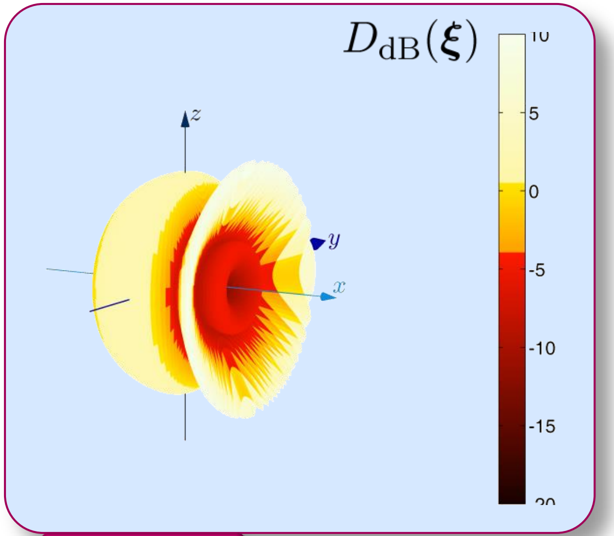
$\varphi_{\text{R}} = 90°$

Numerical examples: ×8 linear array along $\mathcal{O}x$, 60° beam scan

- **Spacing:** $40\sqrt{2}L_\mathrm{R} = \times 10 \left(4\sqrt{2}L_\mathrm{R}\right)$
- **Observations:** accurate beam steering, smooth side-lobes, no grating lobes



$\varphi_\mathrm{R} = 0°$



$\varphi_\mathrm{R} = 90°$

Conclusions

- A complete TD, far-field radiation formalism
- A meaningful TD "radiation pattern" quantity

$$\boldsymbol{\Phi}^{\mathrm{rad}}(\boldsymbol{\xi}) \cdot \boldsymbol{\xi} / 4\pi W^{\mathrm{rad}}$$

- Illustrative numerical experiments:
  - beam narrowing (like FD arrays)
  - beam steering (like FD arrays)
  - smooth side-lobes region (unlike FD arrays)
  - **no grating lobes** (unlike FD arrays)
- Future work: monocycle excitation, fidelity analysis

Thank you